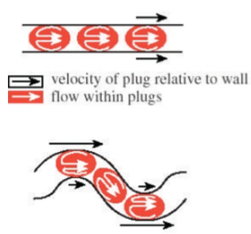
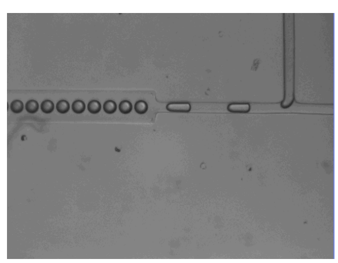
2.674
2016 Spring

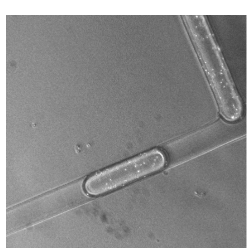
Introduction to Microfluidics III

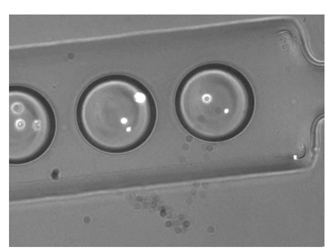
Hydrodynamics and Electrokinetics

Inside droplets: secondary flow

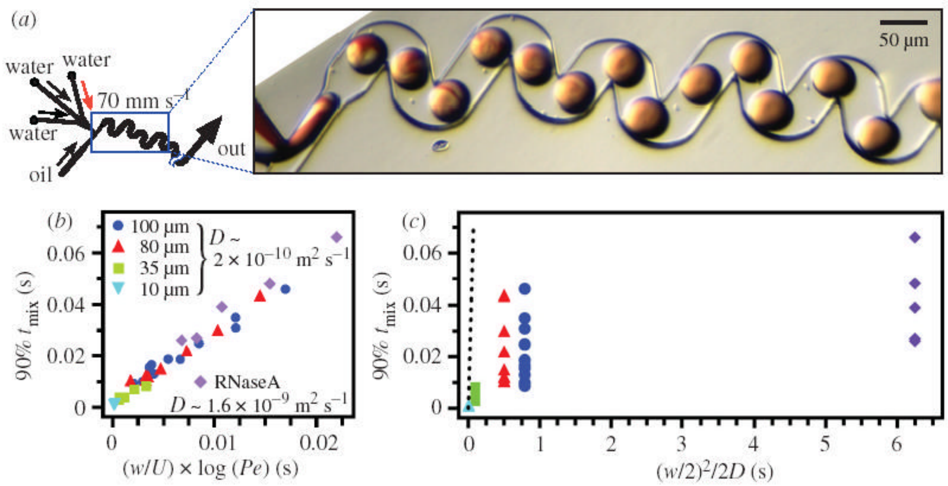








Chaotic Mixing in Droplets



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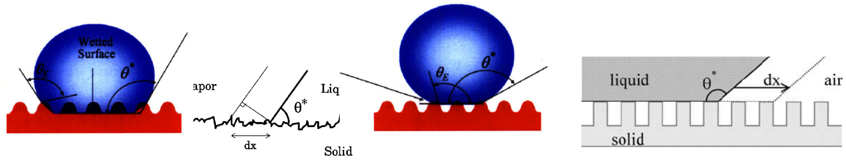
Effect of surface roughness



Wenzel's model

- If the surface has a high free energy, roughness promotes wetting.
- If it has low free energy, roughness promotes hydrophobicity.

$$\cos\theta^* = r\cos\theta$$

$$r = \frac{actual\_area}{projected\_area}$$

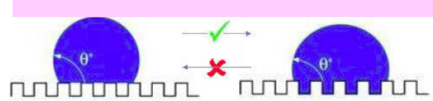
$$\theta^* = apparent\_contact\_angle$$

Cassie's model

- Wettability of heterogeneous (solid+air) surfaces
- Contact angle on air fraction is 180^0.

