

2.14/2.140 Problem Set 3

Assigned: Wed. Feb. 19, 2014

Due: Wed. Feb. 26, 2014, in class

Reading: FPE Chapter 5; Lecture notes on PID control and root locus.

For the root locus problems from FPE, attempt to solve them by hand with the root locus sketching rules. Only use Matlab where requested, or if you can't see an analytical solution. It's very important that you learn how to sketch moderate order root loci with pencil and paper.

The following problems are assigned to both 2.14 and 2.140 students.

Problem 1 FPE 5.2

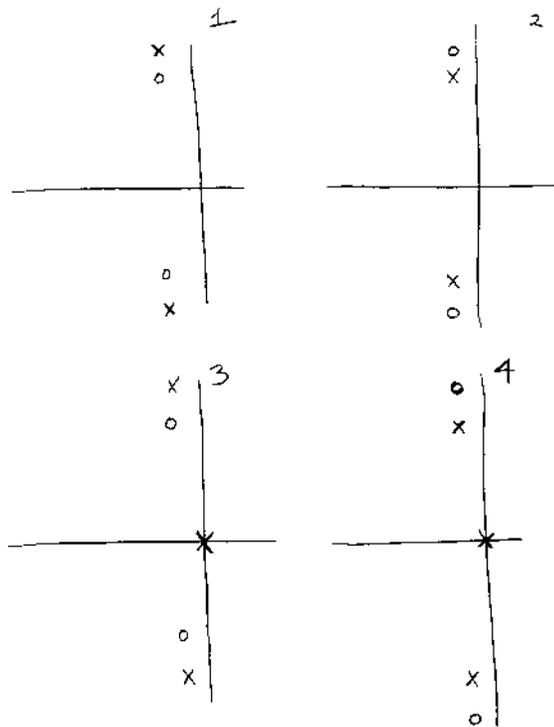
Problem 2 FPE 5.3

Problem 3 FPE 5.13

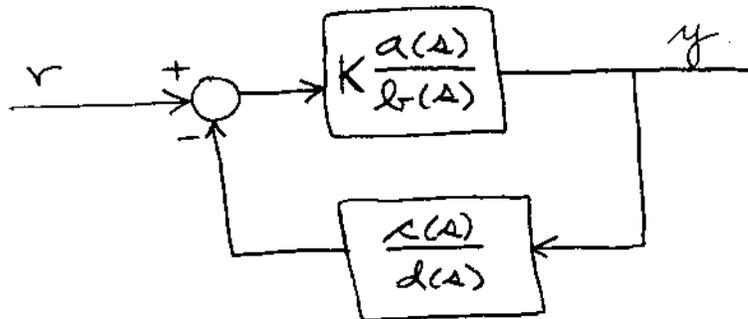
Problem 4 FPE 5.14

Problem 5 FPE 5.22. Use Matlab computational tools to help with all steps in this problem.

Problem 6 The four plots below show the pole and zero locations of the return ratio of a feedback system, informally called the 'open-loop' poles and zeros. Each of these loops also have a variable gain $K > 0$, which is used to move the poles along the root locus branches. For the four systems shown below, sketch the approximate shape of the root locus plot for $K > 0$. Note that you will need to pay particular attention to the angle criteria in the vicinity of the complex poles and zeros. If the complex pairs are lightly damped, which of these systems presents a danger of instability as the loop gain is varied? This analysis has practical relevance for the situation where a notch filter is used to help stabilize a system with a lightly-damped pair of poles.



Problem 7 The block diagram for a feedback loop has a forward path transfer function $G(s) = Ka(s)/b(s)$, and a feedback path transfer function $H(s) = c(s)/d(s)$ as shown below. Prove that the closed-loop zeros are located at: 1) the zeros of the forward path and 2) the poles of the feedback path, independent of the loop gain K .



Problem 8 FPE 4.23

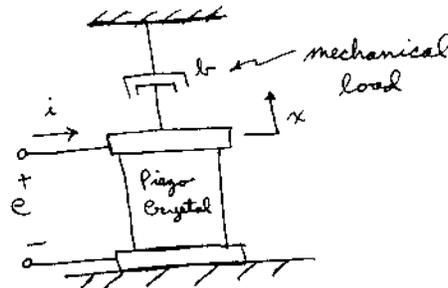
The following problems are assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1 FPE 5.35

