2.43 ADVANCED THERMODYNAMICS

Spring Term 2024
LECTURE 10

Room 3-442
Friday, March 8, 11:00am - 1:00pm

Instructor: Gian Paolo Beretta
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Room 3-351d

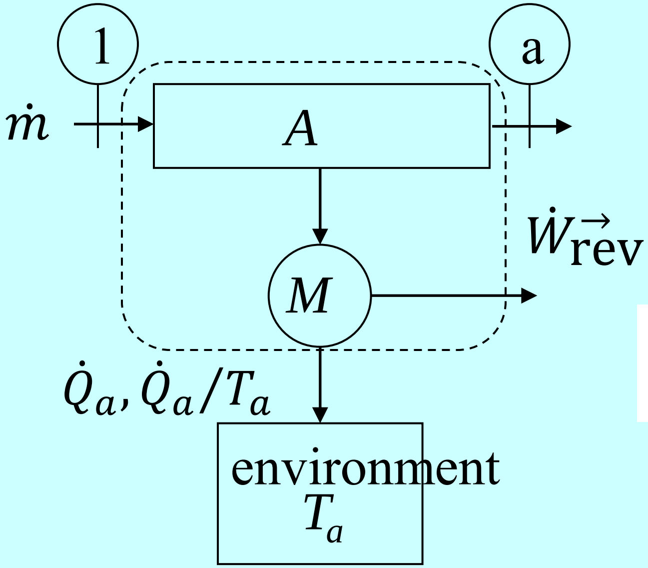
Review of basic concepts:

Exergies
and
first and second law efficiencies

in
energy conversion technologies

Review of basic concepts: Exergy of bulk flow interactions

Exergy associated with an interaction of mass flow in conditions 1



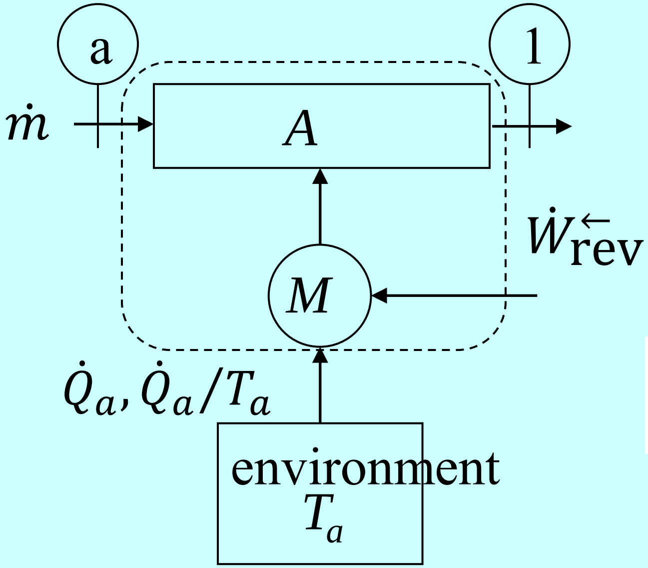
- To obtain the maximum power from the flow in conditions 1, the flow must be taken in a position 'a' in mutual equilibrium with the environment

$$\dot{W}^{\rightarrow}_{\text{rev}} = \dot{E}x_1 = \dot{m}[(h_1 - h_a) - T_a(s_1 - s_a)]$$

From the balance of energy and entropy (per unit time) for AM, the optimal equivalent mechanical power of flow in conditions 1 can be determined; which is therefore the exergy per unit time associated with the flow in conditions 1

Review of basic concepts: Exergy of bulk flow interactions

Exergy associated with an interaction of mass flow in conditions 1



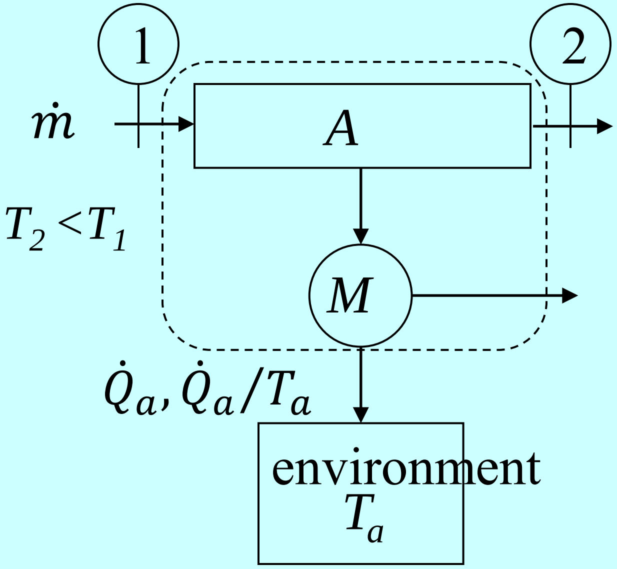
- The lowest power that is necessary to use in order to produce the flow conditions in 1 starting from the condition 'a' in mutual equilibrium with the environment

$$\dot{W}^{\leftarrow}_{\text{rev}} = \dot{E}x_1 = \dot{m}[(h_1 - h_a) - T_a(s_1 - s_a)]$$

From energy and entropy balances (per unit time) for AM the optimal equivalent mechanical power of flow in conditions 1 can be determined, which is therefore the exergy per unit time associated with the flow in conditions 1

Review of basic concepts: Exergy in heating and cooling bulk flows

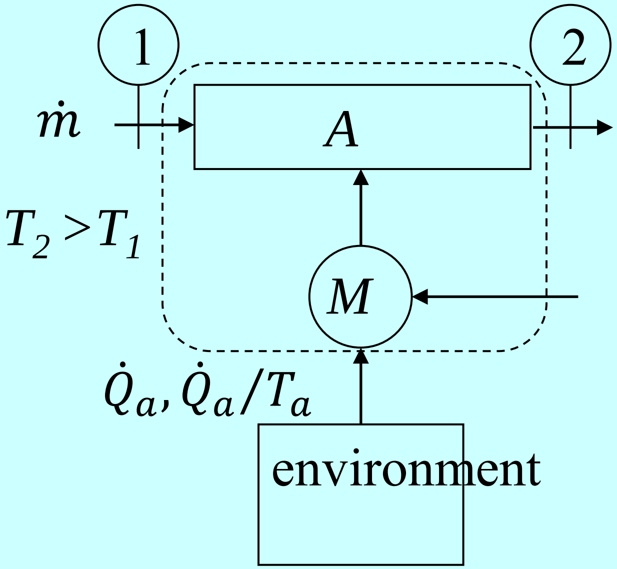
Variation of exergy in heating / cooling a flow



Cooling. From energy and entropy balances (per unit time) for AM; the following is found

$$\dot W_{\text{rev}}^{\rightarrow} = \dot E x_1 - \dot E x_2 = \dot m[(h_1 - h_2) - T_a(s_1 - s_2)]$$

$$\dot W_{\text{rev}}^{\rightarrow} = \dot E x_1 - \dot E x_2$$
$$= \dot m(h_1 - h_2)\left(1 - T_a \frac{s_1 - s_2}{h_1 - h_2}\right)$$

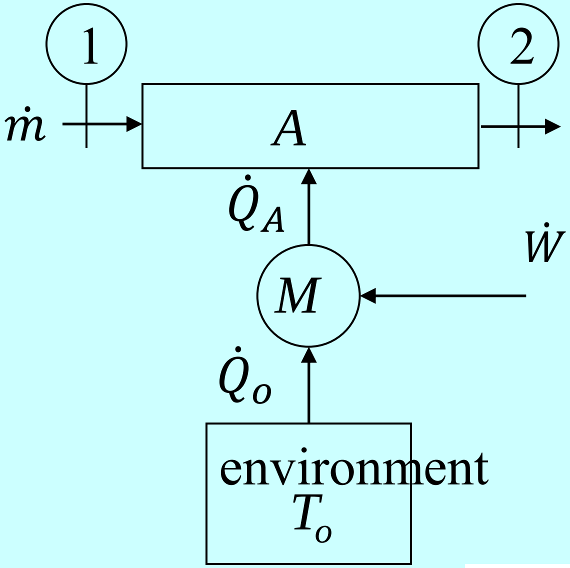


Heating. From the balance of energy and entropy (per unit time) for AM; the following is found:

$$\dot W_{\text{rev}}^{\leftarrow} = \dot E x_2 - \dot E x_1 = \dot m[(h_2 - h_1) - T_a(s_2 - s_1)]$$

$$\dot W_{\text{rev}}^{\leftarrow} = \dot E x_2 - \dot E x_1$$
$$= \dot m(h_2 - h_1)\left(1 - T_a \frac{s_2 - s_1}{h_2 - h_1}\right)$$

Review of basic concepts: Exergy in heating and cooling bulk flows



Heating of a flow with a heat pump

$$\dot{Q}_A = \dot{m}(h_2 - h_1)$$

$$\eta_{\mathrm{I}} = \frac{\dot{m}(h_2 - h_1)}{\dot{W}} = \frac{\dot{Q}_A}{\dot{W}}$$

$$\eta_{\mathrm{II}} = \frac{\dot{m}[(h_2 - h_1) - T_o(s_2 - s_1)]}{\dot{W}} = \frac{\dot{Q}_A\left[1 - T_o\,\frac{s_2 - s_1}{h_2 - h_1}\right]}{\dot{W}}$$

Warning: the use of symbol Q in this case could be misleading, for it is used to represent the energy supplied to the flow, while the interaction is not heat, but a set of continuous heat interactions at temperatures between T_1 and T_2. The entropy supplied to the flow is equal to

$$\frac{\dot{Q}_A}{T_{\mathrm{lm}1\,2}} \quad \text{where } T_{\mathrm{lm}1\,2} = \frac{h_2 - h_1}{s_2 - s_1}$$

Review of basic concepts: Exergy of an hydraulic jump

Energy balance

$$0 = \dot m(h_1 - h_2) + \dot m\left(\cancel{w_1^2/2 - w_2^2/2}\right) + \dot m(gz_1 - gz_2) - \dot W^{\rightarrow}$$

$$\dot m c(T_2 - T_1) = \dot m g(z_1 - z_2) - \dot W^{\rightarrow}$$

Cascata delle Marmore, Italy, 165 m

If the jump is not exploited (no work)

$$c(T_2 - T_1) = g(z_1 - z_2) \quad \rightarrow \quad \frac{\Delta T}{\Delta z} = \frac{g}{c} = \frac{9.8}{4200} \approx \frac{1}{430}\,\frac{°\mathrm{C}}{\mathrm{m}}$$

Entropy balance

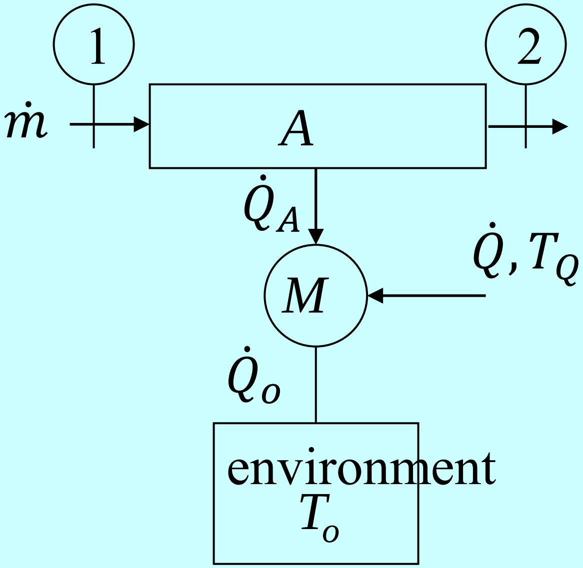
$$0 = \dot m(s_1 - s_2) + \dot S_{\mathrm{irr}}$$

$$\dot S_{\mathrm{irr}} = \dot m c \ln\frac{T_2}{T_1} = \dot m c \ln\left(1 + \frac{T_2 - T_1}{T_1}\right) \approx \dot m c\,\frac{T_2 - T_1}{T_1} = \frac{\dot m g(z_1 - z_2) - \dot W^{\rightarrow}}{T_1}$$

If the jump is exploited in best way (maximum work)

$$\dot W^{\rightarrow}_{\mathrm{max}} = \dot m g(z_1 - z_2) \quad \rightarrow \quad \frac{\dot W^{\rightarrow}_{\mathrm{max}} / \dot m}{z_1 - z_2} = g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = 9.8\,\frac{\mathrm{kJ/ton}}{\mathrm{m}} = \frac{42\ \mathrm{MJ/ton}}{4300\ \mathrm{m}}$$

Review of basic concepts: **Exergy in heating and cooling bulk flows**



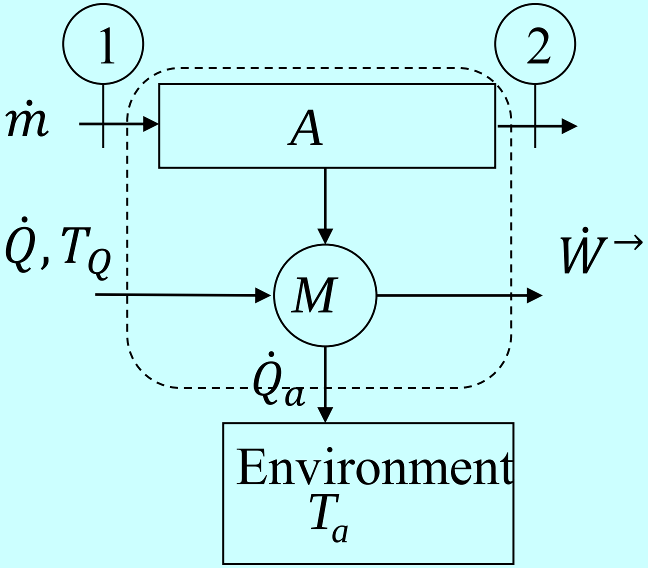
Cooling of a fluid using heat at a temperature different from the environment (refrigeration or absorption machine)

$$\dot{Q}_A = \dot{m}(h_1 - h_2)$$

$$\eta_{\mathrm{I}} = \frac{\dot{m}(h_1 - h_2)}{\dot{Q}} = \frac{\dot{Q}_A}{\dot{Q}}$$

$$\eta_{\mathrm{II}} = \frac{\dot{m}[(h_1 - h_2) - T_o(s_1 - s_2)]}{\dot{Q}\left(1 - \frac{T_o}{T_Q}\right)} = \frac{\dot{Q}_A\left[1 - T_o\,\frac{s_1 - s_2}{h_1 - h_2}\right]}{\dot{Q}\left(1 - \frac{T_o}{T_Q}\right)}$$

Review of basic concepts: Exergy in heating and cooling bulk flows



Again the balances for the composite system AM

$$\begin{cases} 0 = \dot m(h_1 - h_2) + \dot Q - \dot Q_a - \dot W^{\rightarrow} & (1) \\ 0 = \dot m(s_1 - s_2) + \dfrac{\dot Q}{T_Q} - \dfrac{\dot Q_a}{T_a} + \dot S_{\text{irr}} & (2) \end{cases}$$

(1)-T_a (2):

$$\dot W^{\rightarrow} = \dot m(h_1 - h_2)\left(1 - T_a \frac{s_1 - s_2}{h_1 - h_2}\right) + \dot Q\left(1 - \frac{T_a}{T_Q}\right) - T_a \dot S_{\text{irr}}$$

$$\dot W^{\rightarrow}_{\text{rev }12} = \dot E x_1 - \dot E x_2$$

exergy reduction of the flow between inlet and outlet

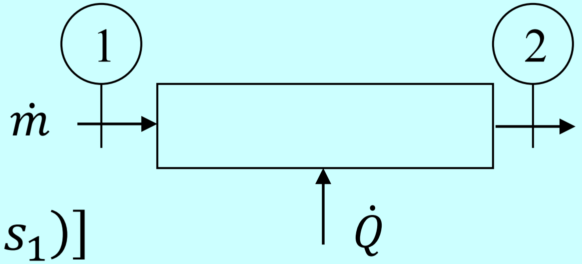
$$\dot W^{\rightarrow}_{\text{rev }\dot{\text{Q}}} = \dot E x_{\dot Q}$$

exergy provided by the heat interaction

exergy destroyed due to irreversibility

Review of basic concepts: Temperature of the equivalent heat interaction in heating / cooling for a **liquid** flow