$$\cos\theta^* = -1 + \phi_s(\cos\theta + 1)$$

$$\phi_s = solid\_fraction\_surface$$



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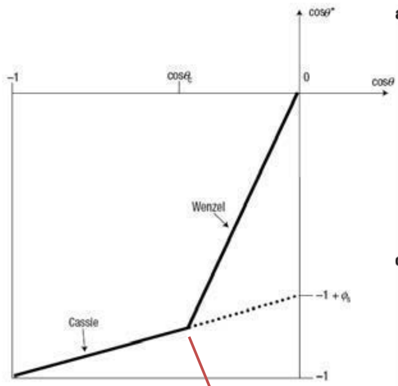
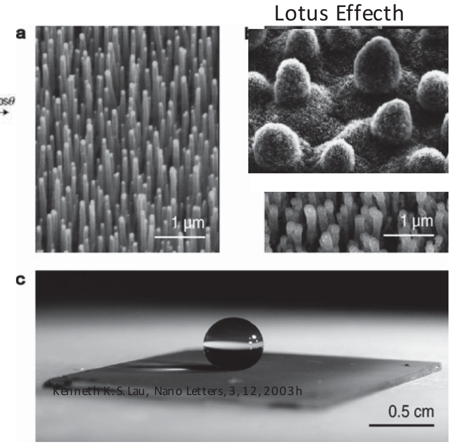
Cassie to Wenzel transition

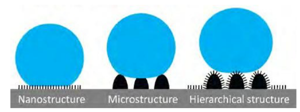


A. Lafuma Nature Materials 2 457-460 (2003)

Lotus Effecth



$$\cos\theta_c = \frac{\phi_s - 1}{r - \phi_s}$$



https://en.wikipedia.org/wiki/Ultrahydrophobicityh

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Electrowetting

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Digital microfluidics

Electrowetting on Dielectrics (EWOD)

Droplets can be manipulated by electrowetting

Application of voltage bias between electrodes used to manipulate droplets

Complex operations can be performed

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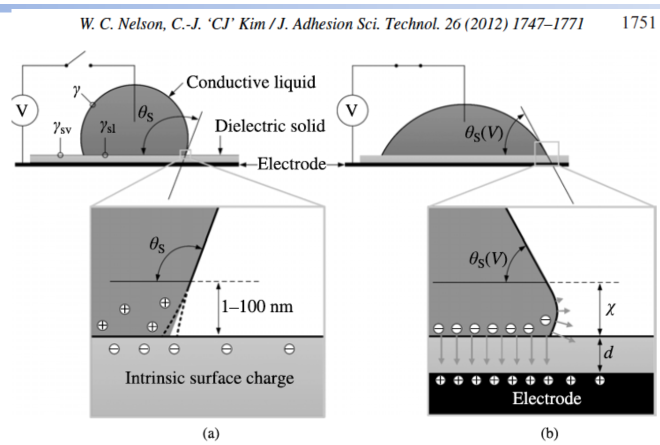
Electrowetting



$$\frac{Interfacial\_Force}{Length} = C\frac{V^2}{2}$$

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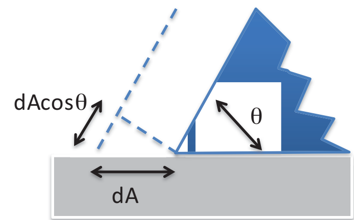
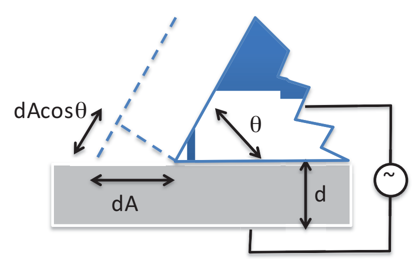
Work of Wetting

