

1) a, b) See pp 1-3 of 13.42 OCEAN WAVE SPECTRA AND DG.

c) SIG. WAVE HT IS VERY CLOSE TO THE WAVEHEIGHT WHICH EXPERIENCED MARINERS WOULD ESTIMATE FROM THEIR OBSERVATIONS

d) B-S USED FREQUENTLY SINCE IT REQUIRES ONLY SIG. WAVE HT AND MODAL FREQUENCY AS INPUTS, AND, UNLIKE P-M, IT CAN BE USED IN SITUATIONS OTHER THAN FULLY DEVELOPED SEAS.

$$2) \text{ B-S: } S_{\eta}^+(\omega) = \frac{1.25}{4} \frac{\omega_m^4}{\omega^5} \xi^2 e^{\left\{-1.25\left(\frac{\omega_m}{\omega}\right)^4\right\}}$$

$$\xi = 5.75 \text{ m} \quad \omega_m = 0.68 \text{ rad/s}$$

$$\eta(t) \longrightarrow \boxed{|H(\omega)|} \longrightarrow \eta_p(t)$$

$$S_{\eta_p}(\omega) = S_{\eta}(\omega) |H(\omega)|^2 \quad |H(\omega)| = \frac{(0.4 - \omega^3)^2 + 0.25}{(0.4 - \omega^3)^2 + 0.15}$$

$$S_{\eta_p}(\omega) = \frac{1.25}{4} \frac{(0.68)^4}{\omega^5} (5.75)^2 e^{\left\{-1.25\left(\frac{0.68}{\omega}\right)^4\right\}} \left[\frac{(0.4 - \omega^3)^2 + 0.25}{(0.4 - \omega^3)^2 + 0.15} \right]^2$$

$$\text{Ave freq of upcrossings } \bar{\eta}(h) = \frac{1}{2\pi} \sqrt{\frac{M_2}{M_0}} e^{-h^2/2M_0}$$

$$\text{and } M_0 = \int_0^{\infty} S_{\eta_p}(\omega) d\omega \approx 4.326$$

$$M_2 = \int_0^{\infty} \omega^2 \cdot S_{\eta_p}(\omega) d\omega \approx 3.139$$

} = solved using MATHCAD.

$$\therefore \bar{\eta}(h) = \frac{1}{2\pi} \sqrt{\frac{3.139}{4.326}} e^{-h^2/2 \cdot 4.326} \text{ SET THIS } \leq \frac{2 \text{ times}}{1 \text{ hr}} \approx \frac{1 \text{ hr}}{3600 \text{ sec}} = \frac{1 \text{ hr}}{1800 \text{ s}}$$

$$e^{-h^2/8.652} \leq \left(\frac{1}{1800}\right) (2\pi) \sqrt{\frac{4.326}{3.139}}$$

$$-h^2 \leq 8.652 \left[\ln\left(\frac{2\pi}{1800} \sqrt{\frac{4.326}{3.139}}\right) \right]$$

$$\boxed{h \geq 6.897 \text{ m}}$$

Homework 6 Solutions, 13.42, Spring 2005

Problem 2:

$$w_m := 0.68 \quad \zeta := 5.75 \quad g := 9.81$$

$$S(w) := \frac{1.25 \cdot w_m^4}{4 \cdot w^5} \cdot \zeta^2 \cdot e^{-1.25 \cdot \left(\frac{w_m}{w}\right)^4} \cdot \left[\frac{\left[\frac{(0.4 - w^3)^2 + 0.25}{(0.4 - w^3)^2 + 0.15} \right]^2}{\left[\frac{(0.4 - w^3)^2 + 0.25}{(0.4 - w^3)^2 + 0.15} \right]^2} \right]$$

$$M_o := \int_0^{\infty} S(w) dw \quad M_2 := \int_0^{\infty} w^2 S(w) dw$$

$$M_o = 4.326 \quad M_2 = 3.139$$

$$n(h) := \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{M_2}{M_o}} \cdot e^{-\frac{h^2}{2 \cdot M_o}}$$

$$h := \sqrt{-2 \cdot M_o \cdot \left(\ln \left(2 \cdot \frac{\pi}{1800} \cdot \sqrt{\frac{M_o}{M_2}} \right) \right)} \quad h = 6.897 \quad \text{meters}$$

