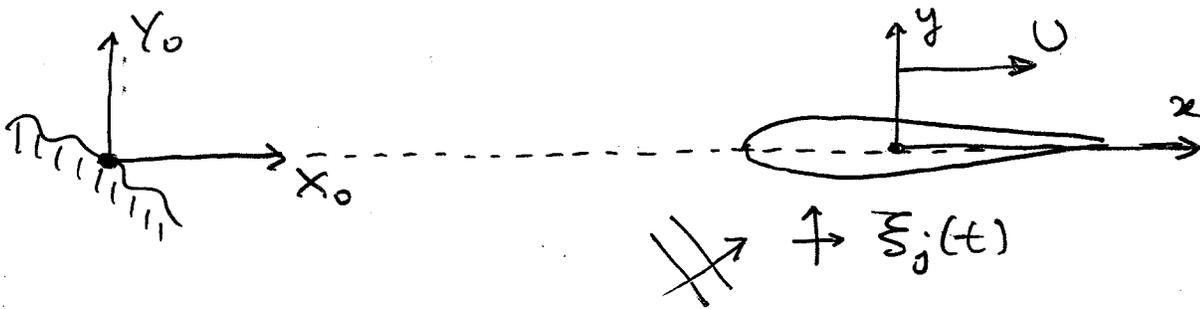


# NEUMANN-KELVIN LINEARIZATION OF $U > 0$ SHIP SEAKEEPING PROBLEM



- LET  $\Phi(x_0, y_0, z_0, t)$  BE THE TOTAL POTENTIAL RELATIVE TO THE INERTIAL COORDINATE SYSTEM

$$x_0 = x + Ut$$

- LET  $\phi(x, y, z, t)$  BE THE SAME POTENTIAL EXPRESSED RELATIVE TO THE TRANSLATING FRAME. IT WAS SHOWN BEFORE THAT

$$\frac{d\Phi}{dt} = \left( \frac{\partial}{\partial t} - U \frac{\partial}{\partial x} \right) \phi(\vec{x}, t)$$

WHERE NOW THE TIME DEPENDENCE OF  $\phi$  WRT TIME WILL BE OF THE  $e^{i\omega t}$  FORM IN THE SHIP SEAKEEPING PROBLEM. —

- THE TOTAL POTENTIAL  $\bar{\Phi}$  CONSISTS OF THE SUM OF TWO COMPONENTS IN A LINEARIZED SETTING

$$\bar{\Phi}_{\text{TOT}} = \bar{\bar{\Phi}} + \bar{\Phi}$$

WHERE  $\bar{\bar{\Phi}}$  IS THE VELOCITY POTENTIAL DUE TO THE VESSEL FORWARD TRANSLATION WITH CONSTANT SPEED  $U$  AND  $\bar{\Phi}$  IS THE SEAKEEPING COMPONENT DUE TO VESSEL MOTIONS IN WAVES.

- RELATIVE TO THE SHIP FRAME:

$$\bar{\bar{\Phi}} = \bar{\phi}(x, y, z)$$

$$\bar{\Phi} = \phi(x, y, z, t) = \text{Re} \{ \varphi(x, y, z) e^{i\omega t} \}$$

WHERE  $\omega$  IS THE ENCOUNTER FREQUENCY AND

$$\varphi = \varphi_I + \sum_{j=1}^7 \varphi_j$$

WITH  $\varphi_j$ ,  $j=1, \dots, 6$  BEING THE RADIATION AND  $\varphi_7$  BEING THE DIFFRACTION POTENTIALS.

## BOUNDARY-VALUE PROBLEM FOR $\bar{\phi}$

- FREE SURFACE CONDITION:

$$U^2 \bar{\phi}_{xx} + g \bar{\phi}_z = 0, \quad z=0$$

- SHIP-HULL CONDITION:

$$\vec{n} \cdot \nabla \bar{\phi} = \vec{n} \cdot \vec{U} = U \eta_1$$

WHERE  $\vec{n} = (\eta_1, \eta_2, \eta_3)$  IS THE UNIT VECTOR POINTING INSIDE THE SHIP HULL