The ratio between exergy and the energy content in heating / cooling of a liquid flow



$$\dot E x_1 = \dot m[(h_1 - h_a) - T_a(s_1 - s_a)]$$
$$\dot E x_2 = \dot m[(h_2 - h_a) - T_a(s_2 - s_a)]$$
$$\Delta \dot E x_{12} = \dot E x_2 - \dot E x_1 = \dot m[(h_2 - h_1) - T_a(s_2 - s_1)]$$
$$= \dot m[c(T_2 - T_1) + (p_2 - p_1)/\rho - T_a c \ln(T_2/T_1)]$$
$$= \dot m c(T_2 - T_1)\left[1 - \frac{T_a \ln(T_2/T_1)}{T_2 - T_1}\right] - \frac{\dot m}{\rho}(p_1 - p_2)$$
$$\Delta \dot E x_{12} \approx \dot Q\left[1 - \frac{T_a \ln(T_2/T_1)}{T_2 - T_1}\right]$$
$$\dot Q\left[1 - \frac{T_a}{T_Q}\right]$$

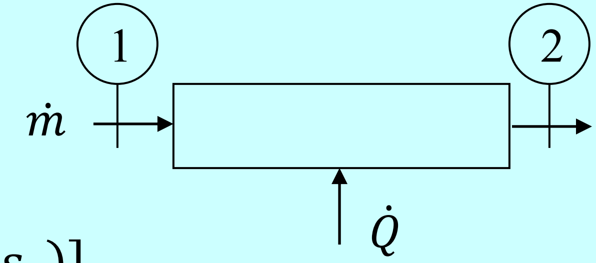
where

$$T_Q = T_{\mathrm{lm}12} = \frac{(T_2 - T_1)}{\ln(T_2/T_1)}$$

Exergy loss due to pressure drop in the duct. It is equal to the minimum pumping work. It is generally negligible with respect to thermal term.

Review of basic concepts: Temperature of the equivalent heat interaction in heating / cooling for an ideal gas flow

The ratio between exergy and the energy content in heating / cooling of a liquid flow



$$\dot{E}x_1 = \dot{m}[(h_1 - h_a) - T_a(s_1 - s_a)]$$
$$\dot{E}x_2 = \dot{m}[(h_2 - h_a) - T_a(s_2 - s_a)]$$
$$\Delta\dot{E}x_{12} = \dot{E}x_2 - \dot{E}x_1 = \dot{m}[(h_2 - h_1) - T_a(s_2 - s_1)]$$
$$= \dot{m}\left[c_p(T_2 - T_1) - T_a c_p \ln(T_2/T_1) + T_a\, R \ln(p_2/p_1)\right]$$
$$= \dot{m}c_p(T_2 - T_1)\left[1 - \frac{T_a \ln(T_2/T_1)}{T_2 - T_1}\right] - T_a\, \dot{m}\, R \ln(p_1/p_2)$$
$$\Delta\dot{E}x_{12} \approx \dot{Q}\left[1 - \frac{T_a \ln(T_2/T_1)}{T_2 - T_1}\right]$$
$$\dot{Q}\left[1 - \frac{T_a}{T_Q}\right]$$

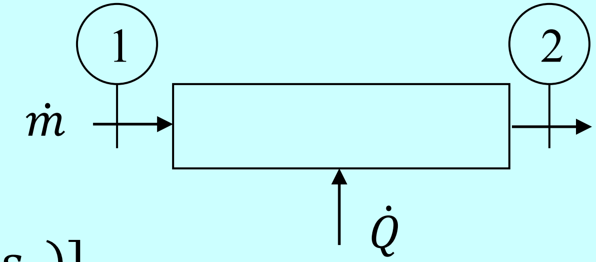
where

$$T_Q = T_{\text{lm}12} = \frac{(T_2 - T_1)}{\ln(T_2/T_1)}$$

Exergy loss due to pressure drop in the duct. It is equal to the minimum pumping work. It is often negligible with respect to thermal term.

Review of basic concepts: Temperature of the equivalent heat interaction in heating / cooling for a boiling or condensing flow

The ratio between exergy and the energy content in heating / cooling of a liquid flow



$$\dot{E}x_1 = \dot{m}[(h_1 - h_a) - T_a(s_1 - s_a)]$$
$$\dot{E}x_2 = \dot{m}[(h_2 - h_a) - T_a(s_2 - s_a)]$$
$$\Delta\dot{E}x_{12} = \dot{E}x_2 - \dot{E}x_1 = \dot{m}[(h_2 - h_1) - T_a(s_2 - s_1)]$$
$$= \dot{m}\left[h_{\mathrm{fg}}(x_2 - x_1) - T_a s_{\mathrm{fg}}(x_2 - x_1)\right]$$
$$= \dot{m}h_{\mathrm{fg}}(x_2 - x_1)\left[1 - \frac{T_a s_{\mathrm{fg}}}{h_{\mathrm{fg}}}\right]$$
$$\Delta\dot{E}x_{12} \approx \dot{Q}\left[1 - \frac{T_a s_{\mathrm{fg}}}{h_{\mathrm{fg}}}\right]$$
$$\dot{Q}\left[1 - \frac{T_a}{T_Q}\right]$$

Here we assumed negligible pressure drop.

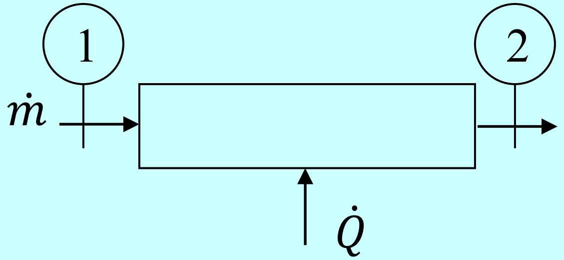
where

$$T_Q = \frac{h_{\mathrm{fg}}}{s_{\mathrm{fg}}}$$

Review of basic concepts: Minimum exergy for low temperature heating a liquid or ideal-gas flow

Ratio between exergy and energy content for a liquid flow:

$$\Delta \dot{E}x_{12} \approx \dot{Q}\left[1 - \frac{T_a \ln(T_2/T_1)}{T_2 - T_1}\right]$$



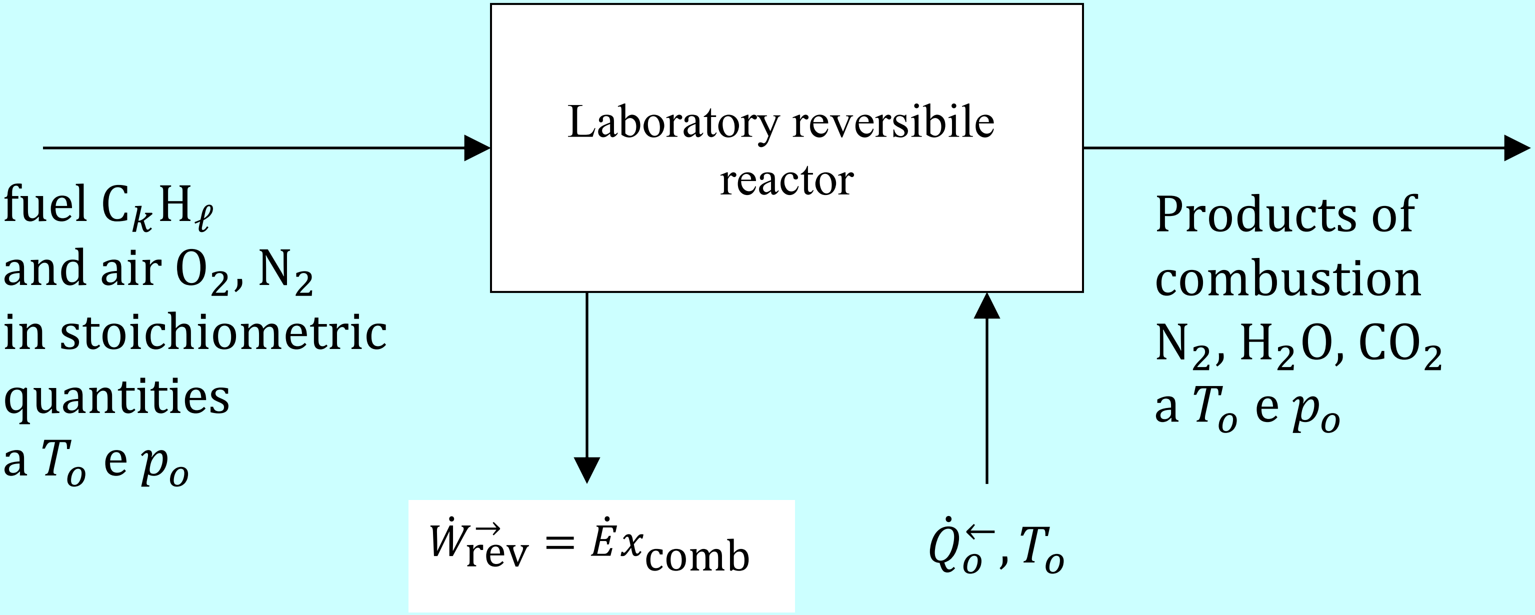
For example, for the heating of pressurized water from 60°C to 120°C (in an environment at 300 K)

$$\frac{\Delta \dot{E}x_{12}}{\dot{Q}} = 1 - \frac{T_a \ln(T_2/T_1)}{T_2 - T_1} = 1 - \frac{300 \ln(393/333)}{60} = 0.172$$

For the heating of water from 20°C to 60°C (in an environment at 300 K)

$$\frac{\Delta \dot{E}x_{12}}{\dot{Q}} = 1 - \frac{T_a \ln(T_2/T_1)}{T_2 - T_1} = 1 - \frac{300 \ln(333/293)}{40} = 0.040$$

Review of basic concepts: Exergy associated with a fossil fuel (definition)



Laboratory reversibile reactor

fuel C_kH_ℓ
and air O_2, N_2
in stoichiometric
quantities
a T_o e p_o

Products of
combustion
N_2, H_2O, CO_2
a T_o e p_o

$\dot{W}^{\rightarrow}_{\mathrm{rev}} = \dot{E}x_{\mathrm{comb}}$

$\dot{Q}^{\leftarrow}_{o}, T_o$

T_o and p_o standard: 25°C and 1 atm

Review of basic concepts: Exergy associated with a fossil fuel (in practice)

In the **second part of the course** we will prove that the exergy of a hydrocarbon $\mathrm{C}_k\mathrm{H}_\ell$ or coal, prior to its combustion, is within ±2.5% of the LHV of the fuel

$$\dot{E}x_{fuel} \approx -\dot{m}(\Delta h^o - T_o \Delta s^o)$$
$$\dot{Q}_{fuel} \approx -\dot{m}\Delta h^o = \dot{m}\,\mathrm{LHV}$$

For all hydrocarbons:

$$|T_o\Delta s^o/\Delta h^o| \leq 0.025$$

Where
Δh^o The enthalpy of combustion T_o and p_o
Δs^o The entropy of combustion T_o and p_o
$LHV = -\Delta h^o$ lower heating value
HHV is the higher heating value of the fuel

Therefore, in practice, we can use the approximation:

$$\dot{E}x_{\mathrm{fuel}} \approx \dot{Q}_{\mathrm{fuel}} \approx \dot{m}\,\mathrm{LHV}$$

$\mathrm{HHV} = \mathrm{LHV} + \ell\, h_{\mathrm{fg,water}}(T_o)$ for example for methane $\mathrm{LHV} = 50.06\,\frac{\mathrm{MJ}}{\mathrm{kg}}$ and $\mathrm{HHV} = 55.54\,\frac{\mathrm{MJ}}{\mathrm{kg}}$

ℓ is the number of hydrogens in the $\mathrm{C}_k\mathrm{H}_\ell$ molecule
for the details see chapter 31 of G&B, Thermodynamics, Dover 2005, Tab.31.7

Review of basic concepts: Lower heating values of some fuels

	kcal/kg	MJ/kg
Firewood	2500-4500	10-19
charcoal	7500	31
Peat	3000-4500	13-19
Lignite	4000-6200	17-26
Bitominous coal	6800-9000	28-38
Coke	7000	29
Anthracite	8000-8500	33-36
Crude oil	10000	42
Combustible oil	9800	41
Kerosene (Aircraft)	10400	44
Gasoline (automative)	10200	43
Petrol (automative)	10500	44
LPG (automative)	11000	46

	kcal/Nm^3	MJ/kg
Natural Gas	8300	47
Methane	8570	50
Water Gas from coke	2700	
Water Gas from carburized	6000	
Blast Furnace Gas	1000	
Gas from oil(cracking)	11500-17500	
	Nm^3 a 0°C	

42 MJ correspond to:

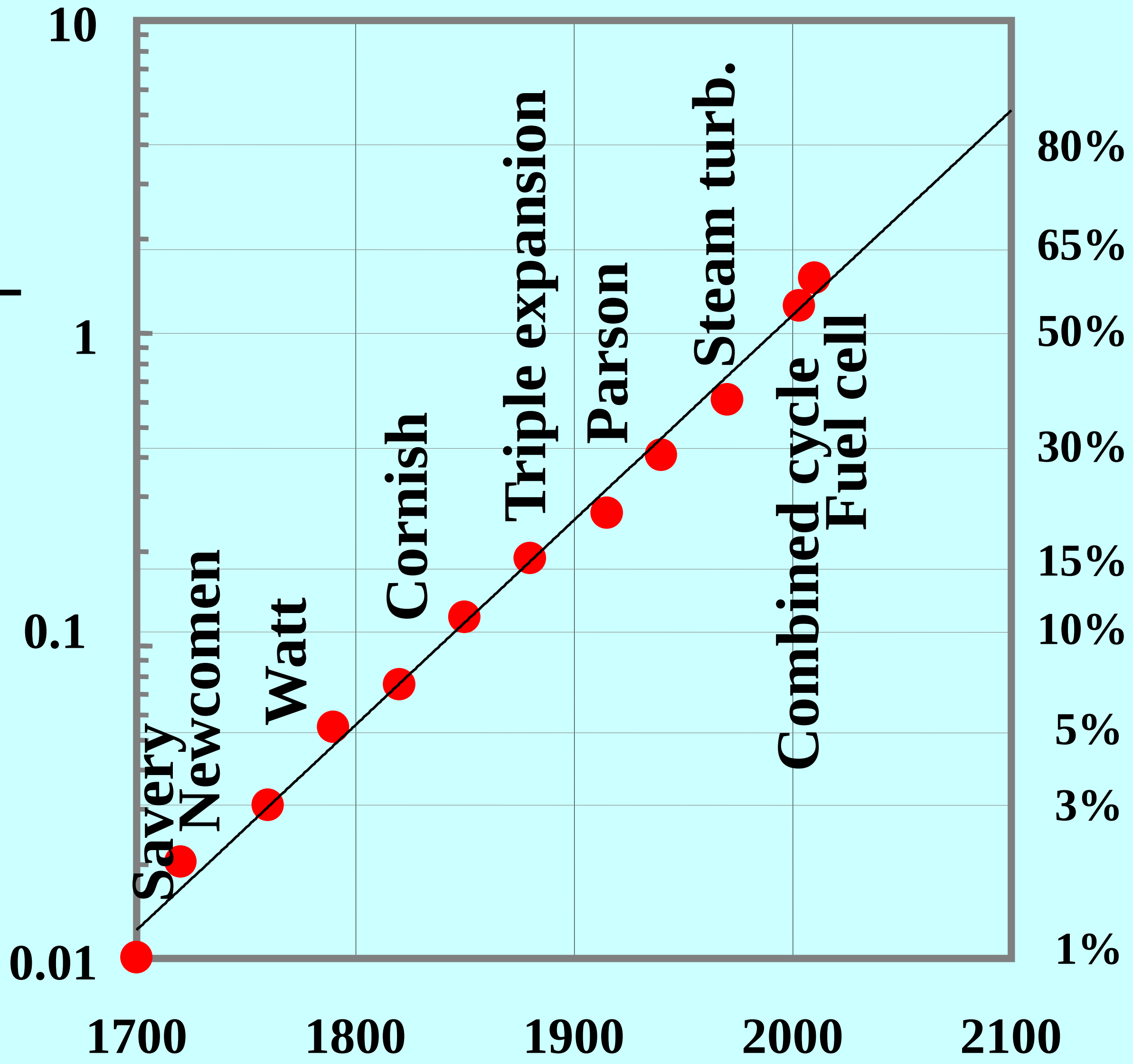
Lift 1 ton of water up to 4300m
Heat 1 ton of water of 10°C
Evaporate 18 kg of water at p_{atm}
Oxidizing 1 kg of oil
Fission of 0.5 mg of uranium-235

