

2.14/2.140 Lab 5

Assigned: Week of Mar. 31, 2014

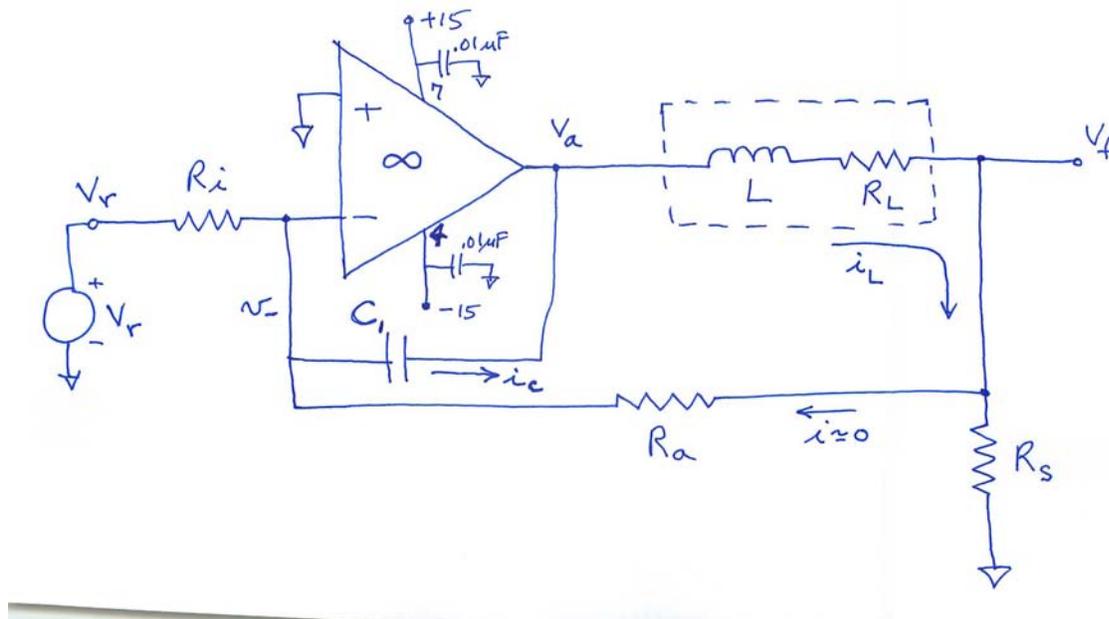
Due: Week of Apr. 7, in your lab session.

Lab 5 concerns the op-amp circuit shown below. This circuit implements a *current-control* loop, which regulates the current i_L passing through the inductor. Such current control circuits are widely used, *e.g.*, to drive actuators such as motors in order to control torque/force. The focus of this lab is on the feedback dynamics of the current control circuit.

Unlike Lab 4, here we will assume that the op amp has *infinite* gain. The reason for this assumption is that the feedback capacitor C_1 sets the op amp dynamics such that the internal $a(s)$ model is not needed to accurately model the loop operation relative to the current control dynamics. Also, since the capacitor looks like a short-circuit at high frequencies, the op-amp feedback has unity gain and zero phase shift at high frequencies, and thus the internal loop will be stable. Thus, the internal amplifier dynamics are stable, and much faster than the current control dynamics, and so we can ignore them for our present purposes.

Real inductors have internal resistance, due to the non-zero resistance of the wire used to form the coil. In this circuit, the inductor resistance is represented by R_L . The inductor current i_L passes through the sense resistor R_S , which gives a voltage $V_f = i_L R_S$. We set $R_a \gg R_s$, and thus we can assume that all of i_L passes through R_S , and thus that $i_a \approx 0$. This lets us ignore the loading from R_a when computing V_f .

You will use the NI myDAQ kit and protoboard to build and test this circuit, and compare your measured results against theory.



