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2.500 Desalination and Water Purification

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2.500 DESALINATION AND WATER PURIFICATION

Homework Set #3
Due 9 April 2009

PROBLEM 1: ELECTRODIALYSIS

A total of 325 m³/day of an aqueous solution of 3,000 ppm of NaCl is to be desalinated to 400 ppm by electrodialysis, with a 40% conversion of this solution to potable water. The system used has four stages in series; all liquid travels from one stage to the next. Each stage has three parallel stacks with 150 cell pairs in each stack. The fractional desalination (or $c_{\text{out}}/c_{\text{in}}$) will be the same for each stage.

The expected current efficiency is $\xi = 90\%$. The applied voltage for the first stage is 220 Vdc. Each cell pair has an area of 1,160 cm². Calculate the current density i in mA/cm², the current I in A, and the power requirement in kW for the first stage.

PROBLEM 2: BOILING POINT ELEVATION AND OSMOTIC COEFFICIENT

Consider a solvent containing a non-volatile solutes (e.g., water with salt in it) at a temperature T_s . The solution is in equilibrium with pure vapor of the solvent at this temperature. The pure solvent has a saturation temperature (or boiling point) T_{bp} at this pressure, with $T_s > T_{\text{bp}}$. Let \bar{G}_w^v be the partial molar Gibbs energy of solvent in the vapor phase and \bar{G}_w^l be the partial molar Gibbs energy of solvent in the liquid phase.

- A. If $\Delta_{\text{vap}}\bar{G}_w$ is the change in molar Gibbs energy of the *pure* solvent when it vaporizes at temperature T_s (which is above the normal boiling point), show that

$$\ln a_w = \frac{\Delta_{\text{vap}}\bar{G}_w}{RT_s}$$

where a_w is the chemical activity of the solvent in the solution at temperature T_s .

- B. Use the result of A to show that

$$-\ln a_w = \frac{\Delta_{\text{vap}}\bar{H}_w}{RT_{\text{bp}}^2} (T_s - T_{\text{bp}})$$

To do this, note that $\Delta_{\text{vap}}\bar{G}_w$ is zero at T_{bp} , but becomes increasingly negative as T increase beyond T_{bp} . In doing this, you may assume that $\Delta_{\text{vap}}\bar{H}_w$ and $\Delta_{\text{vap}}\bar{S}_w$ are independent of temperature for temperatures near T_{bp} .

- C. The osmotic coefficient, ϕ , is related to the chemical activity as

$$\ln a_w \equiv -\phi M_w (10^{-3} \text{ kg/g}) \left(\sum_i m_i \right)$$

where M_w is the molar mass of the solvent in grams/mol and $\sum_i m_i$ is the sum of the molalities of all solutes. Use the data for ϕ that was distributed in class to find the point boiling elevation, $T_s - T_{bp}$, of aqueous sodium chloride solution at molalities of $m_{NaCl} = 0.5, 0.62, 1.0, \text{ and } 2.0$ mol NaCl/kg H_2O and a pressure of 1 bar.

PROBLEM 3: SEA SALT, SODIUM CHLORIDE, AND VAPOR PRESSURE

Values of the osmotic coefficient for seawater are as follow, where w is the mass fraction of sea salt.*

Temperature of 25°C				Temperature of 100°C			
w (%)	ϕ	w (%)	ϕ	w (%)	ϕ	w (%)	ϕ
1.0	0.9052	6.0	0.9249	1.0	0.8902	6.0	0.9122
2.0	0.9007	8.0	0.9479	2.0	0.8855	8.0	0.9360
3.45	0.9048	10.0	0.9754	3.45	0.8904	10.0	0.9636
4.0	0.9080	12.0	1.0065	4.0	0.8940	12.0	0.9938

- A. If we wish to make an aqueous sodium chloride solution have the same chemical activity for water, a_w , as seawater at 25°C and 34,500 ppm, what molality and mass fraction should the sodium chloride solution have? What about at 100°C? For this seawater, $\sum_i m_i = 1.134$ mol/kg.
- B. For low pressures, the chemical activity of solvent in solution, a_w , is equal to the vapor pressure of the solvent divided by the vapor pressure (or saturation pressure) of the pure solvent at the same temperature:

$$a_w = \frac{p_{\text{vap},w}}{p_{\text{sat},w}(T)}$$

(When the solvent is ideal, with $\gamma_w = 1$, this becomes Raoult's law, as discussed in class.) Use the data for ϕ to calculate the vapor pressure of seawater at 34,500 ppm and temperatures of 25°C and 100°C.

- C. Compare the vapor pressures obtained in Part B to those obtained using Raoult's law under the same conditions. What do you conclude?

*Ref: L.A. Bromley et al., "Thermodynamic properties of sea salt solutions," *AIChE J.* **20**(1), 326-335, 1974.