

## 2.62 – Advanced Energy Conversion | Spring 2020

### Homework 1 – Solutions

#### Problem 1

- a) Calculate the energy stored in both tanks between the fully charged state and discharged state:

First, we start by setting a reference state at  $T=T_0=22\text{ C}$  and  $P=1\text{ atm}$ . To simplify the analysis, the specific heats are assumed constant herein. Calculating the energy stored in the thermal mass between its two states:

$$\Delta E_{salt} = M_{salt}c((T_{s2} - T_0) - (T_{s1} - T_0))$$

$$\Delta E_{salt} = (1 \times 10^5 \text{ kg})(1,500 \text{ J/kg}\cdot\text{K})(300 \text{ K})$$

$$\Delta E_{salt} = 4.5 \times 10^{10} \text{ J}$$

Next, we apply a similar approach to calculate the stored energy in the tank:

$$\Delta E_g = ((U_2 - U_0) - (U_1 - U_0))_g$$

$$\Delta E_g = (M_2c_v(T_2 - T_0) - M_0c_v(T_0 - T_0)) - (M_1c_v(T_1 - T_0) - M_0c_v(T_0 - T_0))$$

$$M_0 = M_1 = \frac{P_1V_t}{RT_1} = \frac{(1 \times 10^5 \text{ Pa})(1,000 \text{ m}^3)}{(287 \text{ J/kg}\cdot\text{K})(295 \text{ K})} = 1,181.1 \text{ kg}$$

$$M_2 = \frac{P_2V_t}{RT_2} = \frac{(100 \times 10^5 \text{ Pa})(1,000 \text{ m}^3)}{(287 \text{ J/kg}\cdot\text{K})(873 \text{ K})} = 39,912 \text{ kg}$$

$$\Delta E_g = (39,912 \text{ kg} \times 717.5 \text{ J/kg}\cdot\text{K} \times 851 \text{ K}) - (1,181.1 \text{ kg} \times 717.5 \text{ J/kg}\cdot\text{K} \times 0 \text{ K})$$

$$\Delta E_g = 2.44 \times 10^{10} \text{ J}$$

- b) Calculate the work required by the compressor train to charge the system of the air tank and the salt tank. The compressor train operates adiabatically:

To answer this question, we apply the first law of thermodynamics. Assuming the compressor train operates adiabatically:

$$-W + \Delta m_g(h_{in} - h_0) = \Delta E_g + \Delta E_{salt} = \Delta E_g + \Delta E_{salt}$$

$$-W = (2.44 + 4.5) \times 10^{10}$$

$$W = -6.94 \times 10^{10} \text{ J}$$

- c) Calculate the maximum work that can be extracted from the molten salt tank when fully charged:

To find the maximum work extractable, we use the concept of availability:

$$W_{s,max} = \Xi_2 - \Xi_1$$

where:  $\Xi = (E - U_0) + p(\forall - \forall_0) - T_0(S - S_0)$

Evaluating this for the molten salt:

$$W_{s,max} = (E_2 - E_1) - T_0 \left( C \ln \frac{T_{s2}}{T_{s1}} \right)$$

$$W_{s,max} = M_{salt} c (T_{s2} - T_{s1}) - M_{salt} c T_0 \left( \ln \frac{T_{s2}}{T_{s1}} \right)$$

$$W_{s,max} = (1 \times 10^5 \text{ kg})(1,500 \text{ J/kg}\cdot\text{K}) \left( 300 \text{ K} - 295 \text{ K} \ln \frac{873.15}{573.15} \right)$$

$$W_{s,max} = 2.64 \times 10^{10} \text{ J}$$

- d) Calculate the maximum work that can be extracted from the gas tank when fully charged:

Drawing a control volume around the tank, invoking Equation 2.37 from the text, and following a similar approach:

$$\dot{W}_{g,max} = - \frac{d((E - E_0) - T_0(S - S_0))}{dt} - \dot{m}_{out}((h_{out} - h_0) - T_0(s_{out} - s_0))$$

Maximum work occurs when the air leaves at the dead state (second term goes to zero).

