## STATE EQUATION DERIVATION

## Summary of Basic Bond Graph Elements

A large class of physical systems may be described using the basic lumped parameter elements
— ideal active and passive one-port elements connected by multiport junction elements. The variables and primitive elements of the energy-based formalism are summarized in the following tables.

Table 6.1 Fundamental variables

| Energy and power: | $\mathrm{E}-\mathrm{E}_{\mathrm{O}}=\int \mathrm{P} d t$ |
| :--- | :--- |
| Conjugate power variables: | $\mathrm{P}=\mathrm{e} f$ |
| Conjugate energy variables: | $\mathrm{p}-\mathrm{po}_{\mathrm{o}}=\int \mathrm{e} \mathrm{dt}$ |
|  | $\mathrm{q}-\mathrm{qo}_{\mathrm{o}}=\int \mathrm{fdt}$ |

Table 6.2 Active one-port elements (sources or boundary elements)

|  | effort source | flow source |
| :--- | :--- | :--- |
| constitutive equation | $\mathrm{e}=\mathrm{e}(\mathrm{t})$ | $\mathrm{f}=\mathrm{f}(\mathrm{t})$ |
| bond graph symbol | $\mathbf{S}_{\mathrm{e}}$ | $\leftharpoonup \mathbf{S}_{\mathrm{f}}$ |

Table 6.3 Passive one-port element constitutive equations

|  | capacitor | inertia | resistor |
| :--- | :--- | :--- | :--- |
| general form | $e=\Phi(q)$ | $f=\Psi(p)$ | $e=\Gamma(f)$ |
| linear form | $e=q / C$ | $f=p / I$ | $e=R f$ |
| bond graph symbol |  |  |  |

Table 6.4 Symmetric multi-port junction elements

|  | common flow junction | common effort junction |
| :---: | :---: | :---: |
| constitutive equations | $\begin{aligned} & f_{i}=f_{j}, j=1, n \\ & \sum_{j=1}^{n} \sigma_{j} e_{j}=0 \end{aligned}$ | $\begin{aligned} & e_{i}=e_{j}, j=1, n \\ & \sum_{j=1}^{n} \sigma_{j} f_{j}=0 \end{aligned}$ |
| bond graph symbol | $\xrightarrow[{2^{2} / \sqrt{3}}]{ } \xrightarrow{n}$ | $\xrightarrow{{ }_{2}^{2} \int_{3}} 0 \xrightarrow{n}$ |

where $\sigma_{j}=\left\{\begin{array}{l}+1 \text { if power positive in } \\ -1 \text { if power positive out }\end{array}\right.$
Table 6.5 Asymmetric two-port junction elements

|  | transformer | gyrator |
| :--- | :--- | :--- |
| constitutive equations | $\mathrm{e}_{\mathrm{i}}=\mathrm{T} \mathrm{e}_{\mathrm{j}}$ | $\mathrm{e}_{\mathrm{i}}=\mathrm{G} \mathrm{f} \mathrm{f}_{\mathrm{j}}$ |
|  | $\mathrm{f}_{\mathrm{j}}=\mathrm{T} \mathrm{f}_{\mathrm{i}}$ | $\mathrm{e}_{\mathrm{j}}=\mathrm{G} \mathrm{f}_{\mathrm{i}}$ |
| bond graph symbol |  |  |

## Sign Convention

Half arrows denote the direction of positive power flow. For passive elements power flow is positive inwards. For active elements there is no fixed convention, though it is common to denote power flow as positive outwards. Multiport junction elements may serve to define effort or flow differences (e.g. pressure difference, relative motion). In that case the following sign convention is recommended.