3) a)



$$V_p = \frac{\text{DISTANCE BETWEEN PROBES 1 and 2}}{\text{TIME IT TAKES A WAVE CREST TO GO FROM 1 TO 2}}$$

$$V_g = \frac{\text{DISTANCE BETWEEN PROBES 1 and 3}}{\text{TIME IT TAKES PACKET OF WAVES TO GO FROM 1 TO 3}}$$

NOTE: $V_g = \frac{1}{2} V_p$ FOR DEEP WATER

$V_g = V_p$ FOR SHALLOW WATER

$A = \frac{1}{2}$ [measured distance between ~~high~~ crest + trough of a wave at a single probe]

$\lambda = \frac{\text{time between consecutive wave peaks at a single probe}}{V_p}$

b) $h = 1.5 \text{ m}$ FOR DEEP WATER: $\frac{\lambda}{2} \leq h$

$$\lambda \leq 2h \quad \text{and} \quad k = \frac{2\pi}{\lambda} \quad \therefore \frac{2\pi}{\lambda} \leq 2h \quad \therefore \frac{\pi}{h} \leq k$$

IN DEEP WATER $\omega^2 = kg$, $k = \frac{\omega^2}{g}$

$$\frac{\omega^2}{g} \geq \frac{\pi}{h} \quad \therefore \omega^2 \geq \frac{\pi g}{h} \quad \therefore \omega \geq \sqrt{\frac{\pi g}{h}}$$

$$\therefore \omega \geq \sqrt{\frac{\pi \cdot 9.81 \text{ m/sec}^2}{1.5 \text{ m}}} \geq 4.533 \text{ rad/s}$$

$$\therefore \boxed{\omega \geq 4.533 \text{ rad/s}}$$

c) TAKE WAVE ELEVATION AND HEAVE MOTION DATA IN THE TIME DOMAIN AND DO F.F.T. TO FREQUENCY DOMAIN. RAO [i.e. TRANSFER FUNCTION] IS THEN THE SQUARE ROOT OF THE OUTPUT SPECTRUM DIVIDED BY

Problem 4: Part 1:

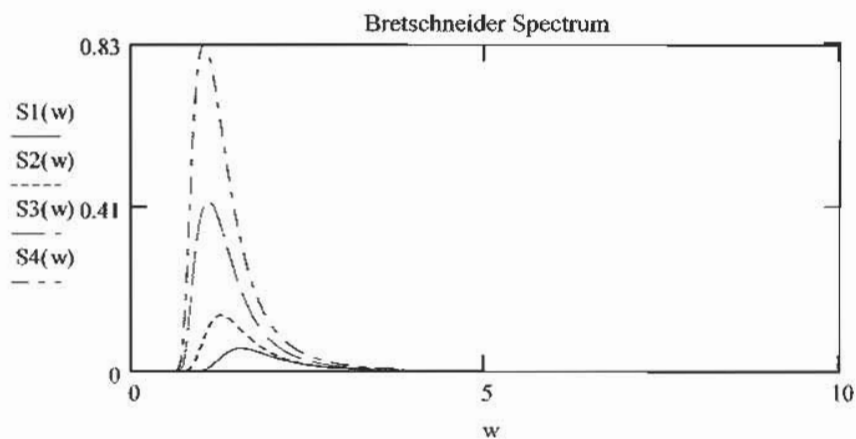
$\zeta_1 := 1$ meter	$T_1 := 4$ sec	$w_{1_m} := \frac{2 \cdot \pi}{T_1}$	$w_{1_m} = 1.571$	$\frac{\text{rad}}{\text{s}}$
$\zeta_2 := 1.4$ meter	$T_2 := 4.8$ sec	$w_{2_m} := \frac{2 \cdot \pi}{T_2}$	$w_{2_m} = 1.309$	$\frac{\text{rad}}{\text{s}}$
$\zeta_3 := 2.2$ meter	$T_3 := 5.4$ sec	$w_{3_m} := \frac{2 \cdot \pi}{T_3}$	$w_{3_m} = 1.164$	$\frac{\text{rad}}{\text{s}}$
$\zeta_4 := 2.5$ meter	$T_4 := 5.6$ sec	$w_{4_m} := \frac{2 \cdot \pi}{T_4}$	$w_{4_m} = 1.122$	$\frac{\text{rad}}{\text{s}}$

