13.49 Homework #2

1. Consider the vibration of a mass-spring-dashpot system:

$$m\ddot{x} + b\dot{x} + kx = 0,$$

wherein all the coefficients are positive and real (i.e., physical).

(a) Set $x(t) = e^{st}$, and find two solutions for s in terms of m, b, and k, using the quadratic formula. The solution pairs depend on whether b exceeds a critical value: what is the critical value and how do the solutions change?

(b) From the initial condition $x(0) = x_o$, and $\dot{x}(0) = 0$, find the coefficients for the general solution

$$x(t) = c_1 e^{s_1 t} + c_2 e^{s_2}.$$

in terms of x_o, s_1, and s_2. What do you think will happen when $s_1 = s_2$?

(c) Write this $x(t)$ for sub-critical b (that is, complex s) in terms of the standard parameters

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m}} \text{ (undamped natural frequency)} \\ \zeta &= \frac{b}{2\sqrt{km}} \text{ (damping ratio)} \\ \omega_d &= \omega_n \sqrt{1-\zeta^2} \text{ (damped natural frequency).} \end{aligned}$$

Give your answer in terms of a real exponential multiplied by a single sine or cosine, and make a sketch.

(d) Sketch the response $x(t)$ for supercritical b, i.e., both s are real, for the same initial conditions. What happens when the roots s_i are far apart vs. when they are close together?

(e) Consider now the case where an input acts to drive the mass from zero initial conditions:

$$m\ddot{x} + b\dot{x} + kx = u.$$

This equation defines a system, with input u, and output x. Using the Laplace transform, write the transfer function for this system, i.e., the impulse response, both in frequency (Laplace) space, and in the time-domain. Make a sketch of the time-domain result. Be sure to cover both the sub-critical and super-critical damping cases.

(f) What is the step response of this system, in both the subcritical and super-critical damping cases? Include sketches.

NOTE: The simulation in Homework #1 can be used to confirm your results. Some transform pairs have also been added to the lecture notes on Laplace transform. Use these in parts e) and f).

2. A submarine has weight $1200t$ (tons) and the center of gravity is $0.5m$ above the center of buoyancy (What can you conclude?). The rolling motion can be assumed to be decoupled from the other motions.

   This submarine has anti-rolling fins to ensure stability. The control hydrodynamic derivative is $K_\delta = -2.8tm$ per degree of fin rotation δ, at a forward speed of $5m/s$.

   (a) Write the equation of motion for roll (K).

   (b) If the automatic control law $\delta = k_1 \psi$ is used, where ψ is the roll angle, what range of k_1 ensures stability?

   (c) If the speed suddenly drops to $2.5m/s$, how does the range of stabilizing k_1 change?

3. Draw curves of rudder angle δ vs. yaw rate r for a dynamically stable surface vessel, and then for an unstable vessel. Indicate areas where the vessel turns against the rudder action.