

13.022

SURFACE WAVES AND THEIR INTERACTION WITH FLOATING BODIES

Quiz 3

Thursday, December 13th, 2001

1:30 Hours - Open Book

1. (50%) The energy loss due to wave breaking may be modeled in linear theory by enforcing the free surface condition

$$\varphi_{tt} + \mu\varphi_t + g\varphi_z = 0, \quad z = 0$$

where μ is a positive dissipation coefficient and the water depth is infinite.

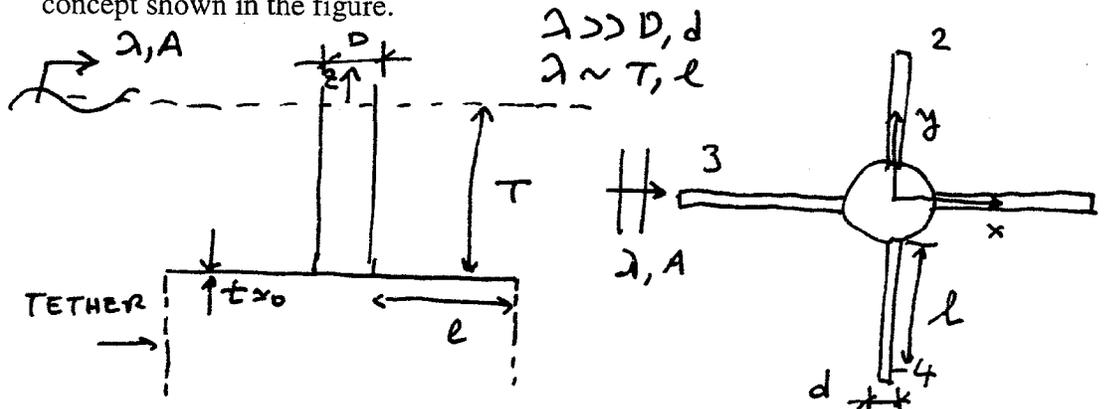
- a) Consider a plane progressive wave with known real frequency ω and possibly complex wavenumber k . Enforce the Laplace equation and the free surface condition define above, and determine the law that governs the rate of decay of the wave amplitude in the direction of the wave propagation due to wave breaking.
- b) Repeat your analysis in a) and b) in water of shallow depth H over a rigid seafloor, using the shallow water equations in the limit as $kH \rightarrow 0$. For the same value of the dissipation coefficient μ as in a), determine the range of frequencies ω for which the rate of energy dissipation in deep water is larger than that in shallow water.
- c) Consider the same wave propagating in water of finite depth H but over a muddy seafloor. Due to the seafloor permeability, the boundary condition that applies over the $z = -H$ plane takes the form

$$p + \lambda\varphi_z = 0, \quad z = -H$$

Where p is the dynamic pressure induced by the wave and λ is positive constant.

Derive the dispersion relation, namely the relation between the wave frequency ω and the wave number k . Comment qualitatively on how you would use it to derive the rate of decay of the wave amplitude due to the energy dissipation on the free surface and the seafloor.

2. (50%) An oil company is in the process of evaluating the design of a new mini-TLP concept shown in the figure.



Assume that both the vertical cylinder and the horizontal plates are slender and their dimensions are small compared to the wavelength λ of the ambient waves.

- Estimate the heave and pitch exciting force and moment amplitude on the platform due to regular waves of length λ propagating towards the platform as shown in the figure.
- Estimate the complex amplitude of the excitation force on tethers 1, 2, 3 and 4 assumed that they are in pre-tension and not allowed to oscillate in the vertical direction.
- Assuming that each tether can be modeled as an elastic spring with effective mass m and spring constant k , determine the natural frequency(ies) of oscillation of the TLP in the heave mode.