$$S_1(w) := \frac{1.25}{4} \cdot \frac{w_{1_m}^4}{w^5} \cdot \zeta_1^2 \cdot e^{-1.25 \cdot \left(\frac{w_{1_m}}{w}\right)^4} \cdot \left[\frac{(0.4 - w^3)^2 + 0.25}{(0.4 - w^3)^2 + 0.15} \right]^2$$

$$S_2(w) := \frac{1.25}{4} \cdot \frac{w_{2_m}^4}{w^5} \cdot \zeta_2^2 \cdot e^{-1.25 \cdot \left(\frac{w_{2_m}}{w}\right)^4} \cdot \left[\frac{(0.4 - w^3)^2 + 0.25}{(0.4 - w^3)^2 + 0.15} \right]^2$$

$$S_3(w) := \frac{1.25}{4} \cdot \frac{w_{3_m}^4}{w^5} \cdot \zeta_3^2 \cdot e^{-1.25 \cdot \left(\frac{w_{3_m}}{w}\right)^4} \cdot \left[\frac{(0.4 - w^3)^2 + 0.25}{(0.4 - w^3)^2 + 0.15} \right]^2$$

$$S_4(w) := \frac{1.25}{4} \cdot \frac{w_{4_m}^4}{w^5} \cdot \zeta_4^2 \cdot e^{-1.25 \cdot \left(\frac{w_{4_m}}{w}\right)^4} \cdot \left[\frac{(0.4 - w^3)^2 + 0.25}{(0.4 - w^3)^2 + 0.15} \right]^2$$



Problem 4: Part 2:

Part a: $U := 7.20222$ m per sec, = 14 kts $x := 51856$ m, = 28 nm

$$x_bar := \frac{g \cdot x}{U^2} \quad a := 0.076 \cdot x_bar^{-0.22}$$

$$\sigma(w) := \begin{cases} 0.07 & \text{if } w \leq w1_m \\ 0.09 & \text{if } w > w1_m \end{cases}$$

$$\delta(w) := \frac{-(w - w1_m)^2}{2 \cdot (\sigma(w))^2 \cdot (w1_m)^2}$$

$\gamma := 3.3$ This is a standard value of gamma.

$$S_J1(w) := \frac{a \cdot g^2}{w^5} \cdot e^{\frac{-5}{4} \left(\frac{w1_m}{w} \right)^4 \cdot \gamma \cdot \delta(w)}$$