For this circuit, we ask you to analyze the loop dynamics as they depend upon the choice of component values. Specifically:

1. Derive and draw a block diagram representation for the circuit. As part of this block diagram, include the signals V_r , i_i , i_a , i_C , V_a , V_f , and the inductor current estimate $\hat{i}_L = V_f/R_S$.
2. Use your block diagram to calculate the closed-loop transfer function $V_f(s)/V_r(s)$. This transfer function should be in terms of the component values as variables. Do not substitute in any numbers at this point. What is an expression for the closed-loop DC gain?
3. Solve for the closed-loop ω_n and ζ of the poles of V_f/V_r in terms of the parameter values.
4. For the remainder of this problem, let $R_i = 20R_a$, $C_1 = 1$ nF, $L = 47$ mH, $R_L = 100$ Ω , and $R_S = 100$ Ω . Show that these values result in a closed-loop DC current gain of $i_L = -0.5$ mA/V $\times V_r$.
5. The only component left unspecified so far is R_a . Compute the three values of R_a which give closed-loop natural frequencies of 10^3 , 10^4 , and 10^5 rad/sec, respectively. For each of these three values, create Bode plots of the loop return ratio. What are the crossover frequency ω_c and phase margin ϕ_m for each of the three values? How do these compare with the closed-loop ω_n and ζ ? (Recall the approximations $\omega_n \approx \omega_c$ and $\zeta \approx \phi_m/100$.)
6. For each of these three values, use Matlab to compute and plot the closed-loop step response of V_f to a 2 volt step in V_r . Also, for each of these three values, compute and plot the closed-loop Bode plot from input V_r to output V_f .

The experimental part of this lab uses the values for $\omega_n = 10^4$, as computed above. For these values, build this circuit on the protoboard of your myDAQ kit. Be sure to connect power to the op amp and use the indicated bypass capacitors. For this circuit, use the 47 mH inductor we have provided, and use a 741 op amp. Measure the coil resistance R_L of the inductor; this should be close to the value given above. Recompute your predicted step and frequency responses for the measured component value. (Extra credit: Design an experiment to measure the inductance value L , and use this measured value in your calculations below.)

1. Use the myDAQ with the scope and signal generator VIs in Labview to measure and plot the closed-loop step response of V_f to a 2 volt step in V_r . Also record the control effort V_a . Be careful that the response is in the linear regime, *i.e.* no saturation in the control effort V_a . If necessary, reduce the amplitude of the input square wave such that the response is in a linear regime.
For this linear response, what are the measured closed-loop 10-90% rise time, percentage peak overshoot, natural frequency ω_n , and damping ratio ζ ? Compare with the Matlab result from above.
2. Use the Bode Analyzer VI in Labview to measure and plot the closed-loop frequency response from input V_r to output V_f . Be sure to use enough frequency points to see any resonance clearly. Be careful that the response is in the linear regime, *i.e.* no saturation in the control effort V_a . If necessary, reduce the amplitude of the input sine wave such that the response is in a linear regime.

From the frequency response, estimate the closed-loop natural frequency ω_n , and damping ratio ζ . Compare with the Matlab result from above.

To learn how to use the Bode analyzer, you might want to first measure a simple circuit such as an RC low-pass filter, which can be easily compared with theory. You also could use the Bode analyzer to measure the plant frequency response V_f/V_a as a way to experimentally measure the inductor resistance and inductance, assuming a known value of $R_S = 100 \Omega$.

Checkoff in lab session: In this week's lab session demonstrate to one of the teaching staff your working circuit and some experimental results. Progress shown in this checkoff will count towards half of your lab grade.

Answer sheets: At the start of your lab session during the week of April 7 you must submit your lab writeup, including answers to all the questions in the lab assignment. These write-ups are due at the start of the lab session, and will not be accepted late. Please include in your lab report clearly labeled answers and plots for all the questions above. This lab report will count towards half of your lab grade.

It is key that you submit your lab report on time at the start of the lab session.

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2.14 / 2.140 Analysis and Design of Feedback Control Systems
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