Review of basic concepts:

The learning curve of fuel-to-power conversion technologies

$$\frac{\eta_{II}}{1-\eta_{II}}$$

logistic scale
(linearity is typical of a learning process)

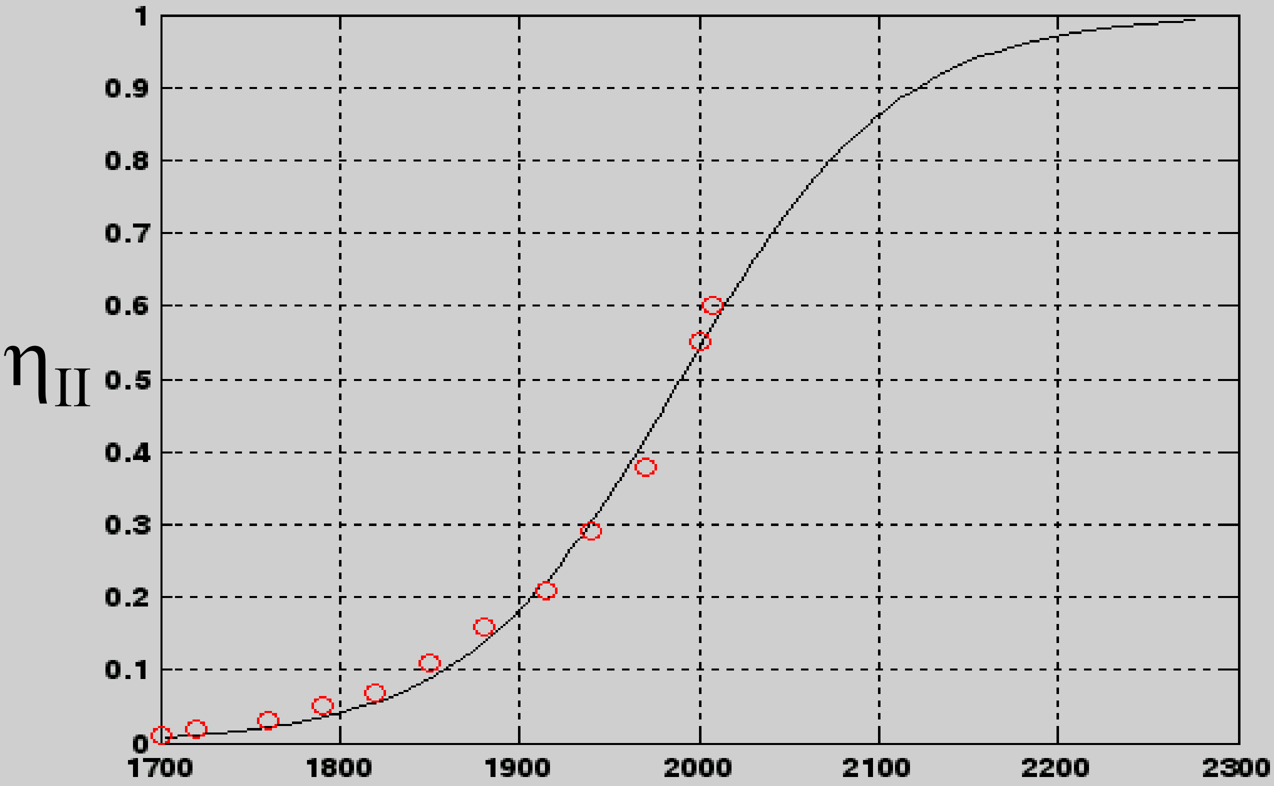


η_{II}

Thermodynamic efficiency of the best-available mature technology for primary energy conversion to work or electricity

Review of basic concepts:

The learning curve of fuel-to-power conversion technologies



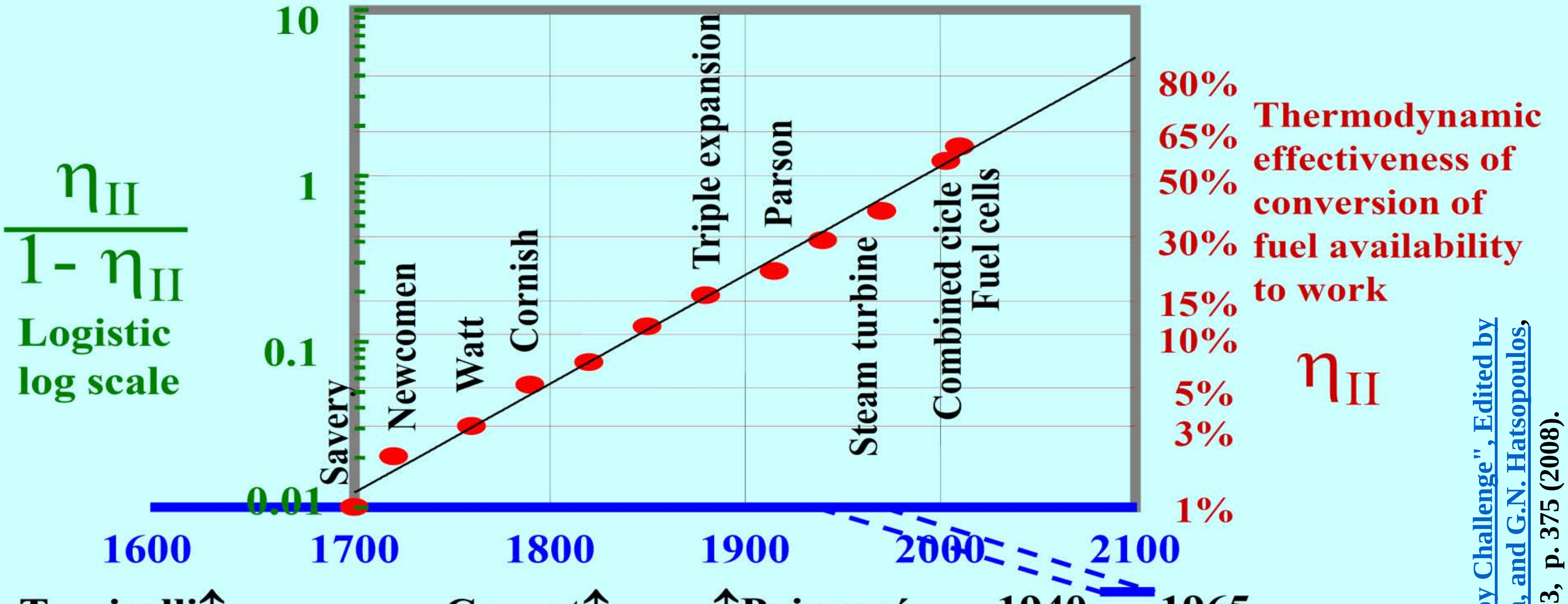
$$\frac{d\eta_{II}}{dt} = \frac{1}{\tau}\eta_{II}(1-\eta_{II}) \quad \text{with} \quad \tau \approx 60\,\text{yr}$$

η_{II}

Thermodynamic efficiency of the best-available mature technology for primary energy conversion to work or electricity

Review of basic concepts:

The learning curve of fuel-to-power conversion technologies



Torricelli↑
Boyle↑
Papin↑
Savery↑
Newcomen↑
Watt↑
Cornish↑

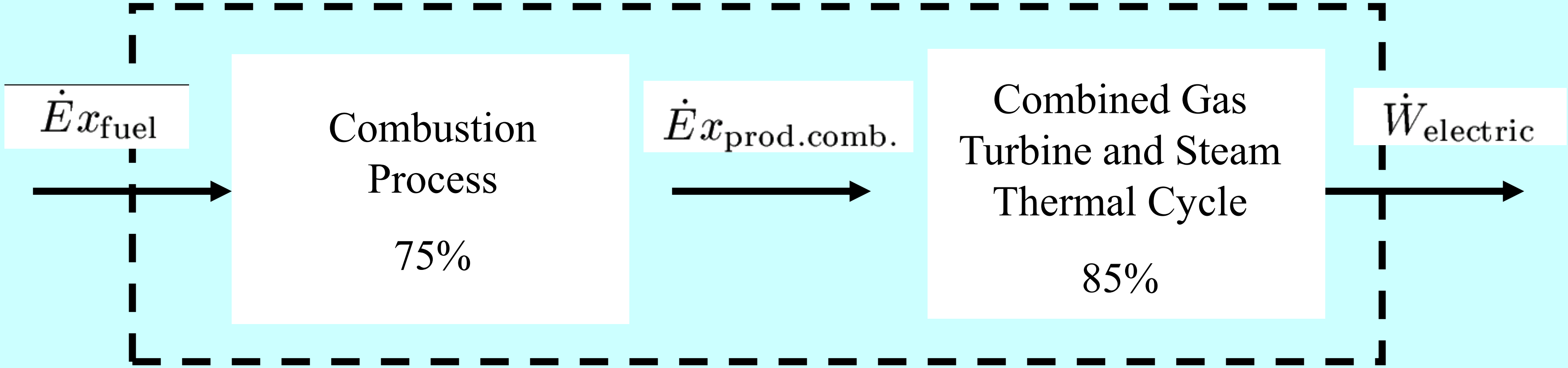
Carnot↑
Joule↑
Rankine↑
Kelvin↑
Clausius↑
Maxwell↑
Boltzman↑
Gibbs↑
Loschmidt↑
Helmholtz↑

↑Poincaré
↑Zermelo
↑Planck
↑Einstein
↑Carathéodory
↑Lewis-Randall
↑Onsager
↑Guggenheim
↑Slater
↑Keenan

↑Schrödinger
↑Prigogine
↑Shannon
↑Brillouin
↑Jaynes
↑Tribus
↑DeGroot-Mazur
↑Feynman
↑Hatsopoulos-Keenan

Form "Meeting the Entropy Challenge", Edited by G.P. Beretta, A.F. Ghoniem, and G.N. Hatsopoulos, AIP CP Series, Volume 2033, p. 375 (2008).

Review of basic concepts: Power-plant philosophy of best available flame-based fuel-to-power conversion technology



$\dot{E}x_{\text{fuel}}$

Combustion Process

75%

$\dot{E}x_{\text{prod.comb.}}$

Combined Gas Turbine and Steam Thermal Cycle

85%

$\dot{W}_{\text{electric}}$

$$\eta_{\text{II}-\text{combustion}} = \frac{\dot{E}x_{\text{prod.comb.}}}{\dot{E}x_{\text{fuel}}} \approx 75\%$$

$$\eta_{\text{II}-\text{thermal}-\text{cycle}} = \frac{\dot{W}_{\text{electric}}}{\dot{E}x_{\text{prod.comb.}}} \approx 85\%$$

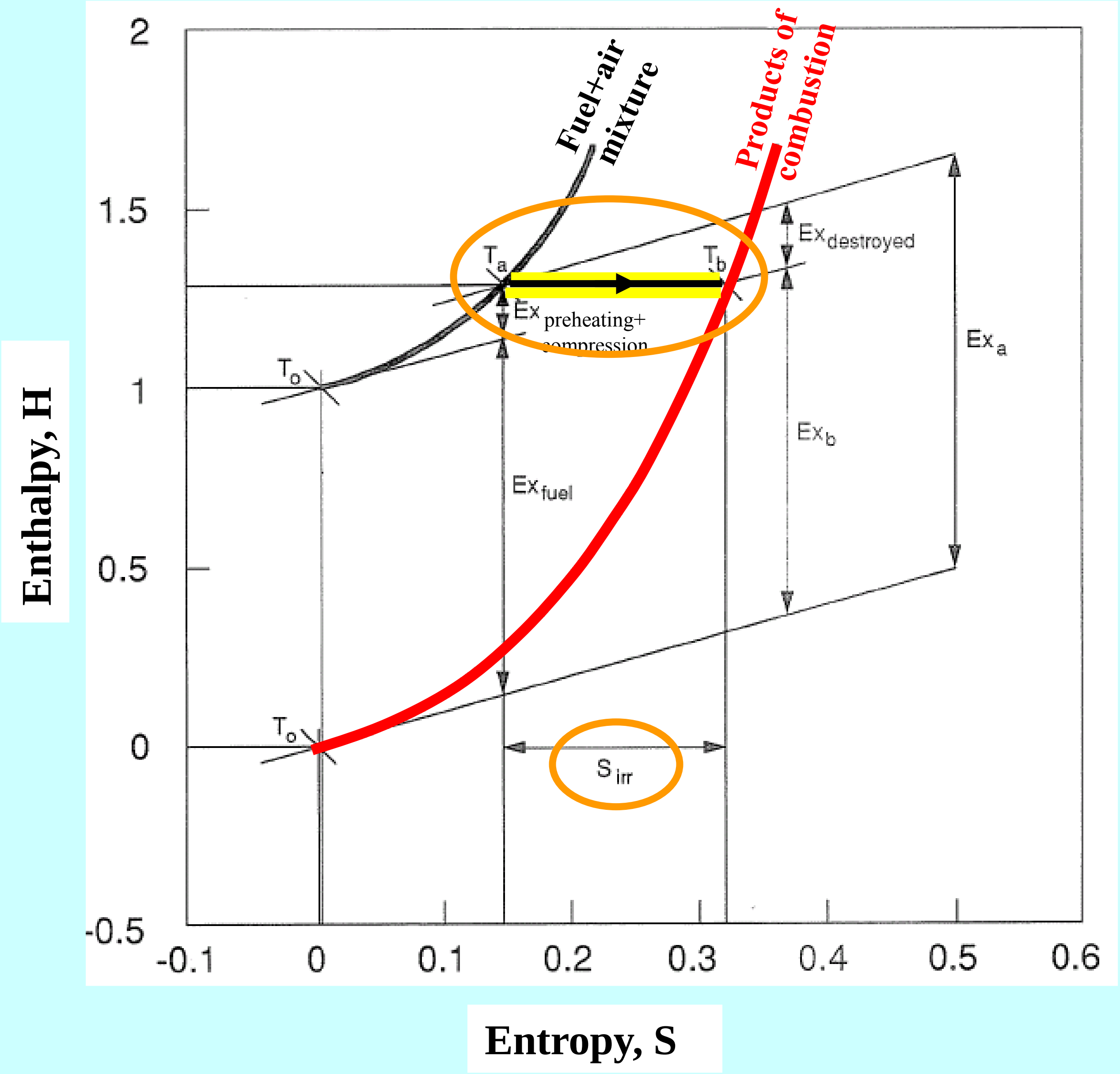
Overall thermodynamic efficiency

64%

$$\eta_{\text{II}-\text{overall}} = \frac{\dot{W}_{\text{electric}}}{\dot{E}x_{\text{fuel}}} = \eta_{\text{II}-\text{combustion}} \times \eta_{\text{II}-\text{thermal}-\text{cycle}} \approx 64\%$$

Review of basic concepts: Power-plant philosophy of best available flame-based fuel-to-power conversion technology

The irreversibility of combustion in the flame dissipates 20/25% of the fuel exergy.