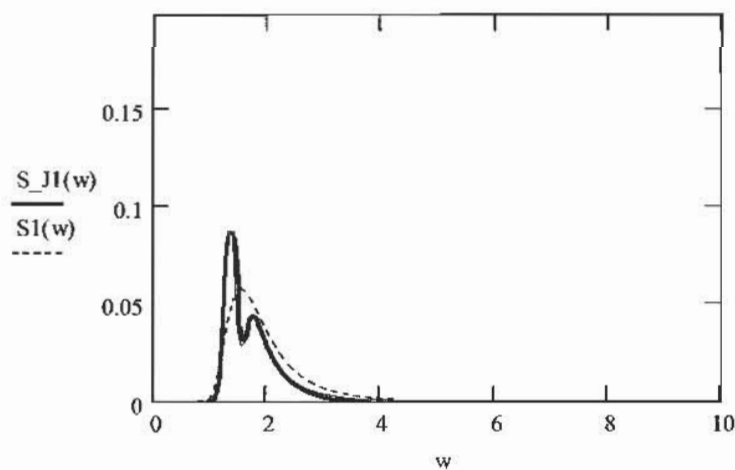
$$a = 0.01$$

$$g = 9.81$$

$$w1_m = 1.571$$

$$\gamma = 3.3$$

$\delta =$ function



b: $\zeta_1 := 1$ meter $T_1 := 4$ sec

$$w1_m1 := \frac{1 \cdot \pi}{T_1} \quad w1_m1 = 0.785 \quad \frac{\text{rad}}{\text{s}} \quad w1_m3 := \frac{3 \cdot \pi}{T_1} \quad w1_m3 = 2.356 \quad \frac{\text{rad}}{\text{s}}$$

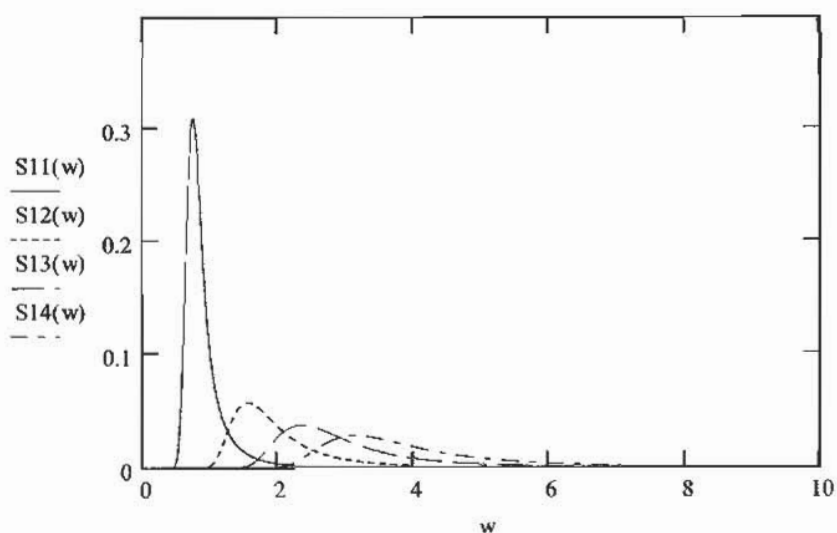
$$w1_m2 := \frac{2 \cdot \pi}{T_1} \quad w1_m2 = 1.571 \quad \frac{\text{rad}}{\text{s}} \quad w1_m4 := \frac{4 \cdot \pi}{T_1} \quad w1_m4 = 3.142 \quad \frac{\text{rad}}{\text{s}}$$

$$S11(w) := \frac{1.25 \cdot w1_m1^4}{4 \cdot w^5} \cdot \zeta_1^2 \cdot e^{\left[-1.25 \cdot \left(\frac{w1_m1}{w} \right)^4 \right]} \cdot \left[\frac{\left(0.4 - w^3 \right)^2 + 0.25}{\left(0.4 - w^3 \right)^2 + 0.15} \right]^2$$

$$S12(w) := \frac{1.25 \cdot w1_m2^4}{4 \cdot w^5} \cdot \zeta_1^2 \cdot e^{\left[-1.25 \cdot \left(\frac{w1_m2}{w} \right)^4 \right]} \cdot \left[\frac{\left(0.4 - w^3 \right)^2 + 0.25}{\left(0.4 - w^3 \right)^2 + 0.15} \right]^2$$

$$S13(w) := \frac{1.25 \cdot w1_m3^4}{4 \cdot w^5} \cdot \zeta_1^2 \cdot e^{\left[-1.25 \cdot \left(\frac{w1_m3}{w} \right)^4 \right]} \cdot \left[\frac{\left(0.4 - w^3 \right)^2 + 0.25}{\left(0.4 - w^3 \right)^2 + 0.15} \right]^2$$

$$S14(w) := \frac{1.25 \cdot w1_m4^4}{4 \cdot w^5} \cdot \zeta_1^2 \cdot e^{\left[-1.25 \cdot \left(\frac{w1_m4}{w} \right)^4 \right]} \cdot \left[\frac{\left(0.4 - w^3 \right)^2 + 0.25}{\left(0.4 - w^3 \right)^2 + 0.15} \right]^2$$



