

2.14/2.140 Problem Set 7

Assigned: Wed. Mar. 19, 2014

Due: Wed. April 2, 2014, in class

Reading: Lecture notes on Operational Amplifiers; section from Chapter 2 of *Operational Amplifiers* which describes static nonlinearities in feedback loops.

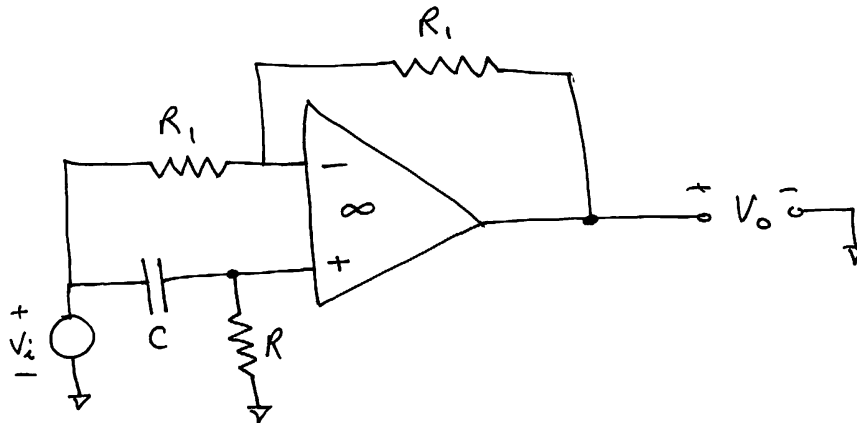
The full text *Operational Amplifiers* can be found online at:

<http://ocw.mit.edu/resources/res-6-010-electronic-feedback-systems-spring-2013/>.

Video lectures by Prof. Roberge can be found at this same location. In particular, for this problem set, video lecture 2 may be helpful to understand the effect of feedback on nonlinearities. There is also a video course manual with blackboards for all the lectures along with solutions to selected problems from the text.

The following problems are assigned to both 2.14 and 2.140 students.

Problem 1 The circuit shown below implements a first-order *all-pass* filter. Note that we assume an infinite gain model for the op-amp.



The term *all-pass* is used because the circuit response magnitude is unity for all frequencies.

a) Show that the circuit transfer function is

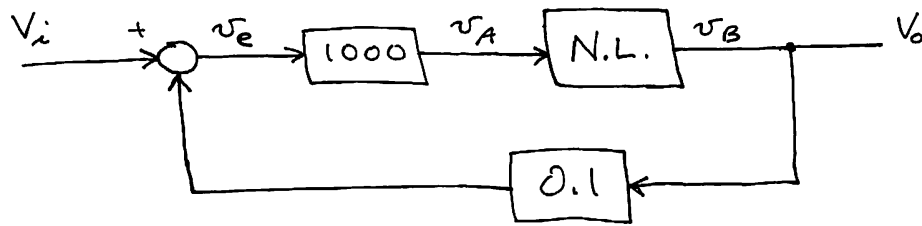
$$\frac{V_o}{V_i}(s) = \frac{RCs - 1}{RCs + 1}.$$

Note the non-minimum phase (RHP) zero.

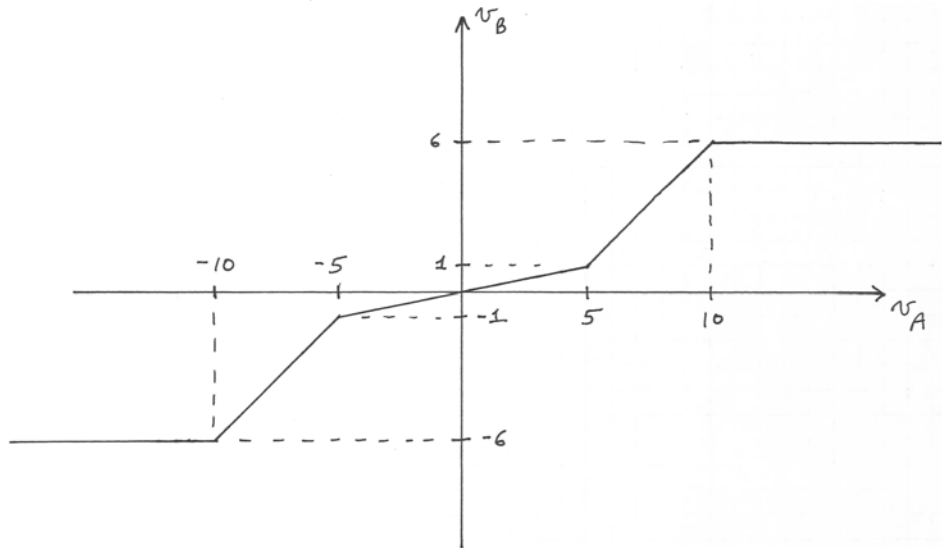
b) Make a hand sketch of the Bode plot for this filter. You should see that the phase varies from $+180^\circ$ at DC to 0° at high frequencies.

c) This circuit can be used as an adjustable phase shifter where R is a variable resistor used to set the phase. Let $C = 0.1\mu\text{F}$. What value of R will set the phase equal to $+90^\circ$ at a frequency of 1000 rad/sec?

