HEAT TRANSFER AND THE SECOND LAW

Thus far we've used the first law of thermodynamics:

Energy is conserved.

Where does the second law come in?

One way is when heat flows.

Heat flows in response to a temperature gradient.

If two points are in thermal contact

and at different temperatures, T_1 and T_2

then energy is transferred between the two in the form of heat, Q.

The rate of heat flow from point 1 to point 2 depends on the two temperatures.

$$\dot{Q} = f(T_1, T_2)$$

If heat flows from hot to cold,

(the standard convention)

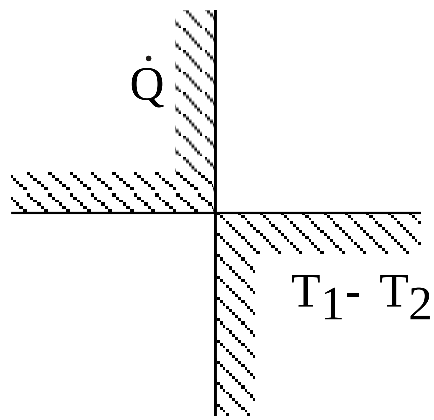
this function must be such that

$$\dot{Q} > 0 \text{ iff } T_1 > T_2$$

$$\dot{Q} < 0 \text{ iff } T_1 < T_2$$

$$\dot{Q} = 0 \text{ iff } T_1 = T_2$$

In other words, the relation must be restricted to 1st and 3rd quadrants of the $\dot{Q}$ vs. $T_1 - T_2$ plane.



NOTE IN PASSING:

It is not necessary for heat flow to be a function of temperature difference alone.

—see example later.

HEAT FLOW GENERATES ENTROPY.

FROM THE DEFINITION OF ENTROPY:

$$dQ = TdS$$

Therefore

$$\dot{Q} = T\dot{S}$$

IDEALIZE THE HEAT TRANSFER PROCESS:

Assume no heat energy is stored between points 1 and 2.

Therefore

$$\dot{Q} = T_1\dot{S}_1 = T_2\dot{S}_2$$

NET RATE OF ENTROPY PRODUCTION:

entropy flow rate out minus entropy flow rate in.

$$\dot{S}_2 - \dot{S}_1 = \dot{Q}/T_2 - \dot{Q}/T_1 = \dot{Q}(T_1 - T_2)/T_1T_2$$

Absolute temperatures are never negative (by definition).

The product of heat flow rate and temperature difference is never negative.

—due to the restrictions on the relation between heat flow rate and temperature

($\dot{Q} > 0$ when $T_1 > T_2$ etc.)

THEREFORE ***THE NET RATE OF ENTROPY PRODUCTION IS NEVER NEGATIVE.***

$$\dot{S}_2 - \dot{S}_1 \geq 0.$$

FURTHERMORE...

In a heat transfer process,

zero entropy production requires zero heat flow.

This requires either

a perfect thermal insulator ($\dot{Q} = 0$) or

a zero temperature gradient ($T_1 = T_2$).

These two constraints on the heat transfer process

— non-negative entropy production

— zero entropy production iff zero heat flow

are consequences of the second law of thermodynamics.

NOTE IN PASSING:

Many processes other than heat transfer also generate entropy

— see example later.

EXAMPLE: CONDUCTIVE HEAT TRANSFER

THERMAL CONDUCTION:

flow of heat from one body to another through direct contact.

A COMMON MODEL OF CONDUCTIVE HEAT TRANSFER:

heat flow rate is proportional to temperature difference
(sometimes called Fourier's Law)

For an insulated rod conducting through it ends

$$\dot{Q} = \frac{kA}{l}(T_1 - T_2)$$

where

A is area

l is length

k is thermal conductivity

WHAT TYPE OF NETWORK ELEMENT DESCRIBES THIS PHENOMENON?

THE MODEL RELATES POWER FLOW TO EFFORT.

Rewrite this relation in terms of efforts, flows, displacements, etc.