Integrating with time:

$$W_{g,max} = -[(E_{dis} - E_{char}) - T_0((S_{dis} - S_0) - (S_{char} - S_0))]$$

Rearranging:

$$W_{g,max} = (E_2 - E_1) - T_0[(S_2 - S_0) - (S_1 - S_0)]$$

Therefore:

$$E_2 - E_1 = M_2 c_v (T_2 - T_0) - M_1 c_v (T_1 - T_0) = 2.44 \times 10^{10} \text{ J}$$

Likewise:

$$T_0(S_2 - S_0) = T_0 M_2 (s_2 - s_0) = 295 \text{ K} \times 39,912 \text{ kg} \times \left( c_p \ln \frac{T_2}{T_0} - R \ln \frac{P_2}{P_0} \right)$$

$$T_0(S_1 - S_0) = T_0 M_1 (s_1 - s_0) = 0$$

Therefore:

$$W_{g,max} = 2.44 \times 10^{10} \text{ J}$$

$$- (295 \text{ K})(39,912 \text{ kg}) \left( 1,004.5 \text{ J/kg}\cdot\text{K} \ln \frac{873.15}{295.15} - 287 \text{ J/kg}\cdot\text{K} \ln 100 \right)$$

$$W_{g,max} = 2.44 \times 10^{10} \text{ J}$$

**e) What is the round-trip efficiency of this storage system:**

$$\eta_{trip} = \frac{W_{out}}{W_{in}} = \frac{0.6W_{s,max} + 0.7W_{g,max}}{W_{in}} = \frac{1.584 + 1.71}{6.94} = 47\%$$

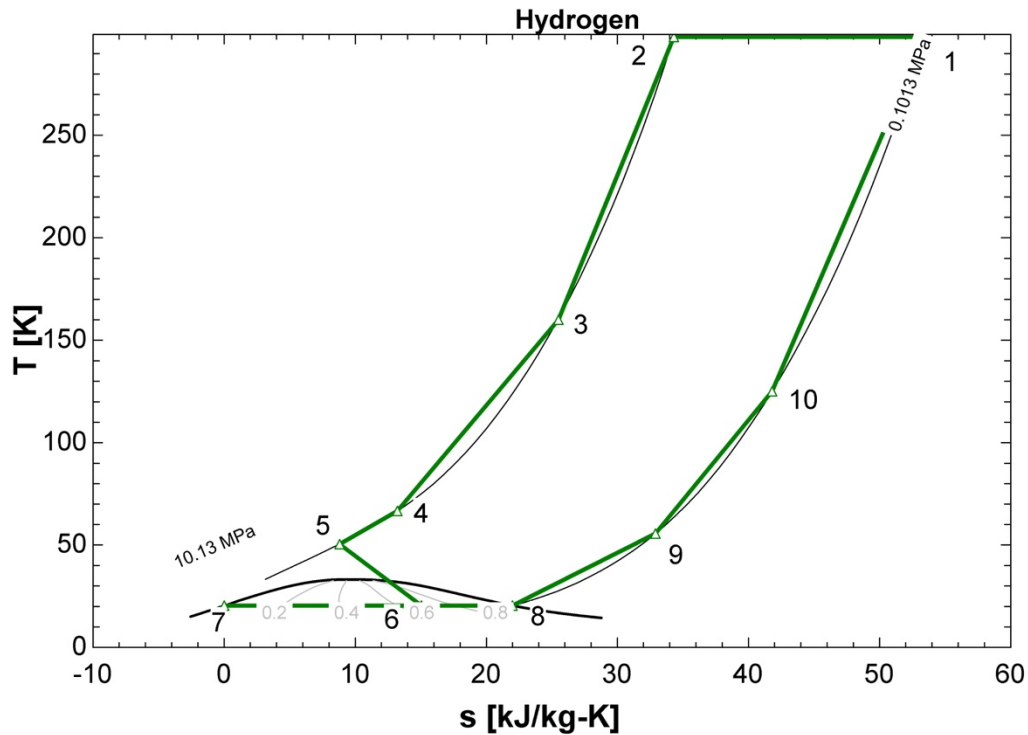
**f) How long does it take to charge the system using a wind turbine operating at 1 MW.**

$$\Delta t = \frac{W_{in}}{\dot{W}} = \frac{6.94 \times 10^{10} \text{ J}}{1 \times 10^6 \text{ J/s}} = 64,000 \text{ s} \approx 19.2 \text{ hrs}$$

## Problem 2

a) Draw schematically the T-s diagram of the Claude cycle:

Using EES, we can plot our states using the Property Plot + Lookup Table (to store states) + Overlay Plot (to add your points from the table):



b) Calculate the mass of liquid H<sub>2</sub> produced in the hydrogen liquid separator per 1 kg of H<sub>2</sub> at state 2:

Start by applying the 1<sup>st</sup> law to Heat Exchanger 1:

$$\dot{m}_2(h_2 - h_3) + \dot{m}'(h_{10} - h_{11}) = 0$$

Note that:

$$\dot{m}' = \dot{m}_2 - \dot{m}_l$$

Therefore:

$$\frac{\dot{m}_l}{\dot{m}_2} = 1 + \left( \frac{h_2 - h_3}{h_{10} - h_{11}} \right) = 0.1587$$

c) Determine the pressure and temperature of H<sub>2</sub> at states 1-9:

We start from state 7 (saturated liquid):

$$P_7 = P_1$$

$$x_7 = 0$$

State 8 (saturated vapor):

$$P_8 = P_1$$

$$x_8 = 1$$

State 6 (liquid-vapor mixture):

$$P_6 = P_1$$

$$x_6 = \frac{m_v}{m_{tot}} = \frac{m_{tot} - m_l}{m_{tot}} = 1 - \frac{m_l}{m_{tot}} = 1 - 2 \left( \frac{m_l}{m_2} \right) = 0.6826$$

State 9 (Use turbine isentropic efficiency). The only unknown is  $h_{out}$ .

$$P_9 = P_1$$

$$\eta_t = \frac{h_{in} - h_{out}}{h_{in} - h_{out,s}}$$

To determine states 4 and 5, we use the other two heat exchangers:

$$0.5\dot{m}_2(h_3 - h_4) + (\dot{m}_2 - \dot{m}_l)(h_9 - h_{10}) = 0$$

$$0.5\dot{m}_2(h_4 - h_5) + (0.5\dot{m}_2 - \dot{m}_l)(h_8 - h_9) = 0$$

Solving the resulting system using EES and summarizing the states in a Table:

State	Pressure [atm]	Temperature [K]	Quality [-]
1	1	298	-
2	100	298	-
3	100	160	-
4	100	66.57	-
5	100	50.42	-
6	1	20.37	0.6826
7	1	20.37	0
8	1	20.37	1
9	1	55.5	-
10	1	125	-

d) Calculate the work produced by the turbine per 1kg of the produced liquid H<sub>2</sub>:

$$\dot{w}_t = 0.5 \left( \frac{\dot{m}_2}{\dot{m}_l} \right) (h_3 - h_9) = 3741 \text{ kJ/kg H}_2$$

e) Determine the total work required to produce 1 kg of liquid hydrogen:

$$\dot{w}_c = \left( \frac{\dot{m}_2}{\dot{m}_l} \right) (T_1(s_2 - s_1) - (h_2 - h_1)) = -36,188 \text{ kJ/kg H}_2$$

$$\dot{w}_{net} = \dot{w}_t + \dot{w}_c = -32,448 \text{ kJ/kg H}_2$$

f) Determine the second law efficiency of the Claude cycle:

$$\eta_{II} = \frac{w_{min}}{w_{net}} = \frac{-([h_7 - h_1] - T_1(s_7 - s_1))}{w_t + w_c} = 36.9\%$$

### Problem 3

We consider the desalination of seawater at standard temperature and pressure (293 K, 1 atm), modeled as a 1% concentration by mass of NaCl and assume that the solution is an incompressible liquid with ideal entropy of mixing, zero enthalpy of mixing and the molar heat capacity of seawater is constant and equal to the molar heat capacity of pure water.

a) We want to produce 1 kg/s of pure water from 2 kg/s of seawater. From mass balance, the brine flow rate equals to 1kg/s. Seawater is modeled as a solution with 1 wt% NaCl; therefore, 2 kg/s of seawater consists 20g/s NaCl and in the outlet all NaCl goes to the brine. As a result, 1 kg/s brine consists 20g/s NaCl and brine has 2% concentration by mass of NaCl. In terms of water, the seawater feeds in 1.98kg/s of pure water, which is split into 1kg/s of product and the remaining 0.98kg/s go into the brine.