G.P. Beretta, A.M. Lezzi, A. Niro and M. Silvestri, "On the concept of a reversible flame," in Energy for the Transition Age, FLOWERS '92, Edited by S.S. Stecco and M.J. Moran, Nova Science Pu. Inc., New York, Additional Proceedings, pp. 165-177 (1992).

Review of basic concepts: Power-plant philosophy of best available flame-based fuel-to-power conversion technology

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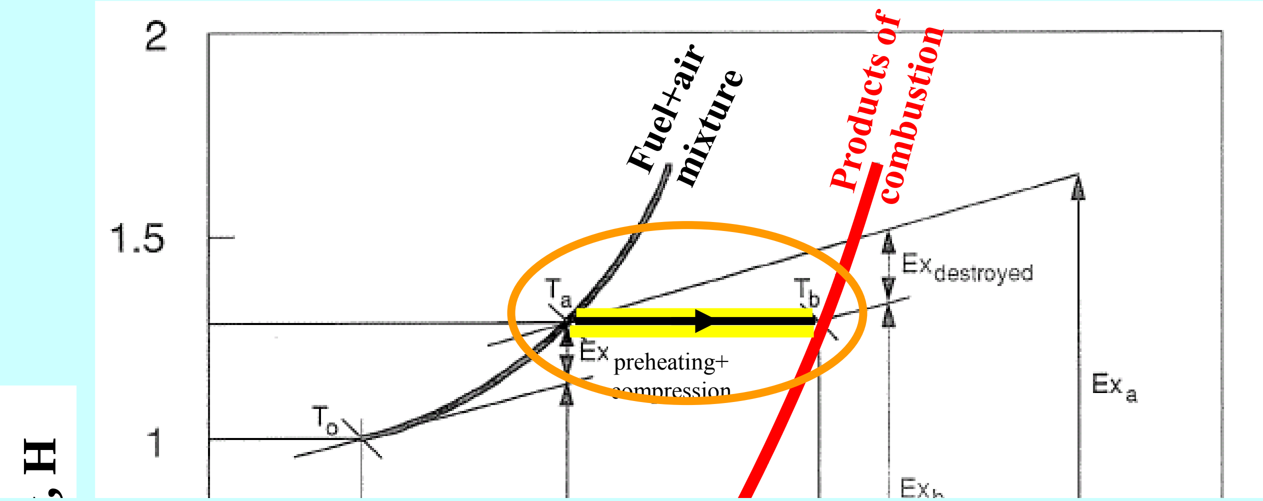
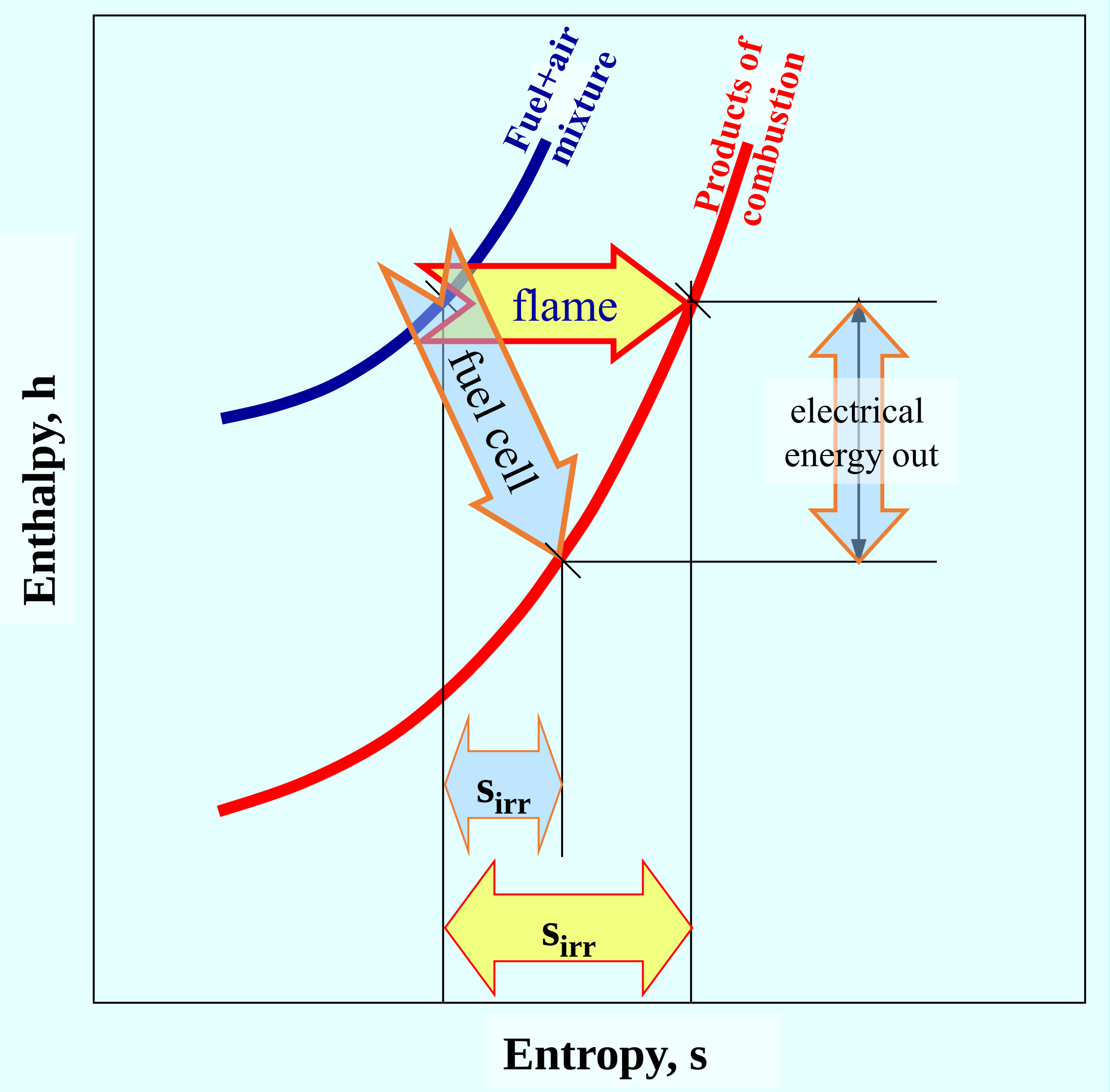


lvestri, "On the concept of a
tion Age, FLOWERS '92,
a Science Pu. Inc., New York,

The role of Thermodynamics? The role of Nonequilibrium?

- History tell us about the future:
  - By the end of this century, power plants will be 85% efficient.
- Thermodynamics tells us:
  - By burning fossil fuels in flames, we cannot exceed 70%.
  - The bottleneck is in the irreversibility of flame combustion.
- History tells us we will overcome that! How?
  - We will gain a better control of fuel oxidation as it occurs or develop alternative oxidation paths that allow better control and less irreversibility,
  - We will gain better control of nonequilibrium states.
  - We will improve our nonequilibrium fluid dynamics, transport phenomena, chemical kinetics models of heterogeneous and multicomponent systems.

Review of basic concepts: one way to get around the inherent irreversibility of flames is by oxidating the fuel in fuel cells



Integrating SOFC in Thermal Cycles (without CO2 sequestration) may yield:

Fuel-Cell Rankine Cycle, 72%

Fuel-Cell Combined Brayton-Rankine Cycle, 75%

Fuel-Cell Regenerative Brayton Cycle, 76%

Methods for the
ALLOCATION

of

energy consumption and **CO_2 production**

in

combined heat and power (CHP) production

and of

heat and/or power production

in

hybrid multi-resource facilities

What fraction of the fuel consumed by a heat-and-power cogeneration facility should be allocated to the heat produced?

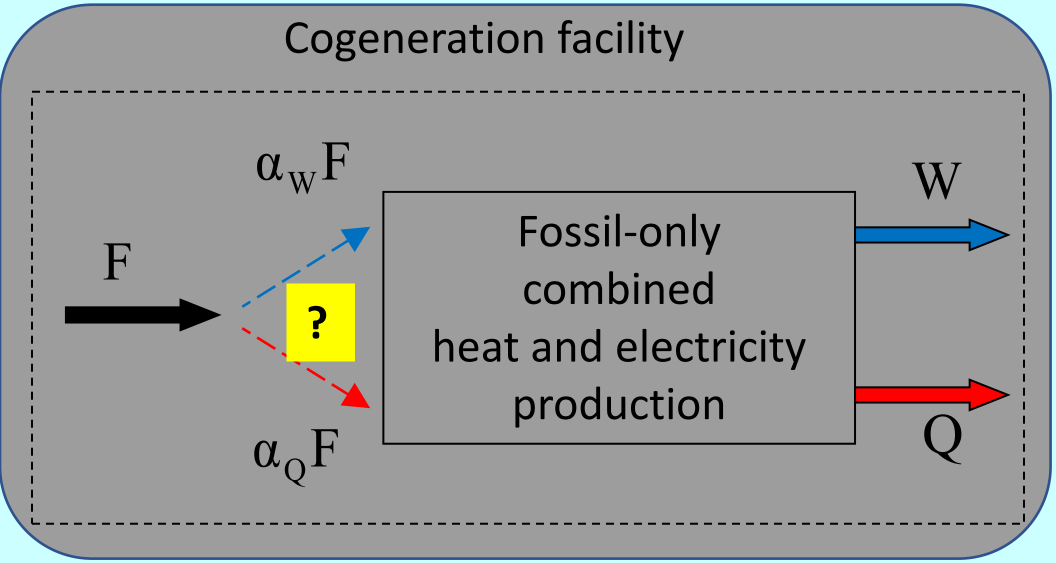
Why is it important?

The question is important in real estate, for buildings served by district heating systems where the heat is produced in CHP facilities.

Each country has its own specific certification process and criteria for evaluating the energy performance of residential buildings. In the United States, the Energy Performance Certificate (EPC)* or Home Energy Score* provides information about a building's energy efficiency and, therefore, it affects the building's commercial value. Among the parameters which determine the Home Energy Score is how much primary energy is consumed for heating.

*Similar indices adopted in various countries:
Italy: Attestato di Prestazione Energetica (APE)
France: Diagnostic de Performance Énergétique (DPE)
Germany: Verbrauchsausweis
United Kingdom: Energy Performance Certificate (EPC)
Australia: Nationwide House Energy Rating Scheme (NatHERS) or Building Sustainability Index (BASIX)
Canada: EnerGuide Rating System
Japan: Comprehensive Assessment System for Building Environmental Efficiency (CASBEE)
China: China Green Building Evaluation Standard (GBES)
India: Energy Conservation Building Code (ECBC) or Star Rating for Energy Efficiency of Buildings

Allocation problem in Heat&Power Cogeneration: α_W, α_Q, partial efficiencies, and PES



Cogeneration facility

F

$\alpha_W F$

?

$\alpha_Q F$

Fossil-only
combined
heat and electricity
production

W

Q

Partial
Efficiencies

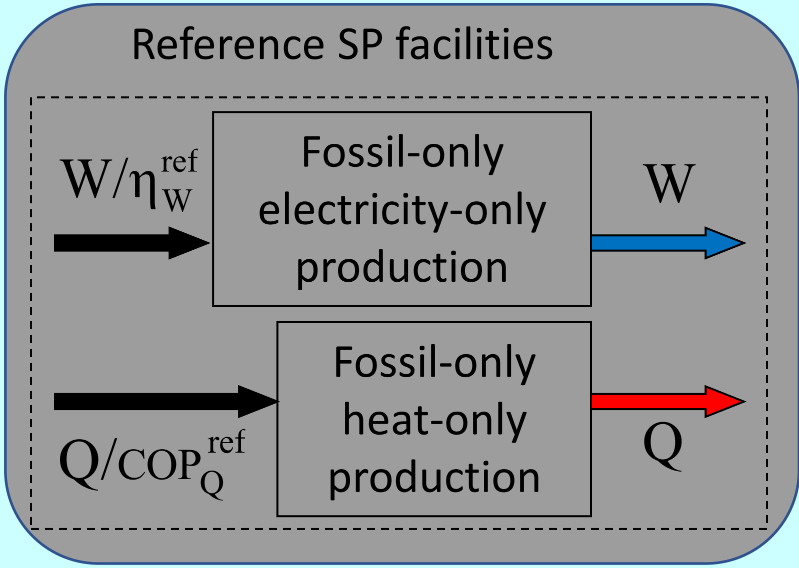
$$\eta_W^{chp} = \frac{W}{\alpha_W F}$$

$$COP_Q^{chp} = \frac{Q}{\alpha_Q F}$$

Primary Energy Savings
vs SP facilities

$$PES_W = \frac{W/\eta_W^{ref} - \alpha_W F}{W/\eta_W^{ref}}$$

$$PES_Q = \frac{Q/COP_Q^{ref} - \alpha_Q F}{Q/COP_Q^{ref}}$$



Reference SP facilities

W/η_W^{ref}

Fossil-only
electricity-only
production

W

Q/COP_Q^{ref}

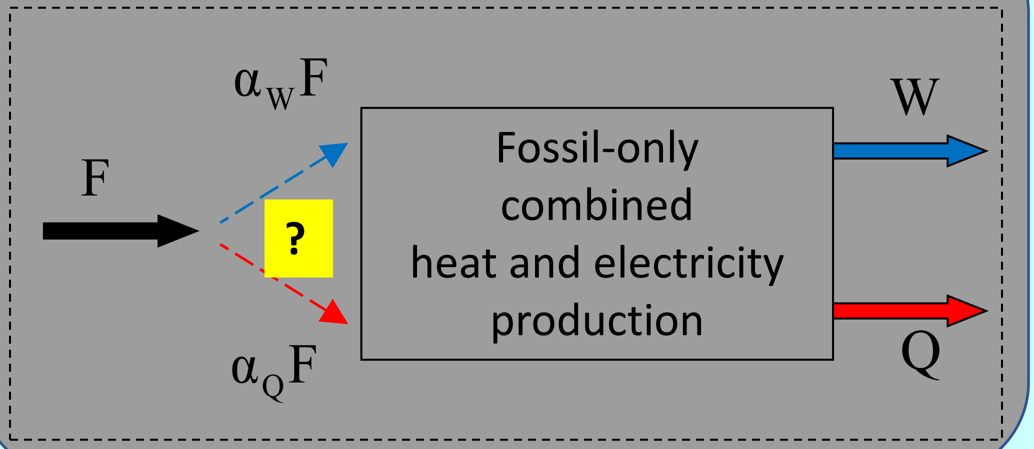
Fossil-only
heat-only
production

Q

$$PES = \frac{W/\eta_W^{ref} + Q/COP_Q^{ref} - F}{W/\eta_W^{ref} + Q/COP_Q^{ref}}$$

Allocation problem in Heat&Power Cogeneration: α_W, α_Q, partial efficiencies, and PES

