2.23 PS #4 Soln

1) a) parabolic mean line $f(x) = f_0\left(1-\left(\frac{x}{c/2}\right)^2\right)$, $\frac{df}{dx} = -\frac{8f_0 x}{c^2} = 4\frac{f_0}{c}\cos\tilde{x}$,

where $x = -\frac{c}{2}\cos\tilde{x}$

$f_0/c = 0.08$, $\alpha = \alpha_{ideal}$, $\rho = 1000\ kg/m^3$, $c = 0.5\ m$, $U = 10\ m/s$

$$L' = \int_{-c/2}^{c/2} \rho U \gamma(x)\,dx = \rho U^2 c\left(\pi a_0 + \frac{\pi}{2}a_1\right)$$

$$a_0 = 0 \quad \text{for} \quad \alpha = \alpha_{ideal} = \frac{1}{\pi}\int_0^\pi 4\frac{f_0}{c}\cos\tilde{x}\,d\tilde{x} = 0$$

$$a_1 = \frac{2}{\pi}\int_0^\pi \frac{df}{dx}\cos\tilde{x}\,d\tilde{x} = \frac{2}{\pi}\frac{4f_0}{c}\int_0^\pi \cos^2\tilde{x}\,d\tilde{x} = 4\frac{f_0}{c} = 0.32$$

$$\boxed{L'} = 1000\cdot 100\cdot 0.5\cdot\left(\pi\cdot 0 + \frac{\pi}{2}\cdot 0.32\right) = \boxed{2.5\times 10^4\ N/m}$$

b) $$M' = \int_{-c/2}^{c/2}\rho U(x-x_{LE})\gamma(x)\,dx = \rho U\int_0^\pi\left(-\frac{c}{2}\cos\tilde{x}+\frac{c}{2}\right)\left[\cancel{-2U(\ldots+a_1\sin\tilde{x}\ldots)}\right]\frac{c}{2}\sin\tilde{x}\,d\tilde{x}$$

$x = -\frac{c}{2}\cos\tilde{x}$, $x_{LE} = -\frac{c}{2}$, $\tilde{x}_{LE} = 0$

$dx = \frac{c}{2}\sin\tilde{x}\,d\tilde{x}$

$-2U\alpha\left(\frac{1+\cos\tilde{x}}{\sin\tilde{x}}\right) - 8U\frac{f_0}{c}\sin\tilde{x}$

$$= 2\rho U^2\left(\frac{c}{2}\right)^2\int_0^\pi(1-\cos\tilde{x})\left(\underbrace{\alpha_{ideal}(1+\cos\tilde{x})}_{0} + 4\frac{f_0}{c}\sin^2\tilde{x}\right)d\tilde{x}$$

$$= 2\rho U^2\left(\frac{c}{2}\right)^2 4\frac{f_0}{c}\underbrace{\int_0^\pi(1-\cos\tilde{x})\sin^2\tilde{x}\,d\tilde{x}}$$

$$= \frac{\tilde{x}}{2} - \frac{1}{4}\sin\tilde{x} - \frac{1}{4}\sin(2\tilde{x}) + \frac{1}{12}\sin(3\tilde{x})\Big|_0^\pi = \frac{\pi}{2}$$

$$\boxed{M'} = 2\cdot 1000\cdot 100\cdot(0.25)^2\cdot 0.32\cdot\frac{\pi}{2} = \boxed{6.28\times 10^3\ \frac{Nm}{m}}$$

c) $$M' = \int_{-c/2}^{c/2}\rho U(x-x_0)\gamma(x)\,dx = \rho U\frac{c}{2}\int_0^\pi(\cos\tilde{x}_0-\cos\tilde{x})\left(-2U\cdot 4\frac{f_0}{c}\sin\tilde{x}\right)\frac{c}{2}\sin\tilde{x}\,d\tilde{x}$$

$\hookrightarrow x_0 = -\frac{c}{2}\cos\tilde{x}_0$

$$0 = \cancel{2\rho U^2\left(\frac{c}{2}\right)^2 4\frac{f_0}{c}}\int_0^\pi\left(\cos\tilde{x}\sin^2\tilde{x} - \cos\tilde{x}_0\sin^2\tilde{x}\right)d\tilde{x}$$

$$0 = \frac{1}{3}\sin^3\tilde{x}\Big|_0^\pi - \cos\tilde{x}_0\left(\frac{\tilde{x}}{2}-\frac{1}{4}\sin(2\tilde{x})\right)\Big|_0^\pi = -\frac{\pi}{2}\cos\tilde{x}_0$$