- FAR FROM THE SHIP  $\bar{\phi}$  REPRESENTS OUTGOING WAVES WHICH ARE KNOWN AS THE KELVIN SHIP WAKE STUDIED EARLIER
- THE SOLUTION FOR  $\bar{\phi}$  BY THE ABOVE FORMULATION KNOWN AS THE NEUNANN-KELVIN PROBLEM AND ITS GENERALIZATIONS DISCUSSED IN THE LITERATURE IS CARRIED OUT BY PANEL METHODS.
- THE PRINCIPAL OUTPUT QUANTITIES OF INTEREST IN PRACTICE ARE :

- FREE-SURFACE ELEVATION

$$\bar{\zeta} = -\frac{1}{g} \left. \frac{d\bar{\Phi}}{dt} \right|_{\text{INERTIAL FRAME}} = \frac{U}{g} \left. \bar{\Phi}_x \right|_{\text{SHIP FRAME}}, z=0$$

- HYDRODYNAMIC PRESSURE (LINEAR)

$$p = -\rho \left. \frac{d\bar{\Phi}}{dt} \right|_{\text{INERTIAL FRAME}} = \rho U \left. \bar{\Phi}_x \right|_{\text{SHIP FRAME}}$$

- HYDRODYNAMIC PRESSURE (TOTAL)

$$\begin{aligned} p_T &= -\rho \left( \left. \frac{d\bar{\Phi}}{dt} + \frac{1}{2} \nabla \bar{\Phi} \cdot \nabla \bar{\Phi} + gz \right) \right|_{\text{INERTIAL FRAME}} \\ &= -\rho \left( -U \bar{\Phi}_x + \frac{1}{2} \nabla \bar{\Phi} \cdot \nabla \bar{\Phi} + gz \right) \Big|_{\text{SHIP FRAME}} \end{aligned}$$

IF  $\bar{S}_w$  IS THE SHIP WETTED SURFACE DUE TO ITS STEADY FORWARD TRANSLATION ON A FREE SURFACE AND  $\vec{n}$  IS THE UNIT NORMAL VECTOR POINTING OUT OF THE FLUID DOMAIN THE SHIP IDEAL-FLUID FORCE IS GIVEN BY

$$\vec{F} = \iint_{\bar{S}_w} p_T \vec{n} ds$$

THE WAVE RESISTANCE IS:  $R_w = \vec{\tau} \cdot \vec{F}$

- WE WILL DERIVE BOUNDARY VALUE PROBLEMS FOR THE POTENTIALS  $\bar{\phi}$  AND  $\phi$  RELATIVE TO THE SHIP FIXED FRAME
- THE PRINCIPAL ASSUMPTION UNDERLYING THE ENSUING DERIVATION IS THAT THE SHIP IS SLENDER, THIN OR FLAT OR IN GENERAL STREAMLINED IN THE LONGITUDINAL DIRECTION  
 MORE EXPLICITLY, IF  $B$  IS THE SHIP BEAM,  $T$  ITS DRAFT AND  $L$  ITS LENGTH WE WILL ASSUME THAT:

$$\frac{B}{L}, \frac{T}{L} = O(\varepsilon), \quad \varepsilon \ll 1$$

WHERE  $\varepsilon$  IS THE SLENDERNESS PARAMETER ASSUMED TO BE SMALL COMPARED TO 1.

- THE SHIP SLENDERNESS JUSTIFIES THE USE OF THE LINEAR FREE-SURFACE CONDITION IN THE FORWARD-SPEED PROBLEM FOR A BROAD RANGE OF SPEEDS AND HULL SHAPES

# BOUNDARY-VALUE PROBLEM FOR TIME-HARMONIC VELOCITY POTENTIAL

FROM THE GALILEAN TRANSFORMATION :

$$\left( \frac{\partial}{\partial t} - U \frac{\partial}{\partial x} \right)^2 \phi + g \phi_z = 0, \quad z = 0$$

RELATIVE TO THE SHIP FRAME. IN TERMS OF THE COMPLEX POTENTIAL :

$$\phi = \text{Re} \{ \psi e^{i\omega t} \}$$

$$\left( i\omega - U \frac{\partial}{\partial x} \right)^2 \psi + g \psi_z = 0, \quad z = 0$$

WHERE  $\omega$  IS THE ENCOUNTER FREQUENCY AND  $\psi$  IS ANY OF THE  $\psi_j$  POTENTIALS.

- THE ABOVE TIME-HARMONIC NEUMANN-KELVIN FREE SURFACE CONDITION IS BEING TREATED BY STATE-OF-THE-ART PANEL METHODS. AN IMPORTANT SIMPLIFICATION FOR SLENDER SHIPS AND LARGE VALUES OF  $\omega$  WILL LEAD TO THE POPULAR STRIP THEORY.