ASSUMING AN IDEAL HEAT TRANSFER PROCESS

(no heat energy is stored between points 1 and 2)

$$\dot{Q} = T_1\dot{S}_1 = T_2\dot{S}_2$$

From the model above (Fourier's Law) we get <u>two constitutive equations</u>.

$$\dot{S}_1 = \frac{kA}{l}(T_1 - T_2)/T_1$$

$$\dot{S}_2 = \frac{kA}{l}(T_1 - T_2)/T_2$$

THE TWO CONSTITUTIVE EQUATIONS ARE COUPLED

$$\dot{S}_1 = \dot{S}_1(T_1, T_2)$$

$$\dot{S}_2 = \dot{S}_2(T_1, T_2)$$

This is a two-port element.

THE EQUATIONS RELATE EFFORTS (TEMPERATURES) AND FLOWS (ENTROPY FLOW RATES)

– this is a <u>two-port resistor</u>.

The bond graph symbol is

$$\xrightarrow[dS_1/dt]{T_1} R \xrightarrow[dS_2/dt]{T_2}$$

NOTE:

Absolute temperatures are never negative, but either (or both) of the two entropy flow rates $\dot{S}_1$ and $\dot{S}_2$ may be negative—e.g., when $T_2 > T_1$

However, the *net entropy production rate is never negative*.

$$\dot{S}_2 - \dot{S}_1 = \frac{kA}{l}(T_1 - T_2)(1/T_2 - 1/T_1)$$

$$\dot{S}_2 - \dot{S}_1 = \frac{kA}{l}(T_1 - T_2)^2/T_1T_2$$

$$\dot{S}_2 - \dot{S}_1 \,\varepsilon\, 0$$

—This is as required by the second law.

—The constitutive equations automatically satisfy the second law.

THIS SIMPLE TWO-PORT RESISTOR

—adds entropy production behavior to our models

—ensures the second law is satisfied

EXAMPLE: RADIATIVE HEAT TRANSFER

THERMAL RADIATION:

the flow of heat from one body to another without direct contact.

A SIMPLE MODEL OF RADIATIVE HEAT TRANSFER

heat flow rate is proportional to the difference of the fourth powers of the temperatures.

(Stefan-Boltzmann Law)

$$\dot{Q} = \sigma(T_1^4 - T_2^4)$$

σ: radiative heat transfer coefficient.

T_1 and T_2: absolute temperatures.

This model satisfies the restrictions on the relation between heat flow rate and temperature difference.

$\dot{Q} = 0$ iff $T_1 = T_2$

$\dot{Q} > 0$ iff $T_1 > T_2$

$\dot{Q} < 0$ iff $T_1 < T_2$

NOTE:

This is a case in which the heat flow rate can *not* be expressed as a function of the temperature difference alone

—as advertised above.

TO FIND CONSTITUTIVE EQUATIONS FOR THIS PROCESS, PROCEED AS BEFORE:

Assume an ideal heat transfer process

(no heat energy stored between points 1 and 2)

$$\dot{Q} = T_1\dot{S}_1 = T_2\dot{S}_2$$

Again we get two constitutive equations

$$\dot{S}_1 = \sigma(T_1^4 - T_2^4)/T_1$$

$$\dot{S}_2 = \sigma(T_1^4 - T_2^4)/T_2$$

As before, these two constitutive equations are coupled and describe a two-port resistor

$$\xrightarrow[dS_1/dt]{T_1} \mathrm{R} \xrightarrow[dS_2/dt]{T_2}$$

As before, either or both individual entropy flow rates $\dot{S}_1$ and $\dot{S}_2$ may be negative—e.g., when $T_2 > T_1$

However, the constitutive equations guarantee non-negative net entropy production.

Net entropy production rate

$$\dot{S}_2 - \dot{S}_1 = \sigma(T_1^4 - T_2^4)(1/T_2 - 1/T_1)$$

Expand this expression

$$\dot{S}_2 - \dot{S}_1 = \sigma(T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)(1/T_2 - 1/T_1)$$

$$= \sigma(T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)^2/T_1T_2 \;\varepsilon\; 0$$

Because absolute temperatures are never negative,

net entropy production rate is never negative.

$$\dot{S}_2 - \dot{S}_1 \;\varepsilon\; 0$$

—as required by the second law.