$\boxed{\tilde{x}_0 = \pi/2}$ or $\boxed{x=0}$

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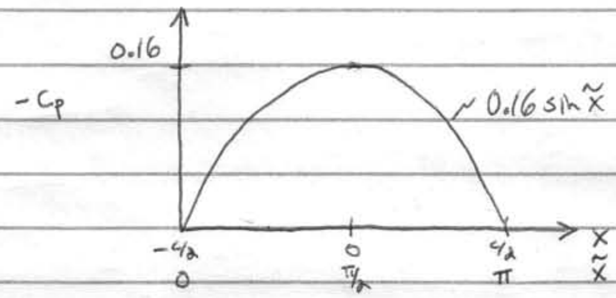
2) 2D, parabolic mean line, $C_L = 0.25$ @ $\alpha = \alpha_{ideal} = 0$, $\sigma = 0.6$

a) $C_L = 2\pi\alpha + 4\pi \frac{f_o}{c} = 0 + 4\pi \frac{f_o}{c} = 0.25$

$$\boxed{f_o/c = 0.02}$$

b) $\boxed{C_p} \approx -2\frac{u}{U} = \frac{\gamma}{U} = -8\frac{f_o}{c}\sin\tilde{x} = \boxed{-0.16 \sin\tilde{x}}$



c) cavitation when $-C_p > \sigma$ at $x = -\frac{c}{2}\cos\tilde{x} = -\frac{c}{4} \Rightarrow \tilde{x} = 60^\circ$

$$2\alpha\left(\frac{1+\cos\tilde{x}}{\sin\tilde{x}}\right) + 8\frac{f_o}{c}\sin\tilde{x} > \sigma$$

$$2\alpha\left(\frac{1+1/2}{\sqrt{3}/2}\right) + 0.16 \cdot \sqrt{3}/2 > 0.6$$

$$\boxed{\alpha > 0.133 \text{ rad} = 7.6^\circ}$$

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3) a) $C_L = 2\pi a_0 + \pi a_1$

$f(x) = 0.3c\left(\frac{x}{c}\right)^3 - 0.12c\left(\frac{x}{c}\right)^2 - 0.18c\left(\frac{x}{c}\right)$, $x_{LE} = -c/2$, $x_{TE} = c/2$

$\frac{df}{dx} = 0.9\left(\frac{x}{c}\right)^2 - 0.24\left(\frac{x}{c}\right) - 0.18 = 0.225\cos^2\tilde{x} + 0.12\cos\tilde{x} - 0.18$, $x = -\frac{c}{2}\cos\tilde{x}$

$$\frac{1}{\pi}\int_0^{\pi} \frac{df}{dx}\,d\tilde{x} = \frac{1}{\pi}\left[0.225\left(\frac{\tilde{x}}{2} + \cancel{\frac{1}{4}\sin(2\tilde{x})}\right) + \cancel{0.12\sin\tilde{x}} - 0.18\tilde{x}\right]_0^{\pi} = -0.0675 = \alpha - a_0$$

$$a_0 = \alpha + 0.0675 = \left(2\deg\cdot\frac{\pi}{180}\right) + 0.0675 = 0.102 \approx 0.1$$

$$a_1 = \frac{2}{\pi}\int_0^{\pi}\Big(0.225\underbrace{\cos^3\tilde{x}}_{\int\Rightarrow 0} + 0.12\underbrace{\cos^2\tilde{x}}_{\int\Rightarrow\frac{\pi}{2}} - 0.18\underbrace{\cos\tilde{x}}_{\int\Rightarrow 0}\Big)d\tilde{x} = \cancel{\frac{2}{\pi}}\,0.12\cdot\cancel{\left(\frac{\pi}{2}\right)} = 0.12$$

$$\boxed{C_L = 1.0}$$

b) $\boxed{\alpha_{ideal}} = -0.0675\text{ rad} = \boxed{-3.9^\circ}$

c) $$\frac{u(x=0)}{U} = -\frac{\gamma(x=0)}{2U} = a_0\left(\frac{1+\cos\tilde{x}}{\sin\tilde{x}}\right) + \sum_{n=1}^{\infty} a_n\sin(n\tilde{x})\Big|_{\tilde{x}=\pi/2} = a_0 + a_1 + \sum_{n=3}^{\infty}\overbrace{a_n\sin(n\tilde{x})}^{0} \quad \cancel{a_0 + \sum_{n=1}^{\infty} a_n}$$

$$a_2 = \frac{2}{\pi}\int_0^{\pi}\Big(0.225\underbrace{\cos^2\tilde{x}\cos(2\tilde{x})}_{\int\Rightarrow \pi/4} + 0.12\underbrace{\cos\tilde{x}\cos 2\tilde{x}}_{\int\Rightarrow 0} - 0.18\underbrace{\cos 2\tilde{x}}_{\int\Rightarrow 0}\Big)d\tilde{x} = 0.1125$$

all other a_n's are zero (see http://integrals.wolfram.com)

$$\boxed{\frac{u}{U}} = a_0 + a_1 + \cancel{a_2/2} = \boxed{\cancel{0.33}} \quad \boxed{0.22}$$

this equation uses $x=0$ at LE

d) $$\frac{q}{U} = \left(1 + \frac{u_t}{U} + \frac{u_c}{U}\right)\sqrt{\frac{x}{x + r_L/2}} + (\alpha - \alpha_{ideal})\sqrt{\frac{c-x}{x + r_L/2}} \quad @\ x = -c/2,\ \tilde{x} = 0$$