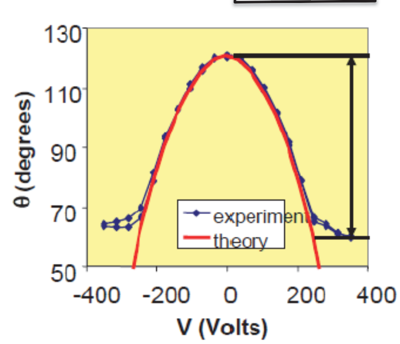
$$W_s = \gamma_{SG} - \gamma_{SL} - \gamma_{LG} \geq 0$$



$$dW = \gamma_{LG} \cos\theta dA + \gamma_{SL} dA - \gamma_{SG} dA = 0$$

Contact Angle vs. V



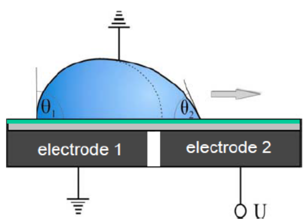


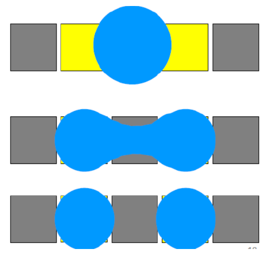
$$\cos\theta = \cos\theta_o + \frac{1}{2}\,\frac{\varepsilon_o \varepsilon_r}{\gamma d} V^2$$

Lippmann Young Equation

Drop Manipulation

- Merge
- Split
- Move





Digital microfluidics

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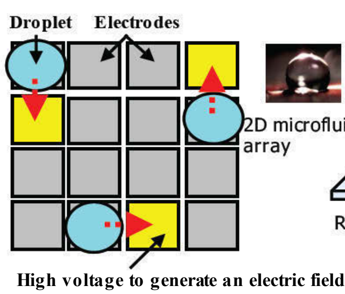

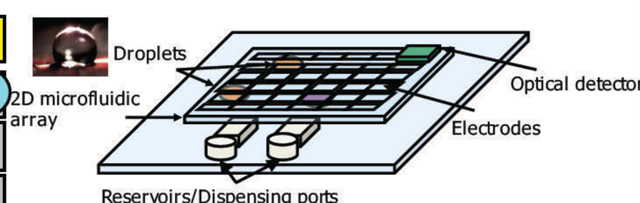

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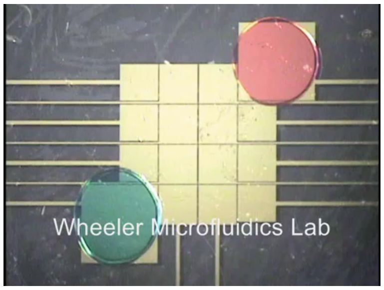
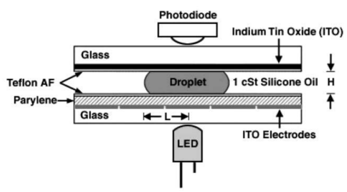
Digital microfluidics





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Electrokinetics

- Electrokinetics deals with electrically driven flow of charges, particles, and fluids typically in the presence of solid-liquid interfaces

Electrokinetics

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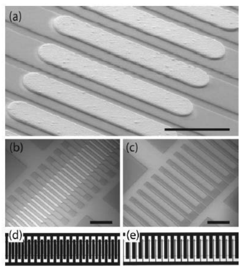
Fluid Conveyor Belt



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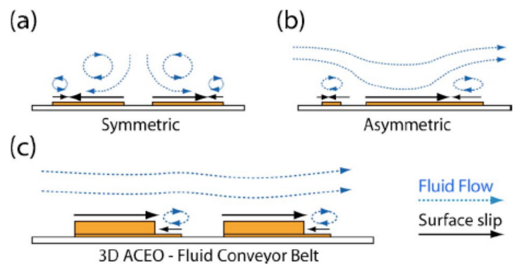
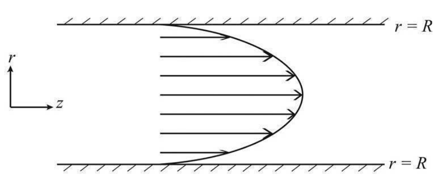


Induced Charge Electro Osmosis (ICEO)