b) The minimal work transfer rate (We will treat NaCl as a single species, which might not be precise since in aqueous solution it dissociates to  $\text{Na}^+$  and  $\text{Cl}^-$ ) is

$$\begin{aligned}\dot{W}_{\min} &= \dot{\Xi}_{sw} - (\dot{\Xi}_{brine} + \dot{\Xi}_{prod}) \\ &= (\dot{H}_{sw} - \dot{H}_{brine} - \dot{H}_{prod}) - T_0 (\dot{S}_{sw} - \dot{S}_{brine} - \dot{S}_{prod})\end{aligned}\quad (1)$$

We note that by assumption the enthalpy of mixing is zero. Hence, the first term is zero. Moreover, for the entropic term we can calculate

$$\begin{aligned}\dot{S}_{brine} &= \dot{n}_{H_2O,brine} (\hat{S}_{H_2O,brine} - \mathfrak{R} \ln X_{H_2O,brine}) + \dot{n}_{NaCl,brine} (\hat{S}_{NaCl,brine} - \mathfrak{R} \ln X_{NaCl,brine}) \\ \dot{S}_{prod} &= \dot{n}_{H_2O,prod} (\hat{S}_{H_2O,brine}) \text{ pure species} \\ \dot{S}_{sw} &= \dot{n}_{H_2O,sw} (\hat{S}_{H_2O,sw} - \mathfrak{R} \ln X_{H_2O,sw}) + \dot{n}_{NaCl,sw} (\hat{S}_{NaCl,sw} - \mathfrak{R} \ln X_{NaCl,sw})\end{aligned}\quad (2)$$

Combining these equations, we note that the standard entropies cancel out and we get

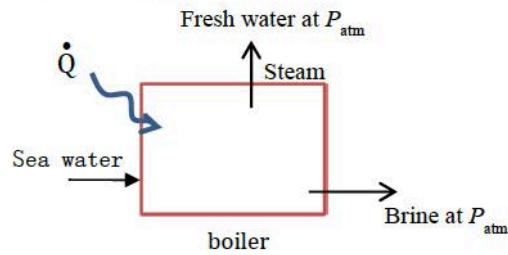
$$\dot{W}_{\min} = -T_0 \mathfrak{R} (\dot{n}_{H_2O,sw} \cdot \ln X_{H_2O,sw} + \dot{n}_{NaCl,sw} \cdot \ln X_{NaCl,sw} - \dot{n}_{H_2O,brine} \cdot \ln X_{H_2O,brine} - \dot{n}_{NaCl,brine} \cdot \ln X_{NaCl,brine}) \quad (3)$$

We now need to calculate the mole fractions of the two species in the three streams. Molar mass of NaCl is 58 g/mol and molar mass of H<sub>2</sub>O is 18 g/mol. Since 2 kg/s of seawater consists 1980 g/s H<sub>2</sub>O and 20 g/s of NaCl. The mole numbers of H<sub>2</sub>O and NaCl in the seawater stream is 110 mol/s and 0.35 mol/s, respectively, and molar fractions are 0.997 and 0.003, respectively. And for the brine, we know that 1 kg/s of brine consist 980 g/s H<sub>2</sub>O and 20 g/s of NaCl. The mole numbers of H<sub>2</sub>O and NaCl in the seawater stream is 54.45 mol/s and 0.35 mol/s respectively and molar fractions are 0.994 and 0.006 respectively. Then the minimal work transfer rate is:

$$\dot{W}_{\min} = -293 \times 8.314 \times (54.45 \times \ln(0.994) + 0.35 \times \ln(0.006) - 110 \times \ln(0.997) - 0.35 \times \ln(0.003)) = -597.8 \text{ W} \quad (4)$$

## Technology I: Once-Through Boiling

f) Flowsheet of the once-through boiling process is presented below:



g) In this process, heat is required to increase the temperature of seawater to boiling temperature and then evaporate it. We assume that the molar heat capacity of seawater is equal to molar heat capacity of pure water and hence equal to  $75.33 \text{ J}/(\text{mol}\cdot\text{K})$  and enthalpy of evaporation for the pure water is  $40.65 \text{ kJ}/\text{mol}$ . Therefore heat transfer rate required is