- THE SOLUTION FOR  $\bar{\phi}$  IS FAR FROM SIMPLE NUMERICALLY. A LOT OF RESEARCH HAS BEEN DEVOTED TO THIS EFFORT, IN PARTICULAR TOWARDS THE EVALUATION OF THE SHIP KELVIN WAKE AND THE SHIP WAVE RESISTANCE
- THE LINEARIZATION OF THE PRESSURE AND VESSEL WETTED SURFACE  $\bar{S}_w$  ABOUT ITS STATIC VALUE IN CALM WATER MUST BE CARRIED OUT CAREFULLY! NONLINEAR EFFECTS ARE KNOWN TO CONTRIBUTE APPRECIABLY TO THE WAVE RESISTANCE
- IF AVAILABLE, A FULLY NONLINEAR SOLUTION OF THE FORWARD-SPEED STEADY SHIP WAVE PROBLEM IS PREFERABLE. NUMERICAL ISSUES MUST BE CAREFULLY TREATED AND ARE THE SUBJECT OF STATE-OF-THE-ART RESEARCH
- COUPLING WITH VISCOUS EFFECTS IS OFTEN STRONG AND IMPORTANT FOR PREDICTING THE TOTAL RESISTANCE OF THE SHIP. —

RELATIVE TO THE SHIP-FIXED COORDINATE SYSTEM THE AMBIENT WAVE ELEVATION OSCILLATES WITH FREQUENCY  $\omega$ .

PROOF:

$$\zeta = -\frac{1}{g} \left( \frac{\partial}{\partial t} - U \frac{\partial}{\partial x} \right) \phi_I$$

WHERE:

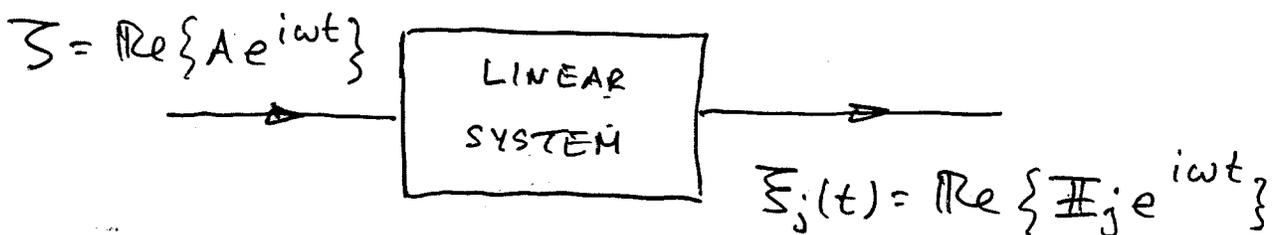
$$\phi_I = \operatorname{Re} \left\{ \frac{igA}{\omega_0} e^{kz - ikx \cos\beta - ik y \sin\beta + i\omega t} \right\}$$

$$\frac{\partial}{\partial t} = i\omega, \quad \frac{\partial}{\partial x} = -ik \cos\beta$$

$$\frac{\partial}{\partial t} - U \frac{\partial}{\partial x} = i(\omega + kU \cos\beta) = i\omega_0$$

THUS:

$$\begin{aligned} \zeta &= \operatorname{Re} \left\{ A e^{-ikx \cos\beta - ik y \sin\beta + i\omega t} \right\} \\ &= \operatorname{Re} \left\{ A e^{i\omega t} \right\} \quad x=y=0. \end{aligned}$$



WHERE  $\Xi_j(\omega)$  IS THE COMPLEX AMPLITUDE OF THE VESSEL MOTION IN MODE- $j$ , A FUNCTION OF THE FREQUENCY OF ENCOUNTER  $\omega$ , KNOWN AS THE RESPONSE AMPLITUDE OPERATOR, RAO.

THE SHIP EQUATIONS OF MOTION FOLLOW AS IN THE  $U=0$  CASE USING LINEAR SYSTEM THEORY:

$$\sum_{j=1}^6 [-\omega^2 (M_{ij} + A_{ij}) + i\omega B_{ij} + C_{ij}] \Xi_j(\omega) = X_i(\omega), \quad i=1, \dots, 6$$

WHERE THE HYDRODYNAMIC COEFFICIENTS  $A_{ij}(\omega)$ ,  $B_{ij}(\omega)$  AND EXCITING FORCES ARE NOW FUNCTIONS OF THE ENCOUNTER FREQUENCY  $\omega$  AND OTHER FORWARD-SPEED EFFECTS.

