

ROLL-SWAY-YAW MOTIONS

IN OBLIQUE WAVES ($\beta \neq 180^\circ$; HEAD, $\neq 0^\circ$), FOLLOWING THE SHIP WILL ALSO OSCILLATE IN ROLL ($i=4$), SWAY ($i=2$) AND YAW ($i=6$) MODES. FOR SHIPS WHICH ARE SYMMETRIC PORT-STARBOARD WHEN CRUISING FORWARD, HEAVE-PITCH ARE NOT COUPLED TO ROLL-SWAY-YAW ACCORDING TO LINEAR THEORY

- AN IMPORTANT EXCEPTION ARE SAILING YACHTS SAILING TO WINDWARD AT A HEEL ANGLE
- THE ROLL-SWAY-YAW EQUATIONS OF MOTION IN THE FREQUENCY DOMAIN ARE OF THE FORM:

$$\left[-\omega^2 (M_{ij} + A_{ij}) + i\omega B_{ij} + C_{ij} \right] \underline{X}_j = \underline{X}_i$$

$$i = 2, 4, 6$$

- THE STRIP THEORY DERIVED ABOVE FOR HEAVE & PITCH EXTENDS TO ROLL-SWAY-YAW WITH VERY FEW MODIFICATIONS OTHER THAN THE DEFINITION OF THE RESPECTIVE MODE SHAPES
- THE SWAY & YAW MODES ARE NOT RESTORED HYDROSTATICALLY

- IN ORDER TO STUDY THE NONLINEARITY INTRODUCED BY VISCOUS EFFECTS, IT IS USEFUL TO ISOLATE THE ROLL EQUATION.
- IT TURNS OUT THAT IN SPITE OF THE SYMMETRY OF THE CROSS COUPLING COEFFICIENTS, THE EFFECT OF ROLL ON SWAY-YAW IS FAR WEAKER THAN THE EFFECT OF SWAY-YAW UPON ROLL.

SO A USEFUL AND ACCURATE DECOMPOSITION IS TO SOLVE SWAY-YAW IN THE FREQUENCY DOMAIN, IGNORING ROLL. THIS LEADS TO:

$$[-\omega^2 (A_{ij} + M_{ij}) + i\omega B_{ij}] \Xi_j = X_i, \quad i, j = 2, 6$$

- SOLVE FOR $\Xi_j(\omega)$ AND DEFINE:

$$\xi_j(t) = \text{Re} \{ \Xi_j(\omega) e^{i\omega t} \}; \quad j = 2, 6$$

- THEN INTRODUCE THE COUPLING FROM SWAY-YAW DEFINED ABOVE INTO THE ROLL EQUATION AS A SPURIOUS EXCITATION IN THE RIGHT-HAND SIDE. IT FOLLOWS THAT:

SO: $C_{ij} = 0, \quad i, j = 2, 6$

WITH THE COORDINATE SYSTEM PLACED ON THE SHIP CENTERPLANE ASSUMED TO BE A PLANE OF SYMMETRY:

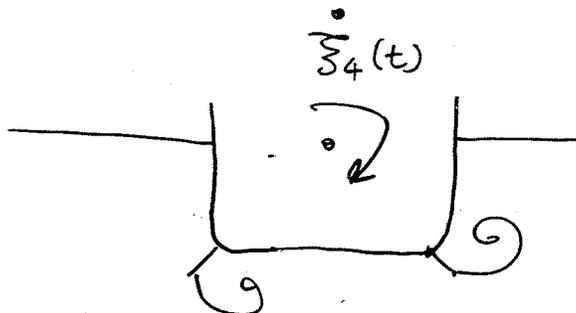
$$C_{ij} = 0, \quad i, j = 2, 4, 6$$

EXCEPT FOR:

$$C_{44} \neq 0$$

SO THE ONLY RESTORED MODE IS ROLL AND NO OTHER CROSS-COUPLING HYDROSTATIC COEFFICIENTS EXIST FOR SHIPS SYMMETRIC PORT STARBOARD.

- THE ROLLING MOTION IS WEAKLY DAMPED DUE TO WAVE EFFECTS AND AT THE ROLL RESONANCE IT IS ESSENTIAL TO INCLUDE VISCOUS OR LIFTING EFFECTS INTRODUCED BY THE FLOW SEPARATION AROUND THE HULL AND ITS BILGE KEELS:



$$\begin{aligned}
 & (\mathbf{I}_{44} + \mathbf{A}_{44}) \frac{d^2 \xi_4}{dt^2} + \mathbf{B}_{44} \frac{d \xi_4}{dt} + \mathbf{C} \left| \frac{d \xi_4}{dt} \right| \frac{d \xi_4}{dt} \\
 & + \mathbf{C}_{44} \xi_4 = X_4(t) - \sum_{j=2,6} [(\mathbf{M}_{4j} + \mathbf{A}_{4j}) \ddot{\xi}_j \\
 & \quad \quad \quad + \mathbf{B}_{4j} \dot{\xi}_j] \cdot - \\
 & \equiv X_4(t) + \tilde{X}_4(t)
 \end{aligned}$$

COMMENTS

- THE ABOVE EQUATION FOR THE ROLL MOTION INCLUDES A MORISON TYPE TERM THAT ACCOUNTS FOR VISCOUS SEPARATION EFFECTS
- THE COUPLING FROM SWAY-YAW \rightarrow ROLL IS INCLUDED AS A SPURIOUS EXCITATION TERM \tilde{X}_4 IN THE RIGHT-HAND SIDE, A FUNCTION OF THE A PRIORI DETERMINED SWAY-YAW MOTIONS
- THE ABOVE ROLL EQUATION IS STRICTLY INVALID IN THE TIME DOMAIN DUE TO MEMORY EFFECTS. YET IT IS OFTEN ACCURATE WHEN ROLL IS HIGHLY TUNED AROUND RESONANCE. OTHERWISE THE FULLY COUPLED SET OF EQUATIONS MUST BE SOLVED, AS IS THE CASE FOR HIGH-SPEED VESSELS WITH MOTION CONTROL APPENDAGES. —