Problem 4, Part 3:

$$M_{0n} := \int_0^{\infty} S1(w) dw \quad M_{0n} = 0.064 \quad M_{0} := \left(\frac{\zeta 1}{4}\right)^2 \quad M_{0} = 0.063$$

$$M_{2n} := \int_0^{\infty} w^2 \cdot S1(w) dw \quad M_{2n} = 0.308 \quad M_{2} := 1.982 \cdot \left(\frac{\zeta 1 \cdot w l_m}{4}\right)^2 \quad M_{2} = 0.306$$

$$M_{4n} := \int_0^{\infty} w^4 \cdot S1(w) dw \quad M_{4} := 7.049 \cdot M_{0} \cdot w l_m^4 \quad M_{4} = 2.682$$

M 4 integral does not converge. The values for M_0 and M_2 are similar to approximations.

$$\varepsilon_w := \sqrt{1 - \frac{M_{2}^2}{M_{0} \cdot M_{4}}} \quad \varepsilon = 0.665 \quad \text{Bandwidth}$$

$$\frac{N_w}{w} := 10$$

$$a_{10} := \sqrt{M_{0}} \cdot \sqrt{2 \cdot \ln \left(\frac{2 \cdot \sqrt{1 - \varepsilon^2}}{1 + \sqrt{1 - \varepsilon^2}} \cdot N \right)} \quad a_{10} = 0.518 \quad \text{meters}$$

$$\frac{N_w}{w} := 50$$

$$a_{50} := \sqrt{M_{0}} \cdot \sqrt{2 \cdot \ln \left(\frac{2 \cdot \sqrt{1 - \varepsilon^2}}{1 + \sqrt{1 - \varepsilon^2}} \cdot N \right)} \quad a_{50} = 0.685 \quad \text{meters}$$

$$\frac{N_w}{w} := 100$$

$$a_{100} := \sqrt{M_{0}} \cdot \sqrt{2 \cdot \ln \left(\frac{2 \cdot \sqrt{1 - \varepsilon^2}}{1 + \sqrt{1 - \varepsilon^2}} \cdot N \right)} \quad a_{100} = 0.746 \quad \text{meters}$$

Problem 5:

$$\omega_n := 1.1 \quad \text{rad/s}$$

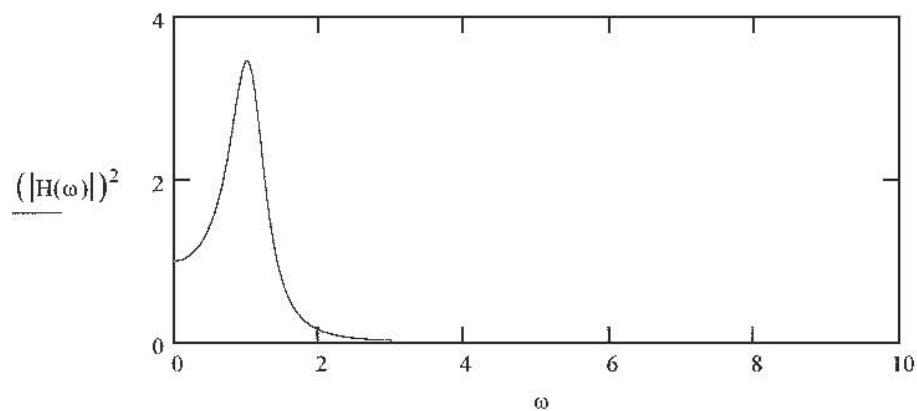
$$\beta := 0.28$$

$$i := \sqrt{-1}$$

Part a:

$$H(\omega) := \frac{\omega_n^2}{-\omega^2 + 2 \cdot i \cdot \beta \cdot \omega_n \cdot \omega + \omega_n^2} \quad H(\omega) \rightarrow \frac{1.21}{-\omega^2 + .616 \cdot i \cdot \omega + 1.21}$$

$$(|H(\omega)|)^2 \rightarrow \frac{1.4641}{(-\omega^2 + 1.21)^2 + .379456 \cdot \omega^2}$$

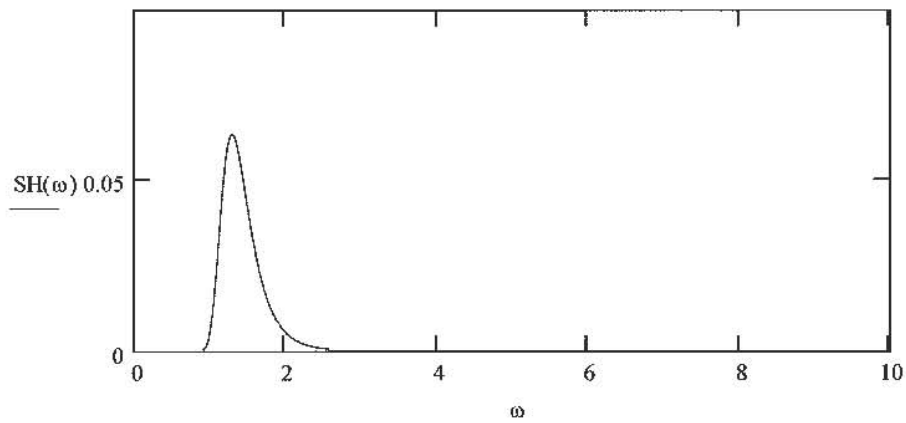


Part b:

$$S1(\omega) \rightarrow 1.953125000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.812500000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{\left[(.4 - \omega^3)^2 + .2\right]}{\left[(.4 - \omega^3)^2 + .1\right]}$$

$$SH(\omega) := S1(\omega) \cdot (|H(\omega)|)^2$$

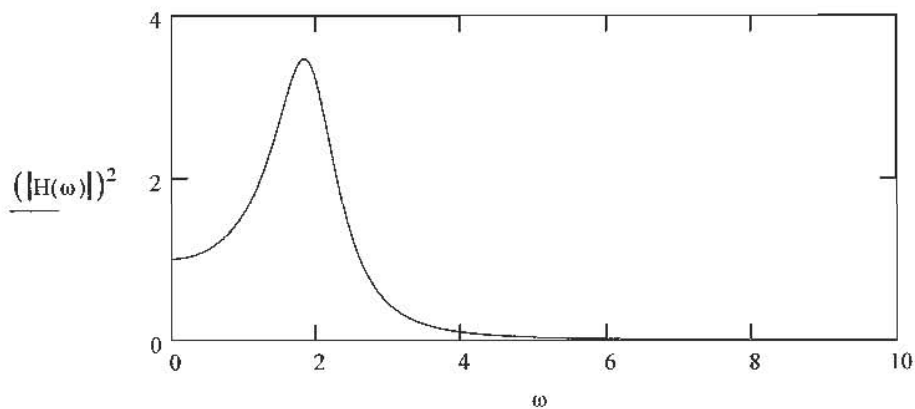
$$SH(\omega) \rightarrow 2.8595703125000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.812500000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{\left[(.4 - \omega^3)^2 + .2\right]}{\left[(.4 - \omega^3)^2 + .15\right]^2}$$



Part c: $\omega_n = 2$ rad/s Repeat above steps with a higher frequency.