Cogeneration facility



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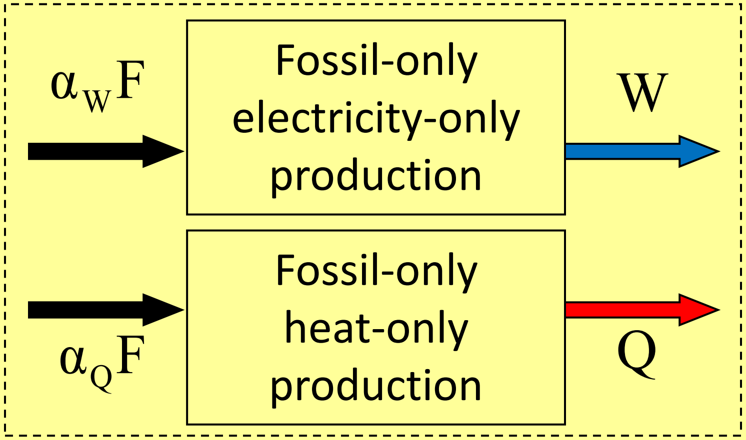
Equivalent SP facilities



Partial Efficiencies

$$\eta_W^{chp} = \frac{W}{\alpha_W F}$$

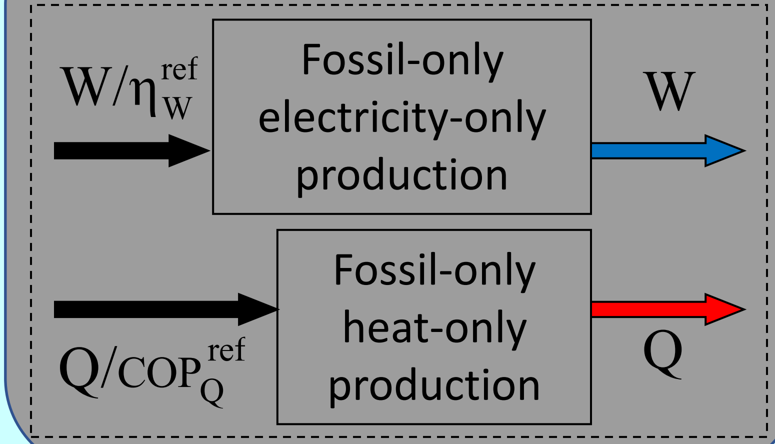
$$COP_Q^{chp} = \frac{Q}{\alpha_Q F}$$

Primary Energy Savings vs SP facilities

$$PES_W = \frac{W/\eta_W^{ref} - \alpha_W F}{W/\eta_W^{ref}}$$

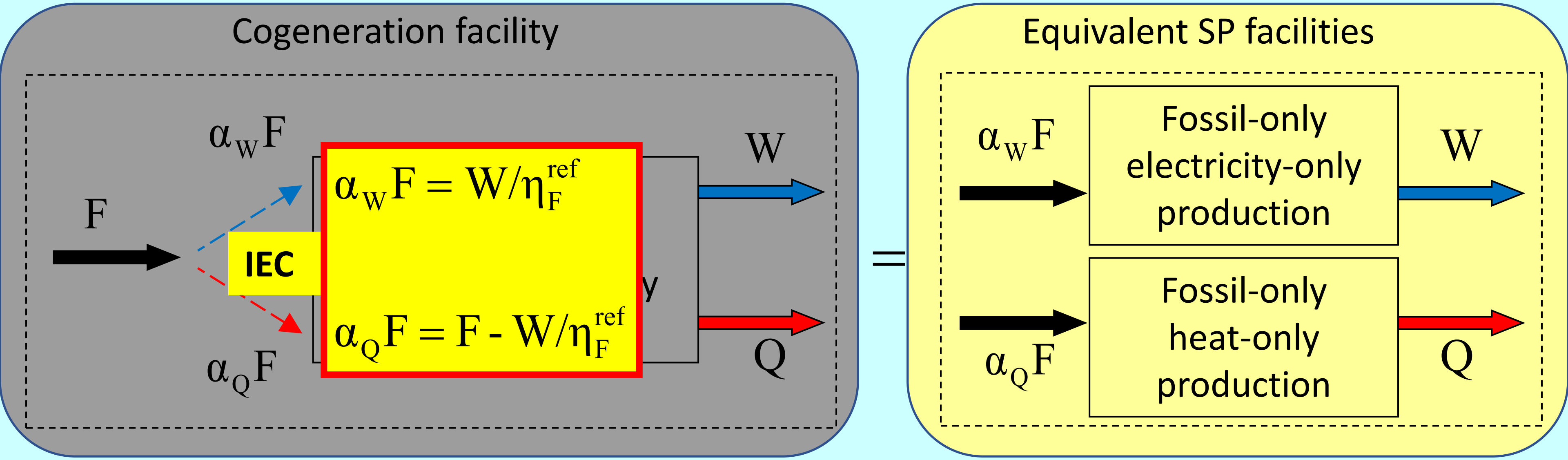
$$PES_Q = \frac{Q/COP_Q^{ref} - \alpha_Q F}{Q/COP_Q^{ref}}$$

Reference SP facilities



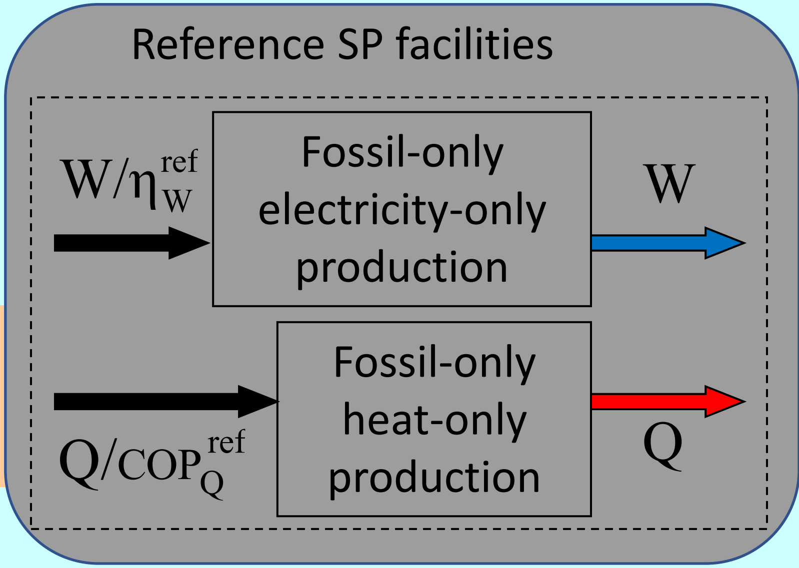
$$PES = \frac{W/\eta_W^{ref} + Q/COP_Q^{ref} - F}{W/\eta_W^{ref} + Q/COP_Q^{ref}} = \frac{W/\eta_W^{ref}}{W/\eta_W^{ref} + Q/COP_Q^{ref}} PES_W + \frac{Q/COP_Q^{ref}}{W/\eta_W^{ref} + Q/COP_Q^{ref}} PES_Q$$

Allocation problem in CHP: Incremental Electricity-Centered Allocation



$$\alpha_W F = W/\eta_F^{ref}$$

$$\alpha_Q F = F - W/\eta_F^{ref}$$



Partial Efficiencies

$$\mathrm{COP}_Q^{chp} = \frac{Q}{F - W/\eta_W^{ref}}$$

$$\eta_W^{chp} = \eta_W^{ref}$$

Primary Energy Savings vs SP facilities

$$\mathrm{PES}_W = 0$$

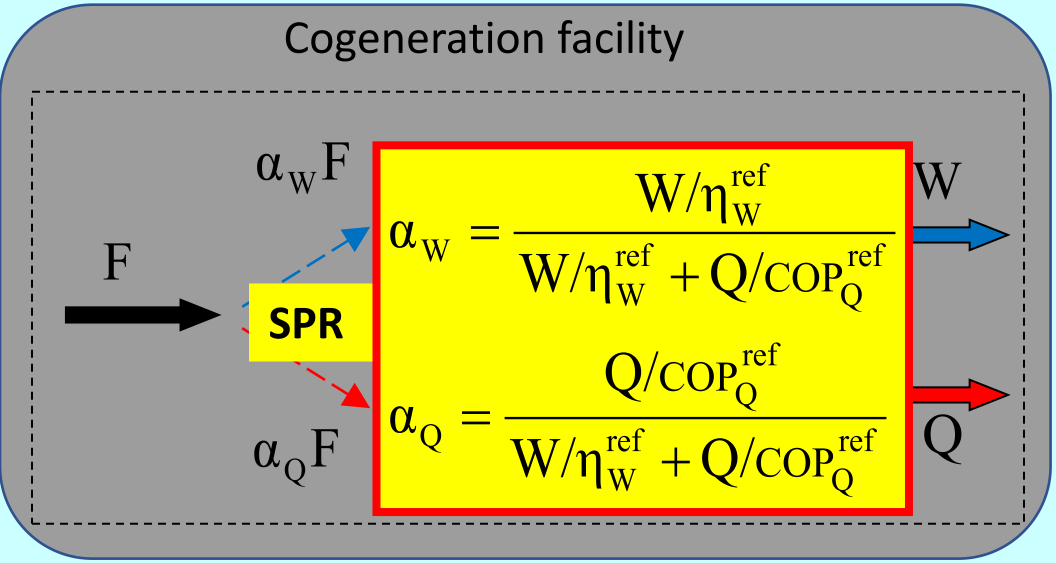
$$\mathrm{PES}_Q = \frac{Q/\mathrm{COP}_Q^{ref} - (F - W/\eta_W^{ref})}{Q/\mathrm{COP}_Q^{ref}}$$

$$\mathrm{PES}_Q = \frac{W/\eta_W^{ref} + Q/\mathrm{COP}_Q^{ref}}{Q/\mathrm{COP}_Q^{ref}}\,\mathrm{PES} > \mathrm{PES}\,!$$

No allotment of the benefits of cogeneration between W and Q! COP_Q may be negative!

Allocation problem in CHP: Separate Production Reference Allocation

Cogeneration facility



F

SPR

$\alpha_W F$

$\alpha_Q F$

$$\alpha_W = \frac{W/\eta_W^{ref}}{W/\eta_W^{ref} + Q/COP_Q^{ref}}$$

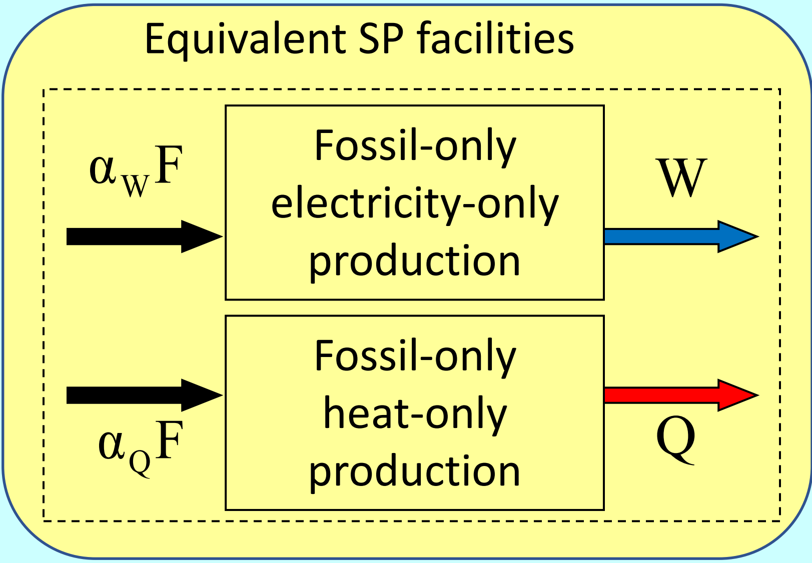
$$\alpha_Q = \frac{Q/COP_Q^{ref}}{W/\eta_W^{ref} + Q/COP_Q^{ref}}$$

W

Q

$=$

Equivalent SP facilities



$\alpha_W F$

Fossil-only
electricity-only
production

W

$\alpha_Q F$

Fossil-only
heat-only
production

Q

Partial Efficiencies

$$\eta_W^{chp} = \frac{W}{\alpha_W F}$$

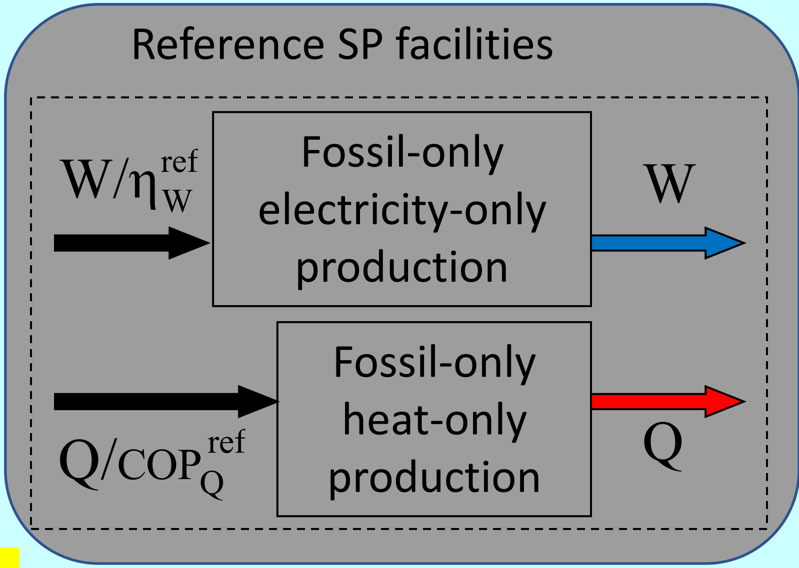
$$COP_Q^{chp} = \frac{Q}{\alpha_Q F}$$

Primary Energy Savings vs SP facilities

$$PES_W = \frac{W/\eta_W^{ref} - \alpha_W F}{W/\eta_W^{ref}}$$

$$PES_Q = \frac{Q/COP_Q^{ref} - \alpha_Q F}{Q/COP_Q^{ref}}$$

Reference SP facilities



W/η_W^{ref}

Fossil-only
electricity-only
production

W

Q/COP_Q^{ref}

Fossil-only
heat-only
production

Q

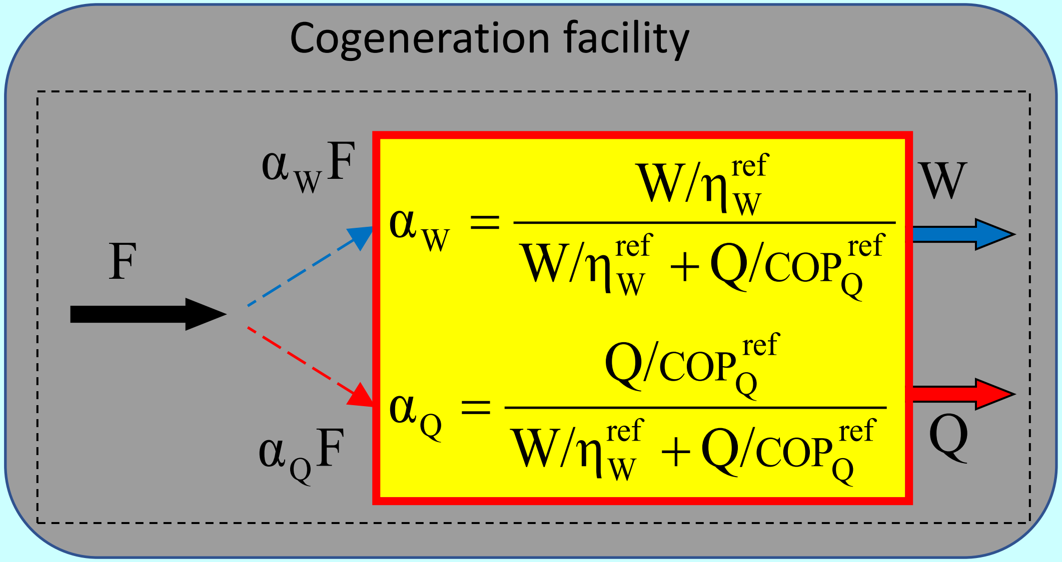
$$PES = \frac{W/\eta_W^{ref} + Q/COP_Q^{ref} - F}{W/\eta_W^{ref} + Q/COP_Q^{ref}} = PES_W = PES_Q$$

Fair allottment of the benefits
of cogeneration between W and Q
provided reference values…!!