$\frac{u_t}{U} = \frac{t_0}{c} = 0.02$, $\frac{u_c}{U}(x=-c/2) = a_0 = 0.1$, $r_L/c = 0.5(0.02^2) = 0.0002$

$$\cancel{\frac{q}{U}(x=-c/2) = (1 + 0.02 + 0.1)\cdot\sqrt{\frac{-\frac{1}{2}c}{-\frac{1}{2}c + 0.0001c}} + (0.1)\sqrt{\frac{3/2\,c}{-c/2}}}$$

$$\boxed{\frac{q}{U}(x=0)} = (\alpha - \alpha_{ideal})\sqrt{\frac{2}{r_L/c}} = (0.1)\sqrt{\frac{2}{0.0002}} = \boxed{\cancel{0.00}} \quad \boxed{10}$$

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4) 2D, parabolic meanline $f_o/c = 0.07$

$\alpha = 3^\circ = 0.0524$ rad

elliptical thickness $t_o/c = 0.04$, $r_L/c = \frac{1}{2}(0.04)^2 = 0.0008$

a) $a_0 = \alpha = 0.0524$

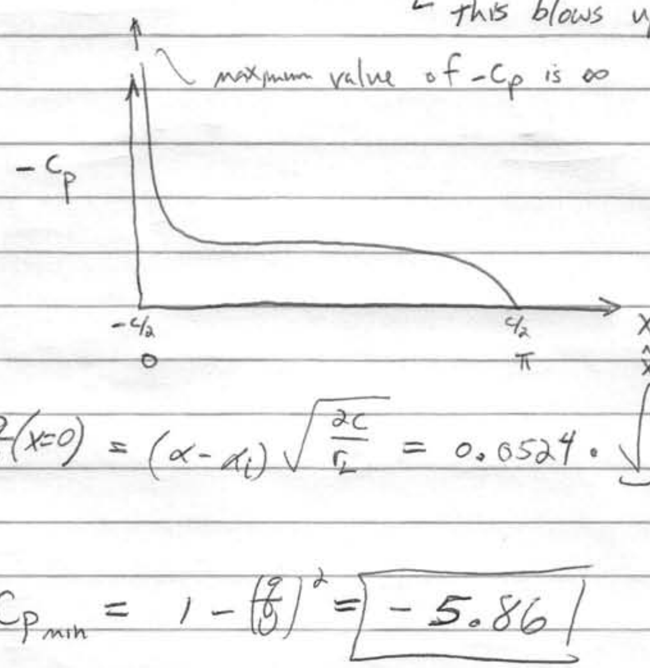
$a_1 = 4 f_o/c = 0.28 \quad \Rightarrow \quad \boxed{C_L} = 2\pi a_0 + \pi a_1 = \boxed{1.21}$

$\frac{x}{c} = \frac{1}{4} = -\frac{1}{2}\cos\tilde{x} \rightarrow \tilde{x} = 120^\circ = \frac{2}{3}\pi \qquad \cos\tilde{x} = -\frac{1}{2}, \ \sin\tilde{x} = \frac{\sqrt{3}}{2}$

$$\boxed{\gamma\left(\tfrac{x}{c} = \tfrac{1}{4}\right)} = -2U \cdot 0.0524 \left(\frac{1 - \frac{1}{2}}{\sqrt{3}/2}\right) - 8U \cdot 0.07 \cdot \frac{\sqrt{3}}{2} = \boxed{0.55\,U}$$

b) $C_p = \frac{-\gamma}{U} = 2\alpha\left(\frac{1+\cos\tilde{x}}{\sin\tilde{x}}\right) + 8\frac{f_o}{c}\sin\tilde{x}$

this blows up at $\tilde{x} = 0$ $(x = -c/2)$



$\Rightarrow$ NEED to use Lighthill correction at the leading edge.

$$\frac{q}{U}(x=0) = (\alpha - \alpha_i)\sqrt{\frac{2c}{r_L}} = 0.0524 \cdot \underbrace{\sqrt{\frac{2}{0.0008}}}_{=50} = 2.62$$

$$C_{p_{min}} = 1 - \left(\frac{q}{U}\right)^2 = \boxed{-5.86}$$