$$\underline{H(\omega)} := \frac{\omega_n^2}{-\omega^2 + 2 \cdot i \cdot \beta \cdot \omega_n \cdot \omega + \omega_n^2} \quad H(\omega) \rightarrow \frac{4}{-\omega^2 + 1.12 \cdot i \cdot \omega + 4}$$

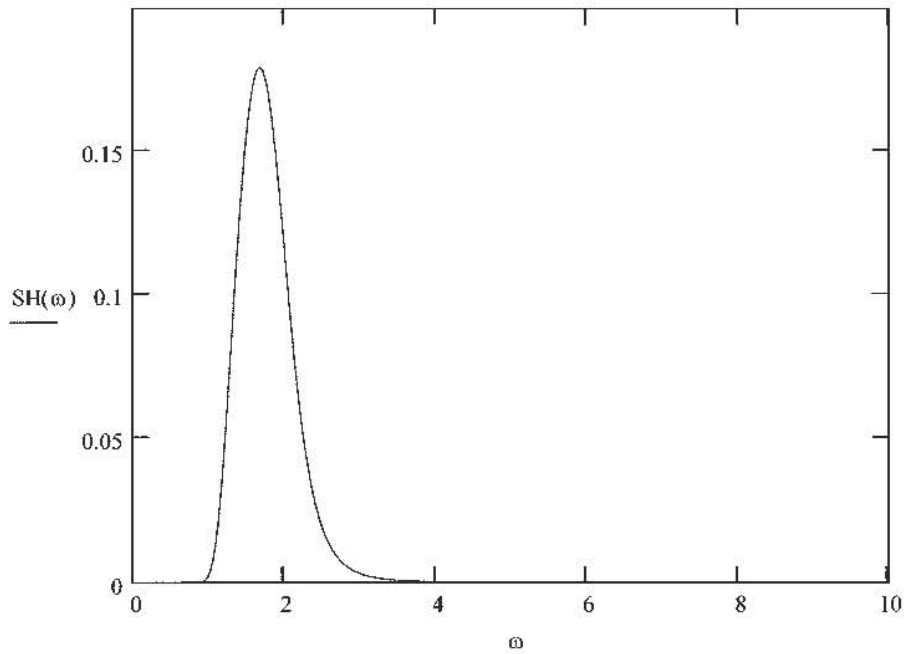
$$(|H(\omega)|)^2 \rightarrow \frac{16}{(-\omega^2 + 4)^2 + 1.2544 \cdot \omega^2}$$



$$SI(\omega) \rightarrow 1.953125000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.812500000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{\left[(4 - \omega^3)^2 + .2\right]}{\left[(4 - \omega^3)^2 + .1\right]}$$

$$\underline{SH(\omega)} := SI(\omega) \cdot (|H(\omega)|)^2$$

$$SH(\omega) \rightarrow .31250000000000000000 \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.812500000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{\left[(.4 - \omega^3) \right]}{\left[(.4 - \omega^3)^2 + .15 \right]^2 \cdot \left[(-\omega \right)}$$

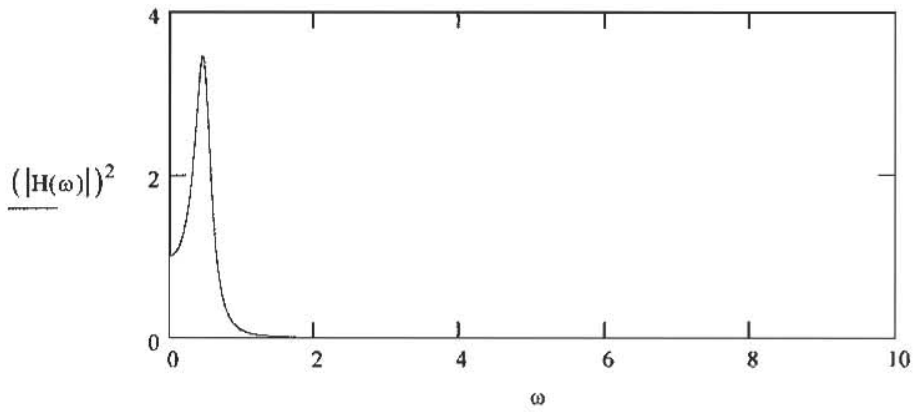


The spectrum is taller with the higher frequency.