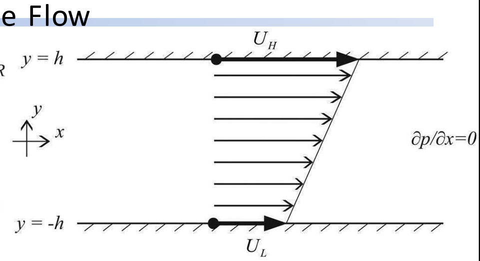
Electroosmotic pump, Urbanski et al, *APL* (2006)

Microchannel flows

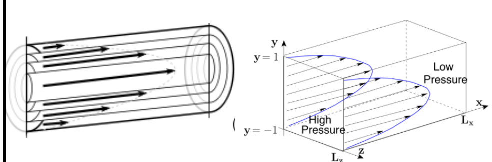
- Poiseuille Flow and Couette Flow



Poiseuille flow in a circular tube.



Couette flow between two parallel, infinite plates.



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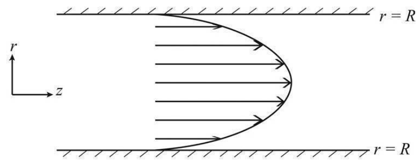
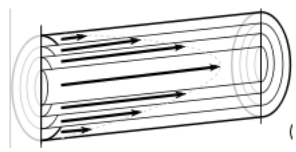
B. Kirby, Micro- and Nanoscale Fluid Mechanics, Cambridge Univ. Press, 2010

Poiseuille Flow

$$\nabla p = \eta \nabla^2 \vec{\boldsymbol{u}}$$



Poiseuille flow in a circular tube.



$$Q = -\frac{\pi R^4}{8\eta} \cdot \frac{\partial P}{\partial z} \qquad Q = \frac{\Delta p}{R_h}$$

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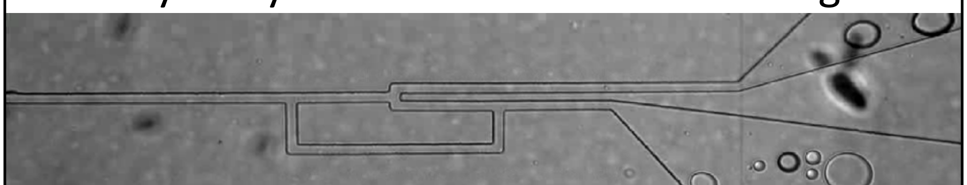
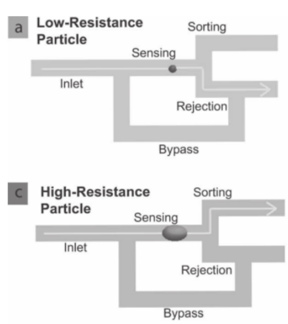
Hydrodynamic Force Assisted Sorting





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Couette Flow

Navier-Stokes

$$\rho\frac{\partial \vec{u}}{\partial t}+\rho\vec{u}\cdot\nabla\vec{u}=-\nabla p+\eta\nabla^2\vec{u} \quad \longrightarrow \quad 0=\eta\frac{\partial^2 u}{\partial y^2}$$

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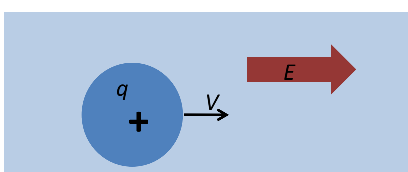
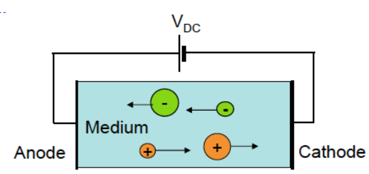
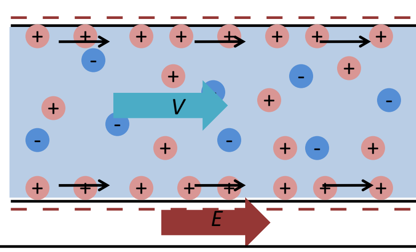
Electrophoresis and Electroosmosis

- Electrophoresis: Movement of charged particles under the influence of electric field