Allocation problem in CHP: Choice of reference values for η_W^{ref} and COP_Q^{ref}

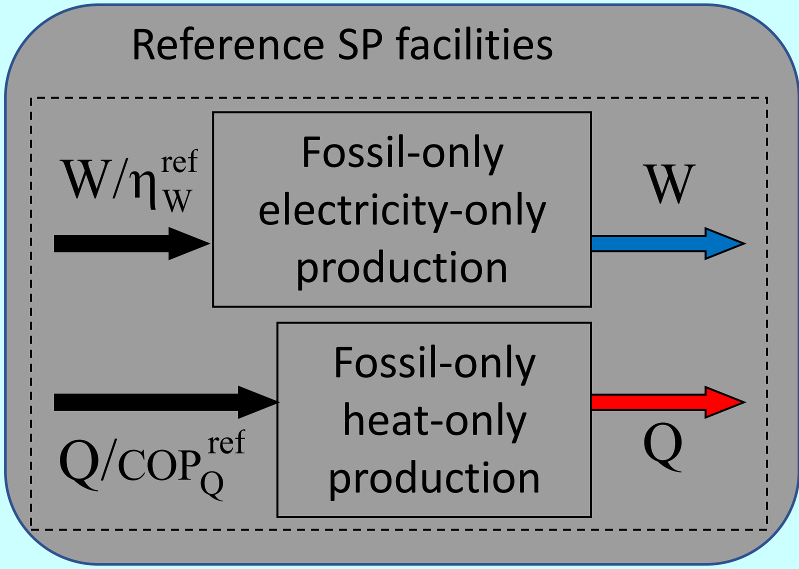
SPR method:

Fixed values set by some local Authority



$$\alpha_W = \frac{W/\eta_W^{ref}}{W/\eta_W^{ref} + Q/COP_Q^{ref}}$$

$$\alpha_Q = \frac{Q/COP_Q^{ref}}{W/\eta_W^{ref} + Q/COP_Q^{ref}}$$



Exergy method:

Fixed values set by Thermodynamics

Effectively takes as references
the REVERSIBLE heat engines!

$$\eta_W^{ref} \approx 1 \qquad COP_Q^{ref} = 1 \Big/ \left(1 - \frac{T_{env}}{T_Q}\right)$$

Allocation problem in CHP: “fair” reference values in a given local area

STALPR Method*: Self-Tuned-Average-Local-Productions-Reference

Adopt reference efficiencies

η_W^{ref} COP_Q^{ref}

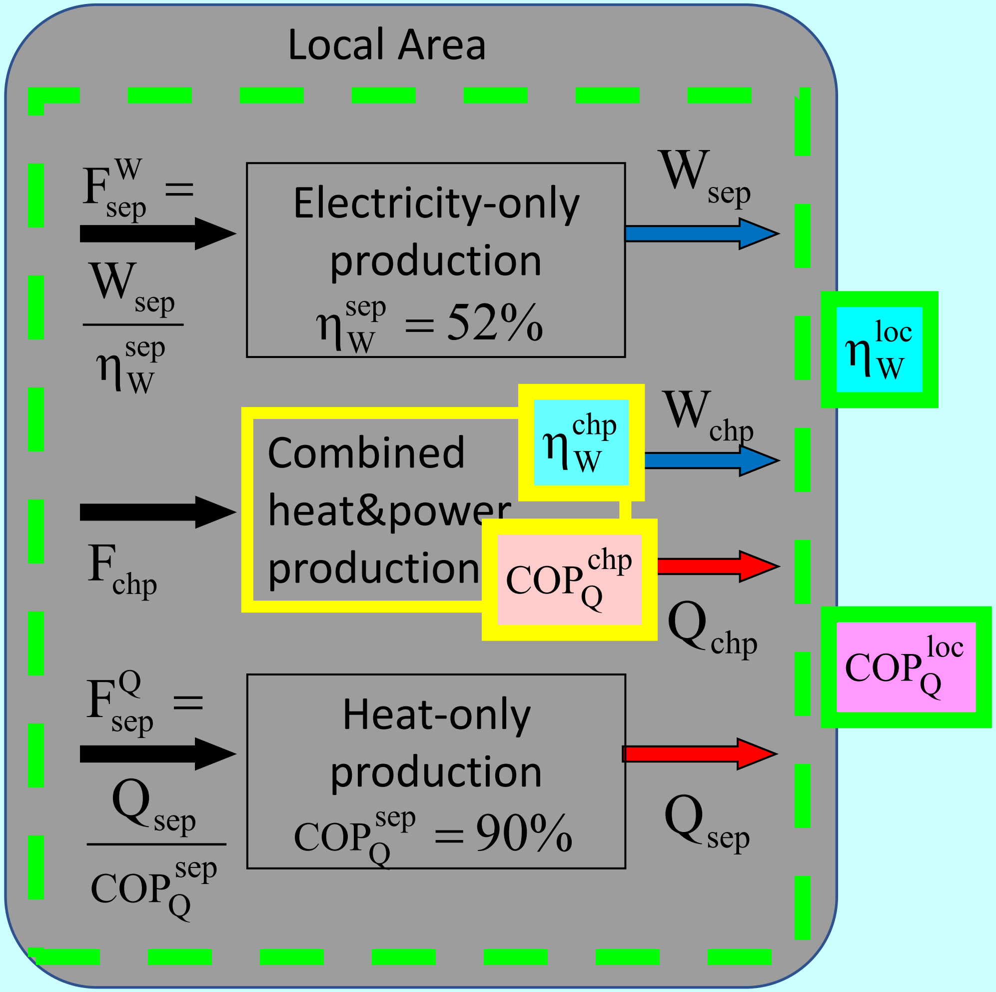
that are representative of the actual average efficiencies

η_W^{loc} COP_Q^{loc}

of the energy production portfolio (typically the local area where the cogenerator itself is located) with which the resulting efficiencies of the cogenerator

η_W^{chp} COP_Q^{chp}

are to be compared.



$$\eta_W^{loc} = \frac{W_{sep} + W_{chp}}{F_{sep}^W + \alpha_W F_{chp}}$$

$$COP_Q^{loc} = \frac{Q_{sep} + Q_{chp}}{F_{sep}^Q + \alpha_Q F_{chp}}$$

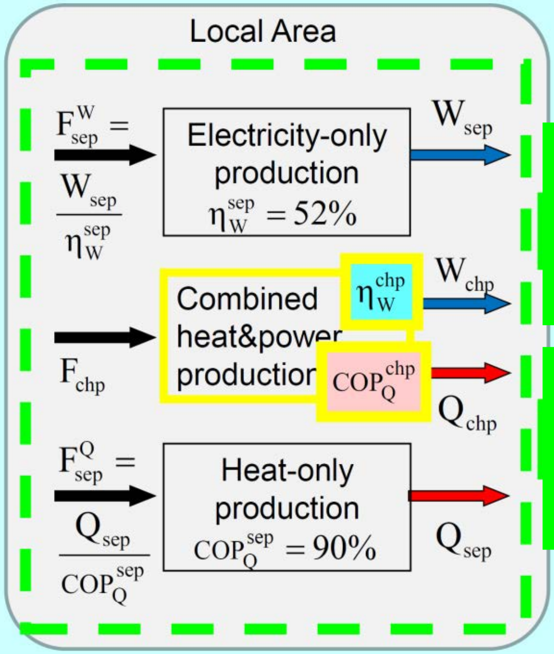
* G.P. Beretta, P. Iora, and A.F. Ghoniem, Energy, Vol. 44, pp. 1107-1120 (2012)

Allocation Example in CHP: a comparison between allocation methods

Parameters of the local area	
η_W^{sep}	52 %
COP_Q^{sep}	90 %

Parameter of the chp plant	CC	BPST
$\sigma_{chp} = \dfrac{W_{chp}}{Q_{chp}}$	1.2	0.2
$\eta_{chp} = \dfrac{W_{chp} + Q_{chp}}{P_{F,chp}}$	78%	85%

Reference values for SPR	
η_W^{ref}	52 %
COP_Q^{ref}	90 %



Local Area

$F_{sep}^W = \dfrac{W_{sep}}{\eta_W^{sep}}$

Electricity-only production $\eta_W^{sep} = 52\%$

W_{sep}

Combined heat&power production η_W^{chp} COP_Q^{chp}

F_{chp}

W_{chp}

Q_{chp}

$F_{sep}^Q = \dfrac{Q_{sep}}{COP_Q^{sep}}$

Heat-only production $COP_Q^{sep} = 90\%$

Q_{sep}

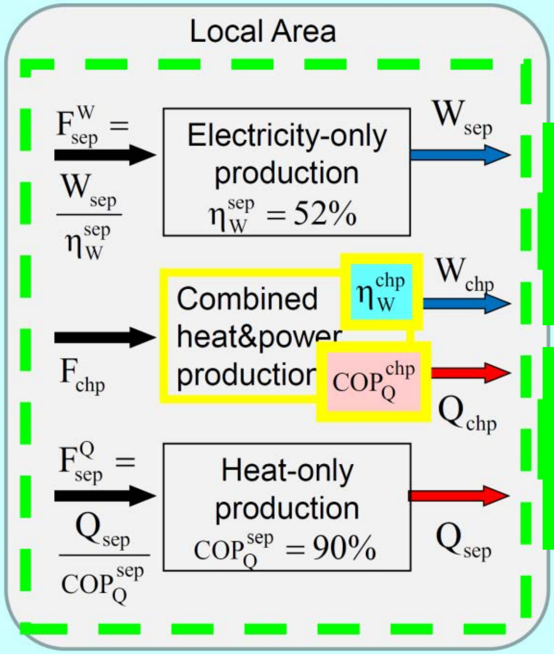
$$\eta_W^{loc} = \frac{W_{sep} + W_{chp}}{F_{sep}^W + \alpha_W F_{chp}}$$

$$COP_Q^{loc} = \frac{Q_{sep} + Q_{chp}}{F_{sep}^Q + \alpha_Q F_{chp}}$$

%	IEC		Exergy		SPR	
	η_W^{chp}	COP_Q^{chp}	η_W^{chp}	COP_Q^{chp}	η_W^{chp}	COP_Q^{chp}
CC	52	354	50	234	63.0	109
BPST	52	101	29	137	55.1	95.4

Allocation Example in CHP: a comparison between allocation methods

Parameters of the local area		
$\sigma_{loc} = \frac{W_{sep} + W_{chp}}{Q_{sep} + Q_{chp}}$	50 %	
$\gamma^{Q}_{chp} = \frac{Q_{chp}}{Q_{sep} + Q_{chp}}$	40 %	
Average efficiencies based on SPR		
	CC	BPST
η^{loc}_{W}	62.5 %	52.4 %
COP^{loc}_{Q}	96.8 %	92.1 %
$\gamma^{W}_{chp} = \frac{W_{chp}}{W_{sep} + W_{chp}}$	96 %	16 %
Reference values for SPR		
η^{ref}_{W}	52 %	52 %
COP^{ref}_{Q}	90 %	90 %



$$\eta^{loc}_{W} = \frac{W_{sep} + W_{chp}}{F^{W}_{sep} + \alpha_{W} F_{chp}}$$

$$COP^{loc}_{Q} = \frac{Q_{sep} + Q_{chp}}{F^{Q}_{sep} + \alpha_{Q} F_{chp}}$$

%	IEC		Exergy		SPR	
	η^{chp}_{W}	COP^{chp}_{Q}	η^{chp}_{W}	COP^{chp}_{Q}	η^{chp}_{W}	COP^{chp}_{Q}
CC	52	354	50	234	63.0	109
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Allocation Example in CHP: a comparison between allocation methods

Parameters of the local area	
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Average efficiencies based on SPR	CC	BPST
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Reference values for SPR	CC	BPST
η^{ref}_{W}	52 %	52 %
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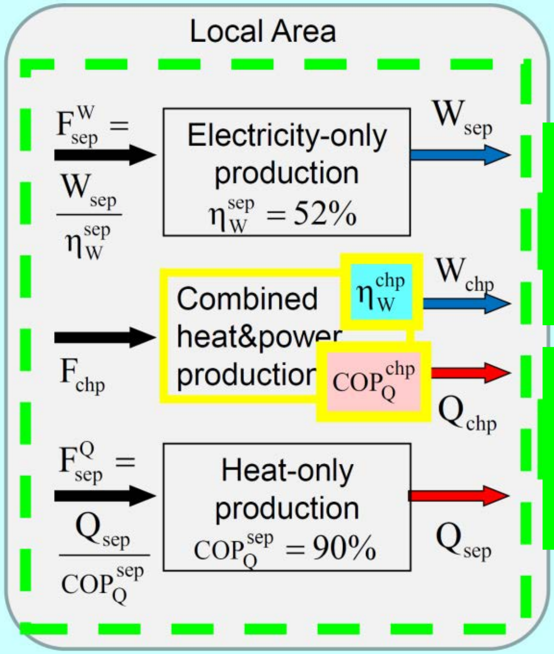


$$\eta^{loc}_{W} = \frac{W_{sep} + W_{chp}}{F^{W}_{sep} + \alpha_{W} F_{chp}}$$

$$COP^{loc}_{Q} = \frac{Q_{sep} + Q_{chp}}{F^{Q}_{sep} + \alpha_{Q} F_{chp}}$$

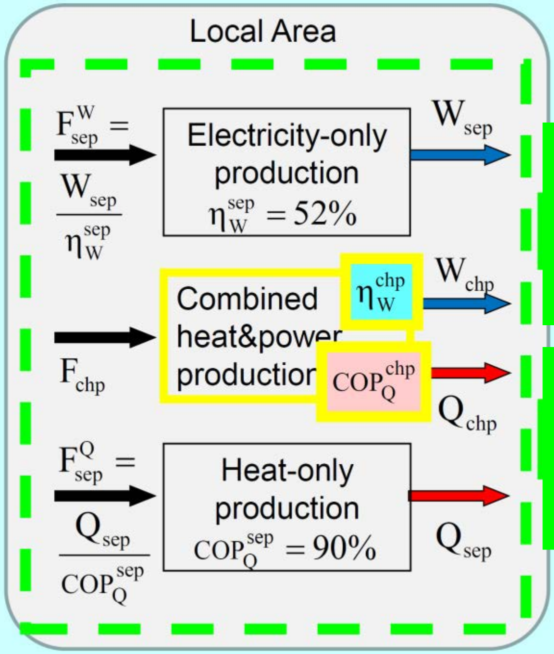
%	IEC		Exergy		SPR	
	η^{chp}_{W}	COP^{chp}_{Q}	η^{chp}_{W}	COP^{chp}_{Q}	η^{chp}_{W}	COP^{chp}_{Q}
CC	52	354	50	234	63.0	109
BPST	52	101	29	137	55.1	95.4

Allocation Example in CHP: a comparison between allocation methods

Parameters of the local area	
$\sigma_{loc} = \dfrac{W_{sep} + W_{chp}}{Q_{sep} + Q_{chp}}$	50 %
$\gamma^{Q}_{chp} = \dfrac{Q_{chp}}{Q_{sep} + Q_{chp}}$	40 %

Average efficiencies based on SPR	CC	BPST
η^{loc}_{W}	%	%
COP^{loc}_{Q}	%	%
$\gamma^{W}_{chp} = \dfrac{W_{chp}}{W_{sep} + W_{chp}}$	96 %	16 %