- INERTIA & HYDROSTATIC MATRICES THE SAME AS IN THE ZERO-SPEED CASE
- WILL DERIVE BVP'S GOVERNING THE COEFFICIENTS  $A_{ij}(\omega)$ ,  $B_{ij}(\omega)$  AND EXCITING FORCES  $X_i(\omega)$ . —

EXPLICITLY:

$$\omega = \omega_0 - U \frac{\omega_0^2}{g} \cos \beta$$

- $\omega \gtrless 0$  : BOTH POSITIVE AND NEGATIVE VALUES OF  $\omega$  ARE POSSIBLE. IN PRACTICE WILL ALWAYS DEAL WITH THE ABSOLUTE VALUE OF  $\omega$ .
- GIVEN THE ABSOLUTE WAVE FREQUENCY  $\omega_0 > 0$  THERE EXISTS A UNIQUE  $\omega$
- GIVEN A POSITIVE ABSOLUTE ENCOUNTER FREQUENCY  $|\omega|$ , THERE EXIST POSSIBLY MULTIPLE  $\omega_0$ 'S SATISFYING THE ABOVE RELATION. MORE DISCUSSION OF THIS TOPIC WILL FOLLOW
- ASSUMING SMALL AMPLITUDE MOTIONS THE SHIP RESPONSES ARE MODELED AFTER LINEAR SYSTEM THEORY. INPUT SIGNAL  $\sim e^{i\omega t} \rightarrow$  OUTPUT SIGNAL  $\sim e^{i\omega t}$ . —

RELATIVE TO THE EARTH-FIXED FRAME  
THE AMBIENT WAVE VELOCITY POTENTIAL  
TAKES THE FORM:

$$\phi_I = \text{Re} \{ \psi_I \}$$

$$\psi_I = \frac{igA}{\omega_0} e^{kz_0 - ikx_0 \cos\beta -iky_0 \sin\beta + i\omega_0 t}$$

WHERE IN DEEP WATER:  $k = \omega_0^2/g$

LET:  $x_0 = x + Ut$

$$y_0 = y$$

$$z_0 = z$$

IT FOLLOWS THAT:

$$\psi_I = \frac{igA}{\omega_0} e^{kz - ikx \cos\beta -iky \sin\beta} e^{i(\omega_0 - Uk \cos\beta)t}$$

LET:

$$\omega \equiv \omega_0 - Uk \cos\beta$$

BE DEFINED TO BE THE ENCOUNTER FREQUENCY  
WHICH ACCOUNTS FOR THE DOPPLER EFFECT  
INCLUDED IN THE SECOND TERM IN THE RHS. -

## COMMENTS ON N-K FORMULATION:

- THE SHIP IS ASSUMED TO BE STREAMLINED IN ORDER TO JUSTIFY THE DECOMPOSITION OF THE STEADY & TIME HARMONIC COMPONENTS
- THE VESSEL MOTIONS ARE ASSUMED SMALL AND OF THE SAME ORDER AS THE AMBIENT WAVE AMPLITUDE. TERMS OMITTED ARE OF  $O(A^2)$ .
- WHEN TAYLOR EXPANDING THE FREE-SURFACE AND BODY-BOUNDARY CONDITION ABOUT  $z=0$  AND  $\bar{S}_B$  RESPECTIVELY, THE STEADY FLOW POTENTIAL  $\bar{\phi} \approx 0$ .
- FOR SHIPS WITH APPRECIABLE THICKNESS A BETTER APPROXIMATION FOR  $\bar{\phi}$  IS THAT OF THE DOUBLE-BODY FLOW DISTURBANCE SUCH THAT  $\bar{\phi}_z = 0$  ON  $z=0$  AND  $\bar{\phi}_n = U\eta_1$  ON  $\bar{S}_B$ . THIS LEADS TO THE STATE-OF-THE-ART LINEAR 3D STEADY FLOW AND SEAKEEPING FORMULATION DISCUSSED LATER IN CONNECTION WITH PANEL METHODS.
- THE N-K FORMULATION IS THE STARTING POINT OF STRIP THEORY →