- Electroosmosis: Flow of fluid under influence of electric field



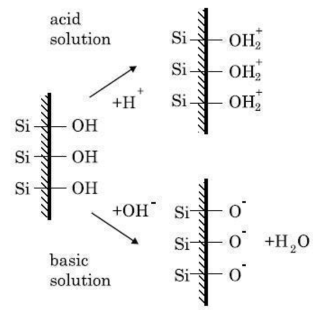
Origins of surface charge

Chemical origins:

- Dissociation of surface groups
- Adsorption of charged species

Physical origins:

- Induced charge



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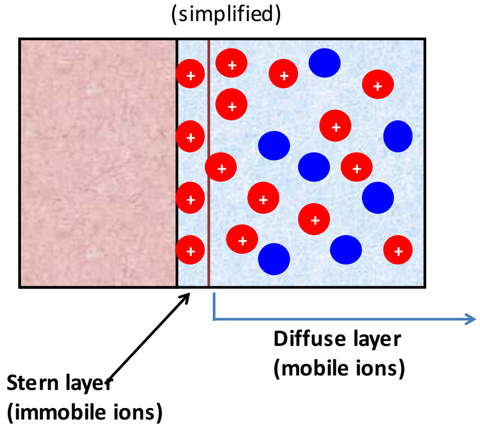
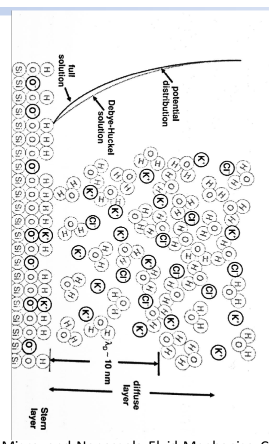
Electrical double layer

(simplified)





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Electroosmosis

Assumptions:h
$u = u(y)$
$v = w = 0$
E_{xy}= constant
p = constant

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- Macroscale Navier-Stokes eqn.h

- N-S equation with negligible h pressure drop, steady flow h $u(y)$, and surface chargeh

- Poisson eqn. (electrostatics)h

B. Kirby, Micro- and Nanoscale Fluid Mechanics for Engineers: Transport in Microfluidic Devices http://www.kirbyresearch.com/index.cfm/wrap/textbook/microfluidicsnanofluidics.html.

Electrical double layer

Gouy-Chapman Modelh

- Boltzmann distribution

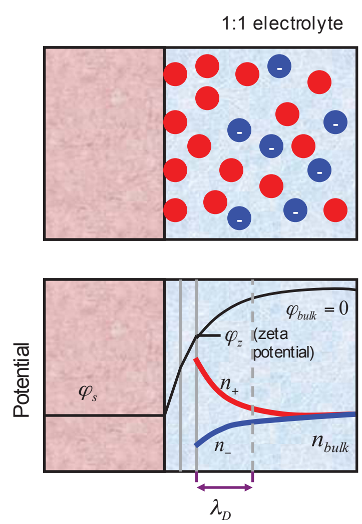
$$n_{\pm} = n_{bulk} \exp\left(\frac{\mp e\varphi}{kT}\right)$$

- Poisson-Boltzmann equation

$$\nabla^2 \phi = \frac{-\rho_E}{\varepsilon} = -\frac{e(n_+ - n_-)}{\varepsilon}$$

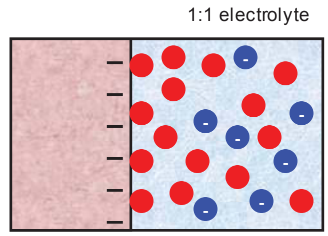
- Debye lengt

$$\lambda_D = \sqrt{\frac{\varepsilon kT}{2 n_{bulk} e^2}} \quad \text{(1-100 nm)}$$

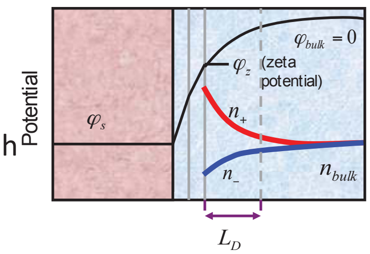


Electrical double layer

- Debye lengt
  - 1   KCl → ~0.3 nmh
  - 100 mM KClh→ 1 nmh
  - 1 m   KClh→ 10 nmh
  - Water → 1 μm