Reference values for SPR	CC	BPST
η^{ref}_{W}	62.5 %	52.4 %
COP^{ref}_{Q}	96.8 %	92.1 %



$$\eta^{loc}_{W} = \frac{W_{sep} + W_{chp}}{F^{W}_{sep} + \alpha_{W} F_{chp}}$$

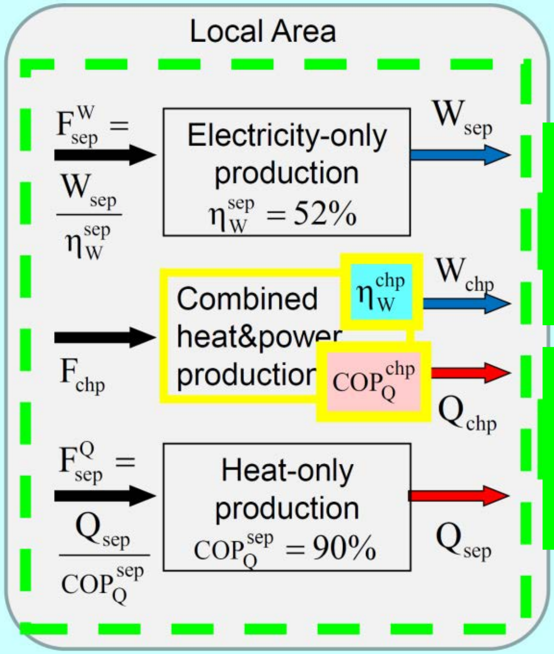
$$COP^{loc}_{Q} = \frac{Q_{sep} + Q_{chp}}{F^{Q}_{sep} + \alpha_{Q} F_{chp}}$$

%	IEC		Exergy		SPR	
	η^{chp}_{W}	COP^{chp}_{Q}	η^{chp}_{W}	COP^{chp}_{Q}	η^{chp}_{W}	COP^{chp}_{Q}
CC	52	354	50	234	65.4	101
BPST	52	101	29	137	55.1	95.4

Allocation Example in CHP: a comparison between allocation methods

Parameters of the local area	
$\sigma_{loc} = \frac{W_{sep} + W_{chp}}{Q_{sep} + Q_{chp}}$	50 %
$\gamma^{Q}_{chp} = \frac{Q_{chp}}{Q_{sep} + Q_{chp}}$	40 %

Average efficiencies based on SPR		
	CC	BPST
η^{loc}_{W}	64.8 %	52.4 %
COP^{loc}_{Q}	94.2 %	92.2 %
$\gamma^{W}_{chp} = \frac{W_{chp}}{W_{sep} + W_{chp}}$	96 %	16 %
Reference values for SPR		
η^{ref}_{W}	62.5 %	52.4 %
COP^{ref}_{Q}	96.8 %	92.1 %



$$\eta^{loc}_{W} = \frac{W_{sep} + W_{chp}}{F^{W}_{sep} + \alpha_{W} F_{chp}}$$

$$COP^{loc}_{Q} = \frac{Q_{sep} + Q_{chp}}{F^{Q}_{sep} + \alpha_{Q} F_{chp}}$$

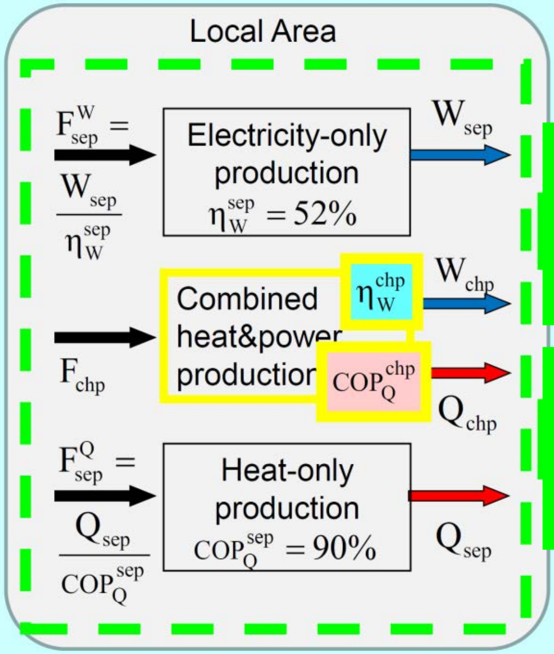
%	IEC		Exergy		SPR	
	η^{chp}_{W}	COP^{chp}_{Q}	η^{chp}_{W}	COP^{chp}_{Q}	η^{chp}_{W}	COP^{chp}_{Q}
CC	52	354	50	234	65.4	101
BPST	52	101	29	137	54.5	95.7

Allocation Example in CHP: a comparison between allocation methods

Parameters of the local area	
$\sigma_{\mathrm{loc}} = \dfrac{W_{\mathrm{sep}} + W_{\mathrm{chp}}}{Q_{\mathrm{sep}} + Q_{\mathrm{chp}}}$	50 %
$\gamma^{Q}_{\mathrm{chp}} = \dfrac{Q_{\mathrm{chp}}}{Q_{\mathrm{sep}} + Q_{\mathrm{chp}}}$	40 %

Average efficiencies based on SPR		
	CC	BPST
η^{loc}_{W}	68.3 %	52.4 %
$\mathrm{COP}^{\mathrm{loc}}_{Q}$	90.8 %	92.2 %
$\gamma^{W}_{\mathrm{chp}} = \dfrac{W_{\mathrm{chp}}}{W_{\mathrm{sep}} + W_{\mathrm{chp}}}$	96 %	16 %

Reference values for SPR		
η^{ref}_{W}	68.3 %	52.4 %
$\mathrm{COP}^{\mathrm{ref}}_{Q}$	90.8%	92.2 %



$$\eta^{\mathrm{loc}}_{W} = \frac{W_{\mathrm{sep}} + W_{\mathrm{chp}}}{F^{W}_{\mathrm{sep}} + \alpha_{W} F_{\mathrm{chp}}}$$

$$\mathrm{COP}^{\mathrm{loc}}_{Q} = \frac{Q_{\mathrm{sep}} + Q_{\mathrm{chp}}}{F^{Q}_{\mathrm{sep}} + \alpha_{Q} F_{\mathrm{chp}}}$$

%	IEC		Exergy		STALPR	
	η^{chp}_{W}	$\mathrm{COP}^{\mathrm{chp}}_{Q}$	η^{chp}_{W}	$\mathrm{COP}^{\mathrm{chp}}_{Q}$	η^{chp}_{W}	$\mathrm{COP}^{\mathrm{chp}}_{Q}$
CC	52	354	50	234	69.2	92.0
BPST	52	101	29	137	54.4	95.8

What fraction of the electrical energy produced in a hybrid solar-fossil power plant should qualify as ‘renewable electricity’?

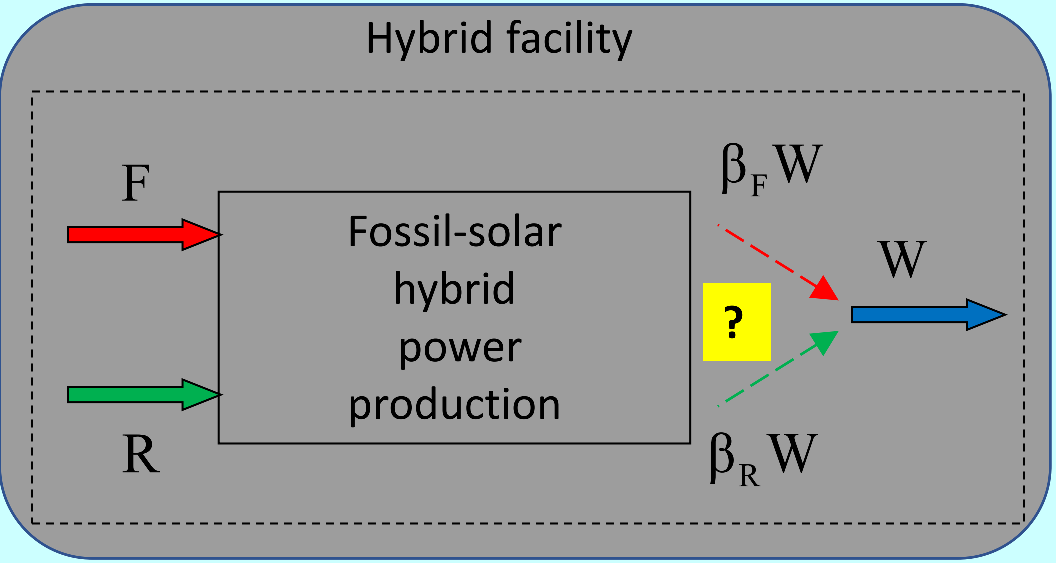
Why is it important?

The question is important because several government programs (in the United States and in most other countries) provide economic incentives* for the production of electricity from solar, wind, and other renewable energy sources. In “hybrid facilities” where these renewable sources are combined/integrated with fossil fuels, the access to these incentives depends on how much of the produced electricity is recognized as renewable.

It is also relevant for multi-fuel power plants or hybrid CHP facilities.

*Examples:
Investment tax credits
Production tax credits
Accelerated depreciation
Cash grants
Loan programs
Grants and loan guarantees to agricultural producers and rural small businesses
Renewable energy tax-credit bonds

Allocation problem in Hybrid Facilities: β_F, β_Q, partial efficiencies, and PES



Hybrid facility

F

R

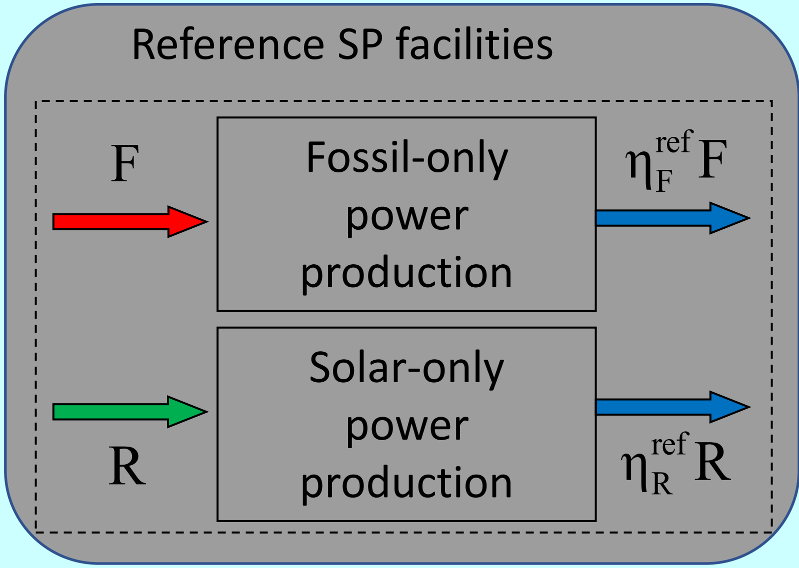
Fossil-solar hybrid power production

?

$\beta_F W$

$\beta_R W$

W



Reference SP facilities

F

Fossil-only power production

$\eta_F^{ref} F$

R

Solar-only power production

$\eta_R^{ref} R$

Partial Efficiencies

$$\eta_F^{hyb} = \frac{\beta_F W}{F}$$

$$\eta_R^{hyb} = \frac{\beta_R W}{R}$$

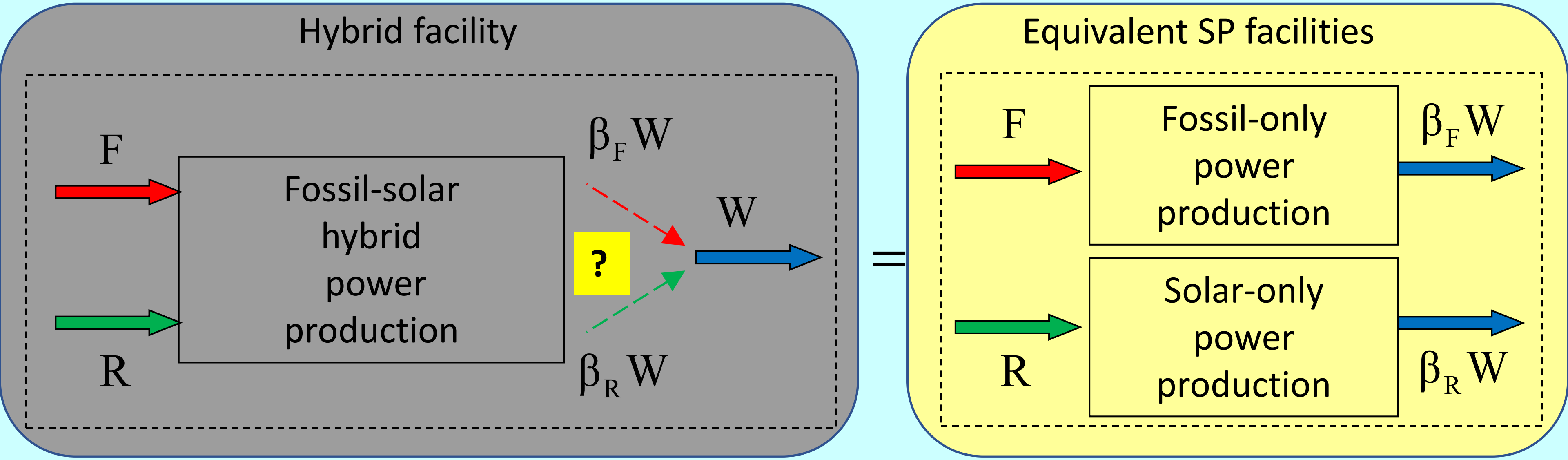
Primary Energy Savings vs SP facilities

$$PES_F = \frac{\beta_F W - \eta_F^{ref} F}{\beta_F W}$$

$$PES_R = \frac{\beta_R W - \eta_R^{ref} R}{\beta_R W}$$

$$PES = \frac{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref} - F - R}{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref}} = \frac{\beta_F W/\eta_F^{ref}}{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref}} PES_F + \frac{\beta_R W/\eta_R^{ref}}{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref}} PES_R$$