Problem G2

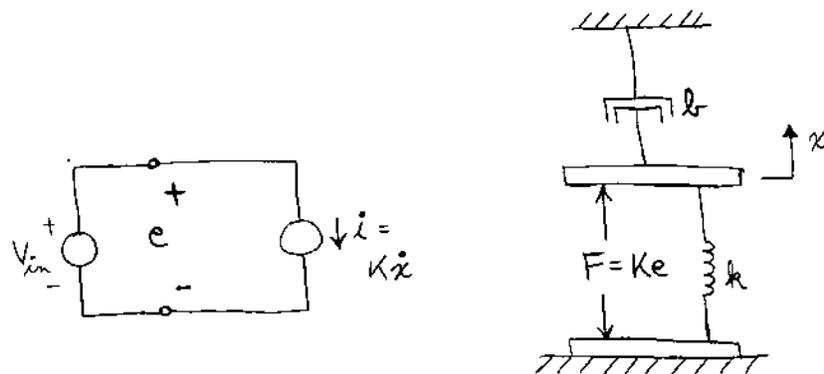
This problem was used as the ME quals systems written exam in 2005. It considers modeling and control issues associated with positioning systems driven with a *piezoelectric* actuator. A piezoelectric positioner is driven with an electrical input in order to produce a mechanical output and vice versa.

The figure below shows a model of a system incorporating a piezoelectric device.



Here the electrical terminals of the device are defined as having an input voltage $e(t)$ and an input current $i(t)$. The piezo crystal is sandwiched between end plates, with the bottom plate connected to mechanical ground. The motion of the upper plate in the vertical direction is defined as $x(t)$. A mechanical load consisting of a damper with value b is connected between the upper plate and mechanical ground. We model the piezoelectric device and plates as massless. Motion is considered to be constrained to the x direction.

The electrical/mechanical coupling of the device is modeled as shown below



Here, the piezoelectric actuator is modeled internally as a dependent force source $F(t)$ in parallel with a spring k . The force depends linearly upon the input voltage as $F(t) = Ke(t)$, where K is a scale factor with units of N/V . The spring k models the internal stiffness of the piezoelectric actuator. The force F is applied to the massless upper plate which connects the damper and spring.

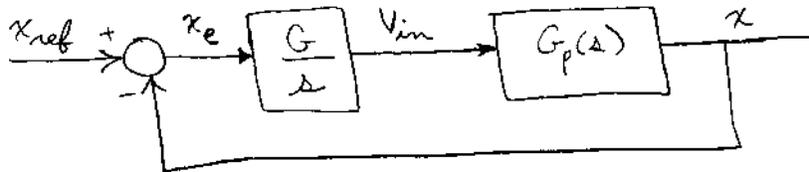
The electrical portion of the model is shown on the left in the figure. Here, a dependent current source has a value $i(t) = K\dot{x}(t)$. The system is driven with a voltage source $V_{in}(t)$.

- Calculate the transfer function $X(s)/V_{in}(s)$. Clearly show the steps in your development.
- Assume initial rest conditions. Let the input voltage be a unit step: $V_{in}(t) = u(t)$. Calculate a closed-form solution for the resulting displacement $x(t)$ and make a graph of $x(t)$ versus time.
- Develop a closed-form expression for the input electrical power $P(t) = e(t)i(t)$ associated with the transient you solved for in part b) above, and make a graph of $P(t)$ versus time.

We now learn that the top plate has finite mass, and so the model developed earlier needs to be augmented. Measurements indicate that the input/output transfer function is now given by

$$G_p(s) \equiv \frac{X(s)}{V_{in}(s)} = \frac{1}{10^{-2}s^2 + 10s + 10^6}$$

This experimentally-adjusted model is to be used to design the feedback loop shown below.



The controller for this loop is an integral controller $G_c(s) \equiv G/s$, where G is an adjustable gain associated with the integrator.

- Sketch a root locus plot for this control loop as the gain $G > 0$ is varied. For what range of gains G is the loop stable?
- The loop transfer function (sometimes called the return ratio) for this loop is given by $L(s) = G_c(s)G_p(s)$. Make a careful hand sketch of the Bode plot for $L(s)$, showing the effect of G as a parameter, and using the numerical representation of $G_p(s)$ given above.
- What value of G will result in a loop crossover frequency $\omega_c = 100$ rad/sec? (Recall that the crossover frequency is the frequency for which the loop transfer function magnitude crosses through unity, that is, we require $|L(j100)| = 1$.)

What are the phase margin and gain margin for the loop with this crossover frequency? Indicate these parameters on a Bode plot for the loop.

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2.14 / 2.140 Analysis and Design of Feedback Control Systems
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