Problem 2 This problem considers a feedback loop with a static nonlinearity as shown below



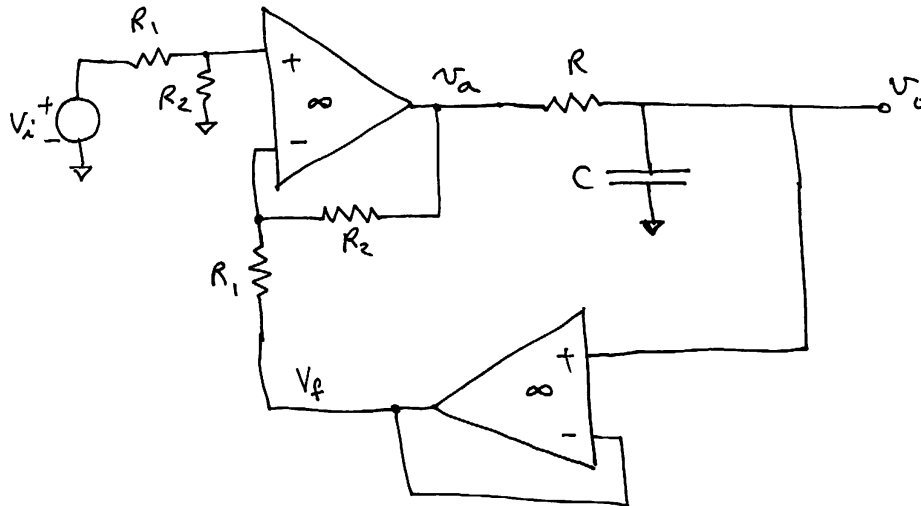
The nonlinear element has an input/output characteristic given by



The example from Chapter 2 of the Roberge textbook will be helpful in understanding this problem.

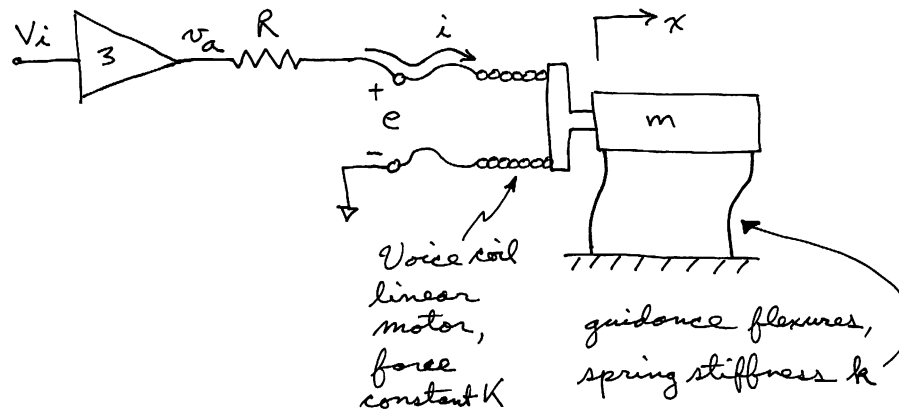
- a) Assume that the input V_i takes the form of a unit ramp $V_i(t) = t$. Calculate and sketch the resulting waveforms $v_A(t)$ and $v_B(t)$. These will be piecewise linear waveforms. Be sure to clearly label the relevant breakpoints. Also label the slope of each line segment.

Problem 3 In the loop shown below, the differential amplifier drives an RC low-pass filter. The output of the filter is fed back through a unity-gain buffer. Note that we assume an infinite gain model for both of the op amps in this loop.



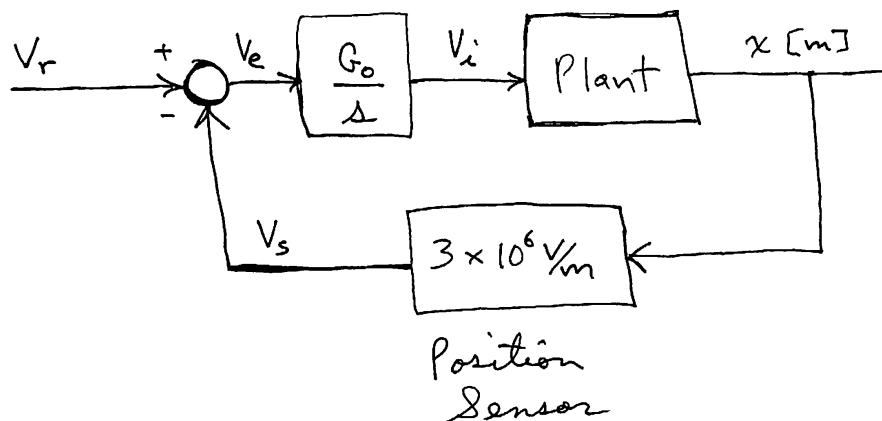
- Draw a block diagram which includes V_i , v_a , v_o , and V_f , and where the blocks have expressions in terms of the system parameters. Solve for the transfer functions $v_o(s)/V_i(s)$ and $v_a(s)/V_i(s)$.
- Let $R_1 = 1\text{k}\Omega$, $R_2 = 100\text{k}\Omega$, $R = 10\text{k}\Omega$, and $C = 1 \mu\text{F}$. Also let the input be a unit step $V_i = u_s(t)$. Calculate analytically and plot the closed-loop step responses $v_a(t)$ and $v_o(t)$.