Allocation problem in Hybrid Facilities: β_F, β_Q, partial efficiencies, and PES



Partial Efficiencies

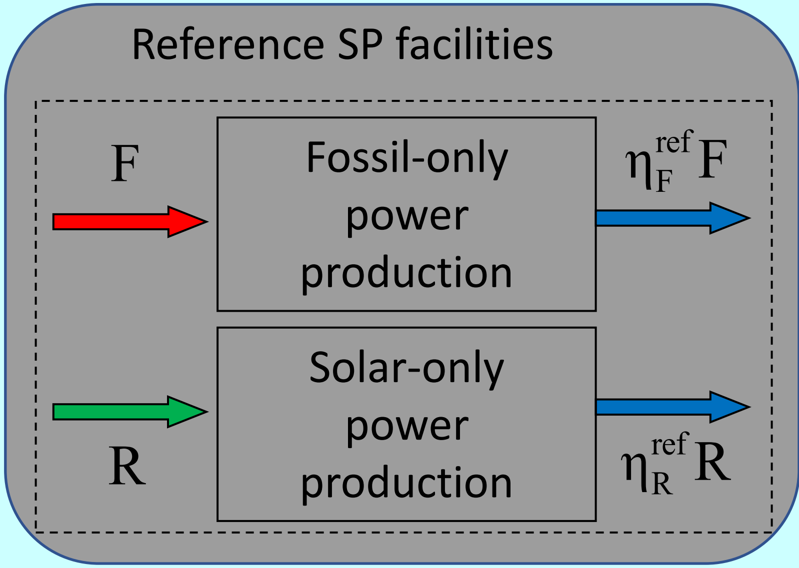
$$\eta_F^{hyb} = \frac{\beta_F W}{F}$$

$$\eta_R^{hyb} = \frac{\beta_R W}{R}$$

Primary Energy Savings vs SP facilities

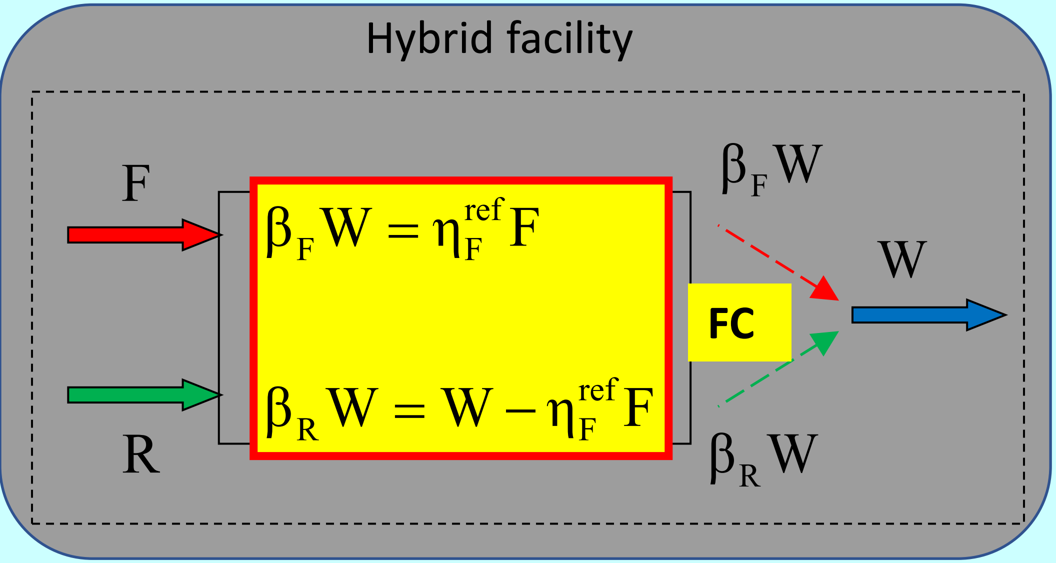
$$PES_F = \frac{\beta_F W - \eta_F^{ref} F}{\beta_F W}$$

$$PES_R = \frac{\beta_R W - \eta_R^{ref} R}{\beta_R W}$$



$$PES = \frac{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref} - F - R}{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref}} = \frac{\beta_F W/\eta_F^{ref}}{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref}} PES_F + \frac{\beta_R W/\eta_R^{ref}}{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref}} PES_R$$

Allocation problem in Hybrid Facilities: Incremental Fossil-Centered Allocation



Hybrid facility

F

R

$$\beta_F W = \eta_F^{ref} F$$

$$\beta_R W = W - \eta_F^{ref} F$$

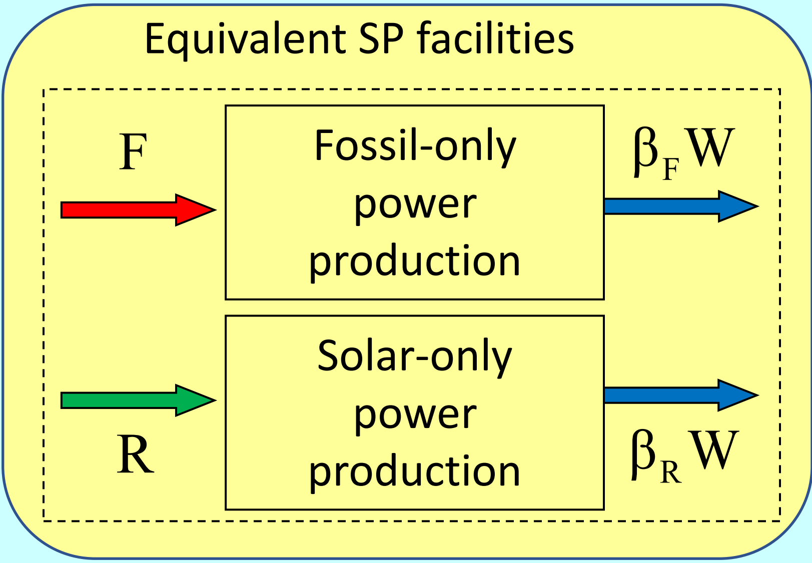
FC

$\beta_F W$

$\beta_R W$

W

=



Equivalent SP facilities

F

Fossil-only power production

$\beta_F W$

R

Solar-only power production

$\beta_R W$

Partial Efficiencies

$$\eta_F^{hyb} = \eta_F^{ref}$$

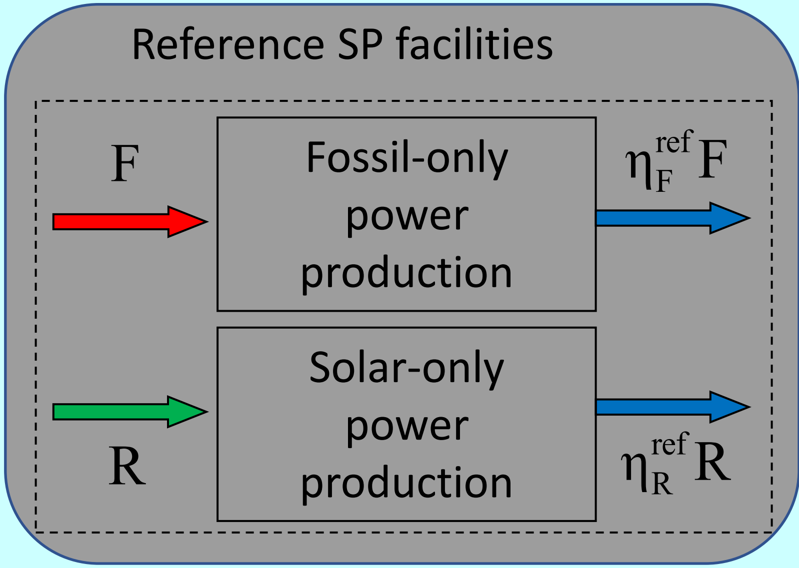
$$\eta_R^{hyb} = \frac{W - \eta_F^{ref} F}{R}$$

Primary Energy Savings vs SP facilities

$$PES_F = 0$$

$$PES_R = \frac{W - \eta_F^{ref} F - \eta_R^{ref} R}{W - \eta_F^{ref} F}$$

$$PES_R = \left(1 + \frac{\eta_R^{ref} F}{\beta_R W}\right) PES > PES\,!$$



Reference SP facilities

F

Fossil-only power production

$\eta_F^{ref} F$

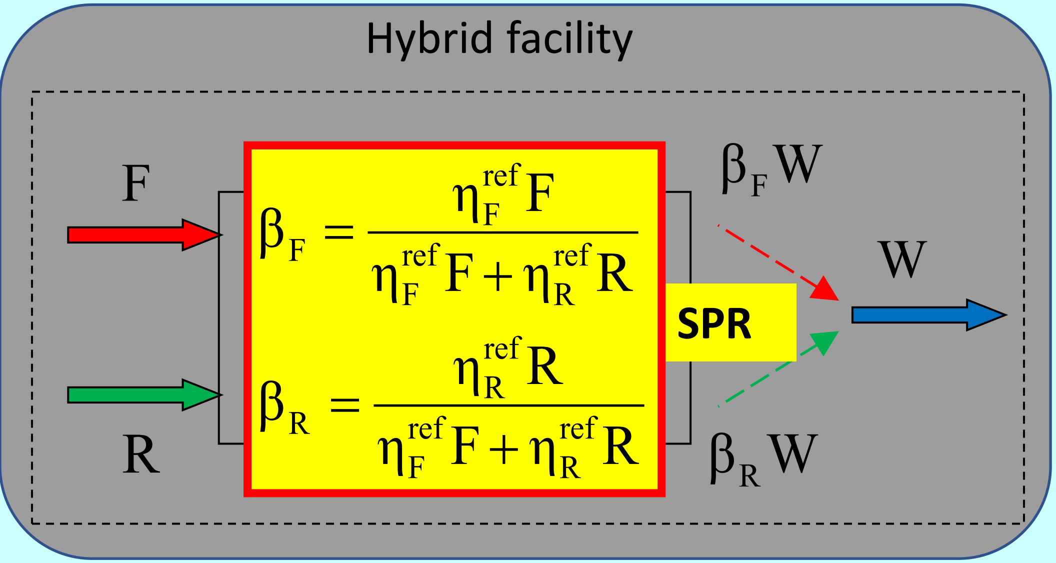
R

Solar-only power production

$\eta_R^{ref} R$

No allottment of the benefits of hybridization between F and R!

Allocation problem in Hybrid Facilities: **Separate Production Reference Allocation**

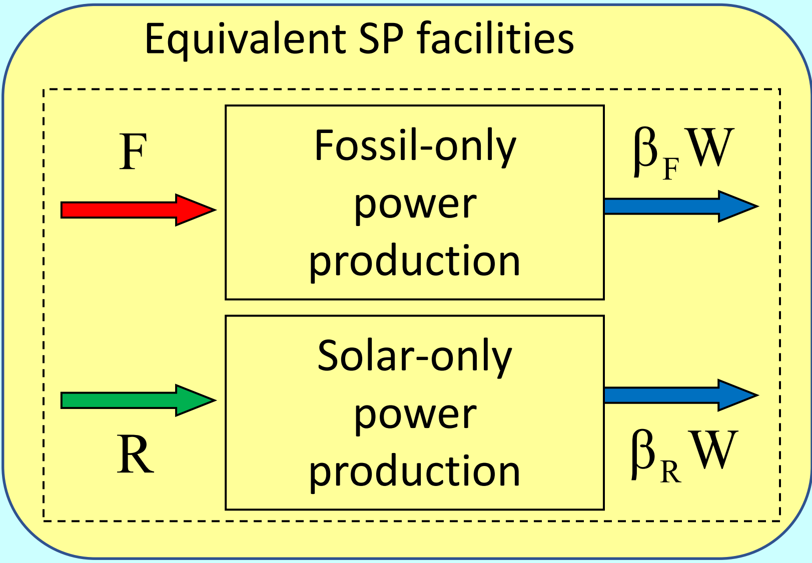


Hybrid facility

$$\beta_F = \frac{\eta_F^{ref} F}{\eta_F^{ref} F + \eta_R^{ref} R}$$

$$\beta_R = \frac{\eta_R^{ref} R}{\eta_F^{ref} F + \eta_R^{ref} R}$$

$=$



Equivalent SP facilities

Partial Efficiencies

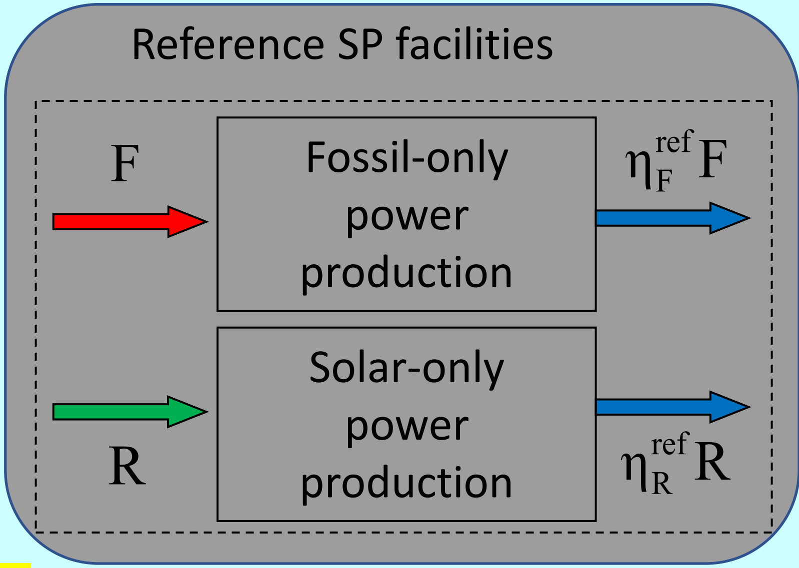
$$\eta_F^{hyb} = \frac{\beta_F W}{F}$$

$$\eta_R^{hyb} = \frac{\beta_R W}{R}$$

Primary Energy Savings vs SP facilities

$$PES_F = \frac{\beta_F W - \eta_F^{ref} F}{\beta_F W}$$

$$PES_R = \frac{\beta_R W - \eta_R^{ref} R}{\beta_R W}$$



Reference SP facilities

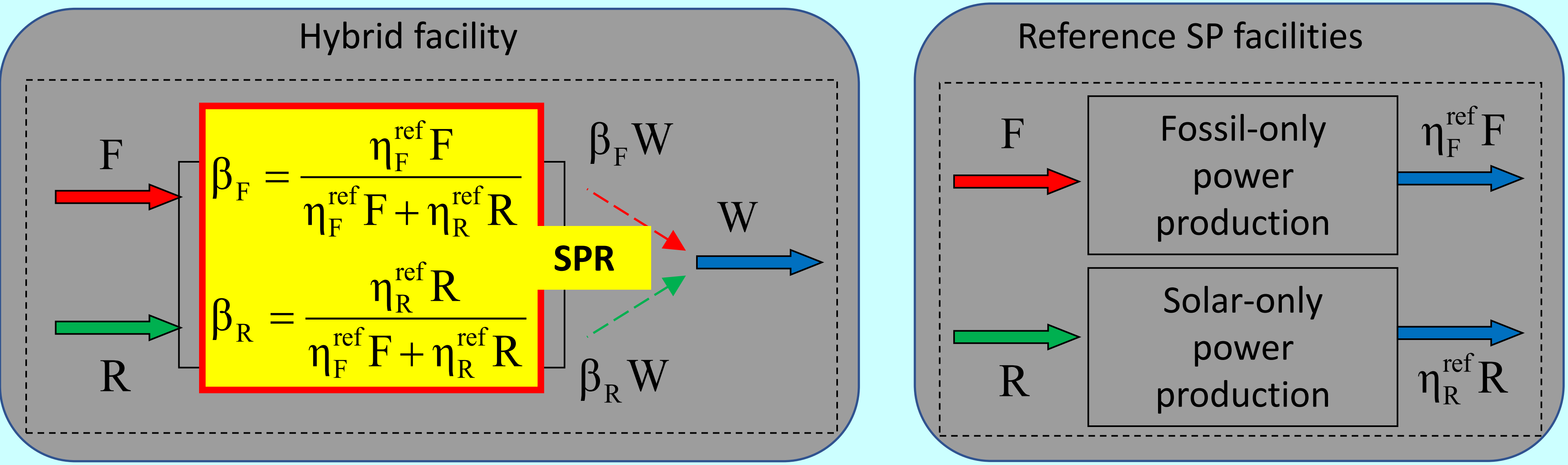
$$PES = \frac{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref} - F - R}{\beta_F W/\eta_F^{ref} + \beta_R W/\eta_R^{ref}} = PES_F = PES_R$$

Allottment of the hybridization benefits between F and R is fair only for fair reference values…!!

Allocation problem in Hybrid Facilities: Choice of reference values for η_F^{ref} and η_R^{ref}

SPR method:

Fixed values set by some <u>local Authority</u>



Hybrid facility

$$\beta_F = \frac{\eta_F^{ref} F}{\eta_F^{ref} F + \eta_R^{ref} R}$$

$$\beta_R = \frac{\eta_R^{ref} R}{\eta_F^{ref} F + \eta_R^{ref} R}$$

Reference SP facilities

Exergy method:

Fixed values set by <u>Thermodynamics</u>

Effectively takes as references
the REVERSIBLE heat engines!

$$\eta_F^{ref} \approx 1 \qquad \eta_R^{ref} \approx 0.93$$

Image Credits

Slide 19:

Slide 21 and **Slide 22**:

2.43 Advanced Thermodynamics
Spring 2024