$$\begin{aligned}\dot{Q} &= \dot{n}(\hat{h}_{out} - \hat{h}_m) = \dot{n}\hat{c}_{water}(T_{boil} - T_0) + \dot{n}\Delta\hat{h}_{evap} \\ &= 110.35 \times 75.33 \times 80 + 55.56 \times 40650 \\ &= 2.92 \text{ MW}\end{aligned}\quad (11)$$

h) Assuming that the heat source is at the boiling temperature of water and we have an ideal heat engine, the “Carnot” efficiency of the heat engine is:

$$\eta_{carnot} = 1 - \frac{293}{373} = 0.214 \quad (12)$$

And the work transfer rate is equal to:

$$\dot{w}_{engine} = \eta_{carnot}\dot{Q} = 0.214 \times 2.92 = 0.626 \text{ MW} \quad (13)$$

The ratio of this work transfer rate to the minimal work transfer rate is

$$\frac{\dot{w}_{engine}}{\dot{w}_{win}} = \frac{626}{0.598} = 1047 \quad (14)$$

and thus the second law efficiency of this system is very low.

i) The most basic improvement that can be performed on this design is to recover thermal energy from the outlet streams. Without any heat recovery, the pure water is evaporated and enthalpy of evaporation is totally lost. What we can do is simply to pass the evaporated water from a heat exchanger and recover the heat while the pure water is condensed (which need to do anyways to sell/use the water) and then use the recovered heat to increase the temperature of seawater before feeding it to the boiler. Even further improvement is to reduce the temperature of liquid pure water more after the condensation to recover more thermal energy from the outlet stream.

### References

Chase, M.W., Jr. (1998), NIST-JANAF Thermochemical Tables, Fourth Edition, J. Phys. Chem. Ref. Data, Monograph 9: 1:1951.

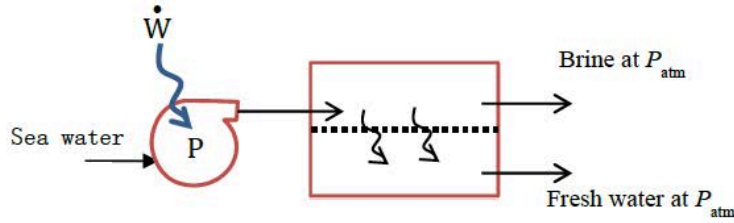
[http://en.wikipedia.org/wiki/Osmotic\\_pressure/](http://en.wikipedia.org/wiki/Osmotic_pressure/)

<http://webbook.nist.gov/chemistry/>



## Technology II: Reverse Osmosis

c) Flow diagram of the reverse osmosis process is presented below:



d) The minimum pressure required for the outlet of the pump is the osmotic pressure. The osmotic pressure can be calculated by the Morse equation as

$$P_{osm} = 2\hat{\rho}_{brine}RT, \quad (5)$$

where  $\hat{\rho}_{brine}$  is the molar density of the solute in the brine,  $R$  is the ideal gas constant and  $T$  is the temperature. And  $\hat{\rho}_{brine} \approx X_{NaCl}\hat{\rho}_{water}$

Thus, the minimal pressure for the pump is given by:

$$P_{H,min} = 2 \times 0.006 \times 1000 \frac{kg}{m^3} \cdot \frac{1000 mol}{18 kg} \times 8.314 \frac{J}{K \cdot mol} \times 293 K = 16.24 \text{ bar} \quad (6)$$

And the corresponding work transfer rate is

$$\dot{W}_{min} = \Delta\dot{H} = \dot{m}_{seawater} \frac{(P_{H,min} - P^0)}{\rho} = 2 \times \frac{(16.24 - 1) \times 10^5}{1000} = 3048 \text{ W} \quad (7)$$

e) For the actual plant we have

$$\dot{W} = \Delta\dot{H} = \dot{m}_{seawater} \frac{(P_H - P^0)}{\rho} = 2 \times \frac{(30 - 1) \times 10^5}{1000} = 5800 \text{ W} \quad (8)$$

For the second law efficiency for this plant we compare the minimum work in d) and e)

$$\eta_{II} = \frac{\dot{W}_{min}}{\dot{W}} = \frac{3048}{5800} = 52.6\% \quad (9)$$

Or, we can compare the minimum work in b) and e)

$$\eta_{II} = \frac{\dot{W}_{min}}{\dot{W}} = \frac{598}{5800} = 10.3\% \quad (10)$$

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