Problem 4 The system shown below is a micropositioner, in which a stage guided by flexural legs moves in the x direction as driven by the force from a voice coil actuator.



The combined moving mass of the stage and voice coil is m . The flexural legs have a spring stiffness k . The actuator is assumed ideal, with a force constant K , and thus the actuator force is $F = Ki$, and $e = K\dot{x}$. The actuator is driven by a power amplifier with a gain of 3, through a series resistor R .

- Draw a block diagram which includes the signals V_i , v_a , e , i , x , and \dot{x} .
- Use this block diagram to solve for the transfer function $X(s)/V_i(s)$ in terms of the system variables.
- Assume the system parameters take the following values: $K = 10 \text{ N/A}$, $R = 10 \Omega$, $k = 10^7 \text{ N/m}$, and $m = 0.1 \text{ kg}$. Calculate the numerical values of the transfer function $X(s)/V_i(s)$.
- Let the input V_i take a 1 Volt step, i.e. $V_i(t) = 1u_s(t) \text{ V}$. Use Matlab to calculate and plot the output step response. What are the system natural frequency and damping ratio?
- Calculate and plot the system Bode plot for $X(s)/V_i(s)$.
- We now design a closed-loop controller for this system of the form



Here, position is measured with a capacitive position sensor, with a gain of $3 \text{ V}/\mu\text{m}$. Choose the value of G_o which allows a gain margin of 2. What is the corresponding crossover frequency and phase margin? Construct a Nyquist plot which shows these values. For this controller, also use Matlab to make a plot of the sensitivity $S = (1 - L.T.)^{-1}$.

- g) Because the resonance is so lightly damped, the system closed-loop bandwidth determined above is quite limited. However, it is possible to *phase-stabilize* this loop by adding an additional high-frequency pole. That is, we do not require that the loop transmission remain below unity magnitude in the vicinity of the resonance if the loop phase is such that the corresponding loop in the Nyquist plot does not encircle the -1 point. As a way to understand this, assume we add a compensator pole to the loop. That is, let the compensator become $G_o/s(\tau s + 1)$. For the present analysis, let $\tau = 5/\omega_n$, where ω_n is the plant natural frequency. What is the largest value of G_0 for which the sensitivity curve does not exceed a peak value of 10 dB? (You can use Matlab to help find this value.) Make a Bode plot of the negative of the loop transmission $L(j\omega)$, the sensitivity $S(j\omega)$, and also make a Nyquist plot showing how the -1 point is avoided. How many crossings of unity gain are there in the loop transmission? Compare the accuracy of command following for this phase-stabilized loop with the pure integral compensated loop from part f). (Hint: the sensitivity curve tells this story.)

The following problems are assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1 The OP77 data sheet is available separately from the course web page. In particular, the device transfer function $a(s)$ is shown in Bode plot form in Figure 10 on page 8 of the data sheet. For this problem, assume we connect the OP77 as a unity gain buffer.

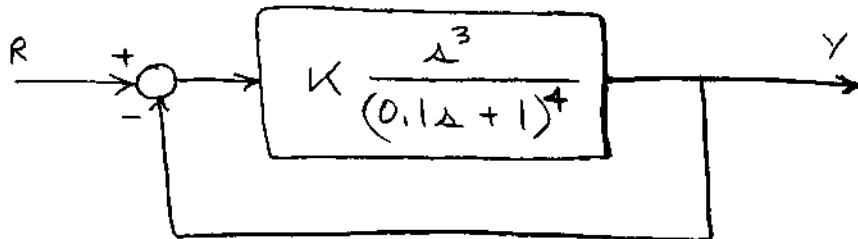
- a) Consider a model of the form

$$a(s) = \frac{G_0}{(\tau s + 1)} e^{-sT}$$

where the e^{-sT} term is a time delay used to model the excess high-frequency phase. What values of G_0 , τ , and T allow the best fit to the data sheet Bode plot?

- b) For the unity-gain buffer configuration, using this model, what are the loop crossover and phase margin?
- c) Use Matlab to simulate the closed-loop step response for this configuration. How is this time response consistent with the loop crossover and phase margin?

Problem G2 This problem considers the AC-coupled feedback loop shown below.



The loop is referred to as AC-coupled since it has zero gain at DC, due to the zeros at $s = 0$.

- a) Sketch a root locus for $K > 0$. Be sure to indicate key values on the locus, and accurately show departure and arrival angles.
- b) For the remainder of the problem assume that $K = 1$. Make a carefully-dimensioned sketch of the Bode plot of the return ratio.
- c) Conduct a Nyquist analysis which indicates whether the loop is stable for the value of $K = 1$. How does this analysis correlate with your root locus from part a)?

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2.14 / 2.140 Analysis and Design of Feedback Control Systems
Spring 2014

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