Part c cont $\omega_n := .5$ rad/s Repeat above steps with a lower frequency.

$$H(\omega) := \frac{\omega_n^2}{-\omega^2 + 2 \cdot i \cdot \beta \cdot \omega_n \cdot \omega + \omega_n^2} \quad H(\omega) \rightarrow \frac{.25}{-\omega^2 + .280 \cdot i \cdot \omega + .25}$$

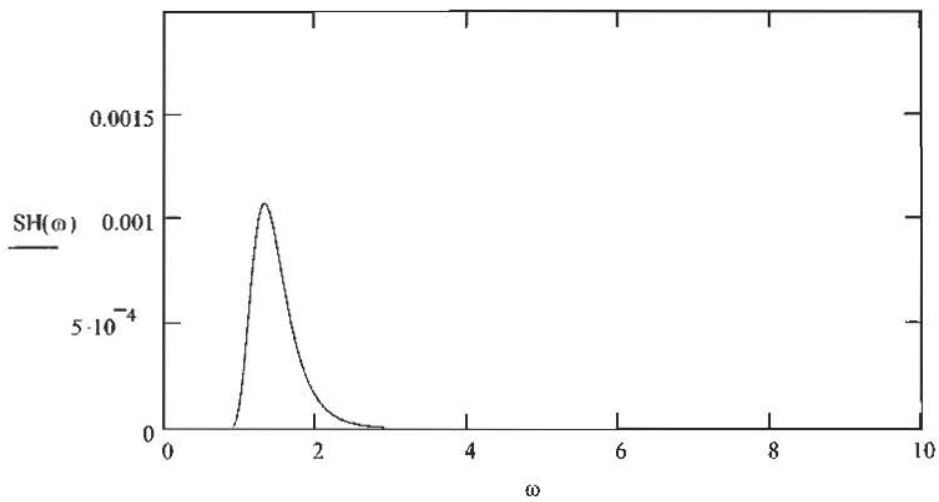
$$\left(|H(\omega)| \right)^2 \rightarrow \frac{6.25 \cdot 10^{-2}}{\left(-\omega^2 + .25 \right)^2 + 7.8400 \cdot 10^{-2} \cdot \omega^2}$$



$$S1(\omega) \rightarrow 1.9531250000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.8125000000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{\left[(.4 - \omega^3)^2 + . \right]}{\left[(.4 - \omega^3)^2 + . \right]}$$

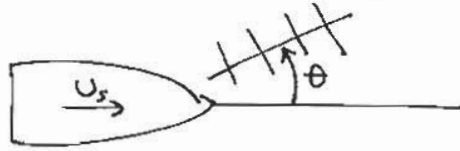
$$\underline{SH(\omega)} := S1(\omega) \cdot (|H(\omega)|)^2$$

$$SH(\omega) \rightarrow 1.2207031250000000000 \cdot 10^{-3} \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.8125000000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{\left[\right]}{\left[(.4 - \omega^3)^2 + .15 \right]}$$



The spectrum is much shorter for the this frequency.

6) FOR A SHIP WITH FORWARD SPEED



$$\omega_e = \omega + \frac{U_s \cos \theta}{g} \omega^2$$

$\omega_e \equiv$ ENCOUNTER FREQUENCY

FOR HEAD SEAS, $\theta = 0$ so $\cos \theta = 1$

$$\omega_e = \omega + \frac{U_s}{g} \omega^2$$

The AMPLITUDE of heave motion is unchanged from the stationary to moving ship. However, the ship moving into head seas experiences this motion at a faster frequency.