CAUSALITY AND THE SECOND LAW

CAUSAL ANALYSIS — INTRODUCED EARLIER — IS AN IMPORTANT ASPECT OF PHYSICAL SYSTEM MODELING.

To model physical processes, we represent relations between quantities as operations on variables.

The choice of which variables may be used as input and which as output is not arbitrary.

The definition of an element may prohibit some choices.

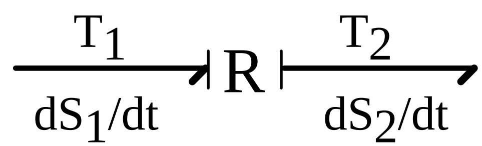
e.g., its constitutive equation may have no inverse.

There may be a "natural" preference for some choices.

e.g., the integral causal form is preferable for an energy-storage element.

THERE IS A RELATION BETWEEN CAUSAL FORMS AND THE SECOND LAW OF THERMODYNAMICS.

Regarded as assignment operators, the heat transfer constitutive equations have temperature inputs and entropy flow rate outputs on both ports.



—"conductance" causality (effort in, flow out)

(rather than "resistance" causality)

THIS CAUSAL FORM SATISFIES THE SECOND LAW DUE TO

(a) the constitutive equations and

(b) the sign-definiteness of absolute temperature.

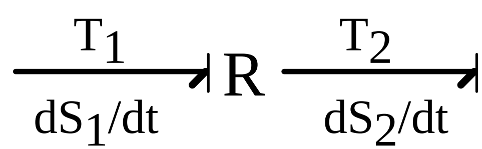
THE ALTERNATIVE CAUSAL FORMS DO NOT HAVE THIS OBVIOUS RELATION TO THE SECOND LAW.

For example:

Re-consider the model of conductive heat transfer

change one port to a “resistance” causal form

(entropy flow rate in, temperature out)



A little algebra shows the corresponding constitutive equations to be:

$$\dot{S}_1 = \left(\frac{kA}{l}\right)\left(\frac{\dot{S}_2}{\dot{S}_2 + \frac{kA}{l}}\right)$$

$$T_2 = \frac{\left(\frac{kA}{l}\right) T_1}{\dot{S}_2 + \frac{kA}{l}}$$

The first equation suggests that entropy flow $\dot{S}_1$ is independent of temperature.

That seems odd—we usually think of entropy (and heat) as flowing in response to a temperature gradient.

The second equation suggests that temperature T_2 may be

infinite if $\dot{S}_2 = -kA/l$ or negative if $\dot{S}_2 < -kA/l$

Those values of temperature would have little physical meaning.

FURTHERMORE...

ACCORDING TO THESE EQUATIONS, THE NET ENTROPY PRODUCTION RATE IS

$$\dot{S}_2 - \dot{S}_1 = \frac{\dot{S}_2^2}{\dot{S}_2 + \frac{kA}{l}}$$

This equation suggests that net entropy production may be negative if $\dot{S}_2 < -kA/l$

— that would violate the second law.

These problems might be solved by restricting the value of the input entropy flow rate.

$$\dot{S}_2 > -kA/l$$

THAT WOULD BE UNSATISFACTORY.

1. Even with that constraint, this form still suggests that entropy flow $\dot{S}_1$ is independent of temperature

   — that does not facilitate insight.

2. We don't know *a-priori* which other elements in a system generate the input entropy flow

   — it is not clear where to apply the constraint.

3. The constraint on entropy flow rates is due to the physical properties of heat transfer.

   If our model is to reflect our understanding of the physical process,

   the constraint should be associated with the element used to model heat transfer,

   — not with the rest of the system.

In contrast, the “conductance” causal form

(temperature inputs, entropy flow rate outputs)

satisifies the constraints of the second law through the constitutive equations of the resistive multiport element itself.

In fact, this is the only causal assignment which will guarantee that the resistive multiport will satisfy the second law by virtue of its own constitutive equations.

Thus there is a “natural” preference for the conductance causal form for an entropy-producing multiport resistor.

(Analogous to the preferred integral causal form for an energy storage element.)