- Zeta potential
  - Typically 0-100 mVh
  - Determined by surface charge h and ionic concentration



Electroosmosish

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Assumptions:h
u = *u(y)*
v = *w* =y0
E_{xy}= constant
p = constant

$$\eta \frac{d^2u}{dy^2} = \varepsilon \frac{d^2\phi}{dy^2} E_x$$

$$\eta u = \varepsilon E_x \phi + C_1 y + C_2$$

BCs: h
Velocity is finite as y →∞
u(0) = 0 h

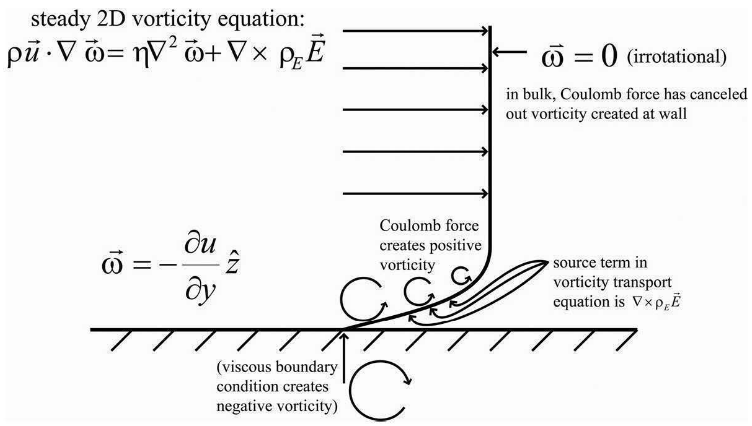
$$u = \frac{\varepsilon E_x}{\eta}\left(\phi - \phi_0\right)$$

$$u_{eo} = -\frac{\varepsilon\zeta}{\eta} E_x$$

(Hǝlmholtz-Smoluchowski equation)h

B. Kirby, hicro- and Nanoscale Fluid hechanics for Engineers: Transport in hicrofluidic Devices http://www.kirbyresearch.com/index.cfm/wrap/textbook/microfluidicsnanofluidics.html.

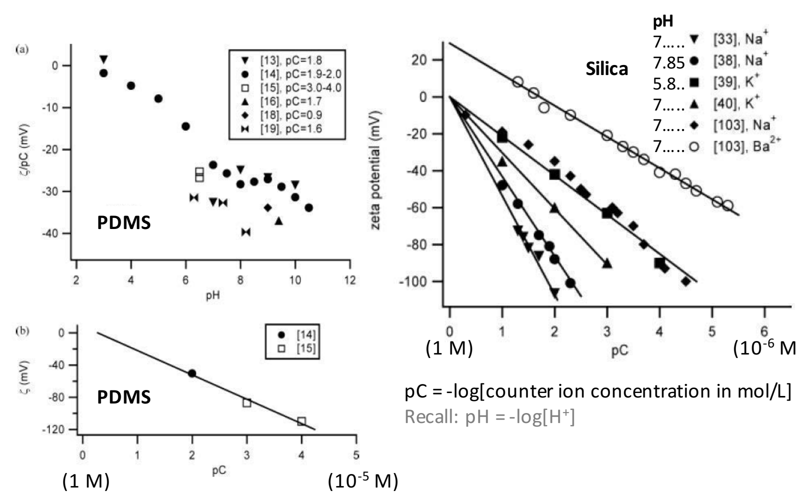
Vorticity Generation



Vorticity generation and cancellation in EDLs.

B. Kirby, Micro- and Nanoscale Fluid Mechanics for Engineers: Transport in Microfluidic Devices
http://www.kirbyresearch.com/index.cfm/wrap/textbook/microfluidicsnanofluidics.html

Zeta potentials of glass and PDMS



pC = -log[counter ion concentration in mol/L]

Recall: pH = -log[H^+]

Kirby & Hasselbrink, *Electrophoresis* (2004)

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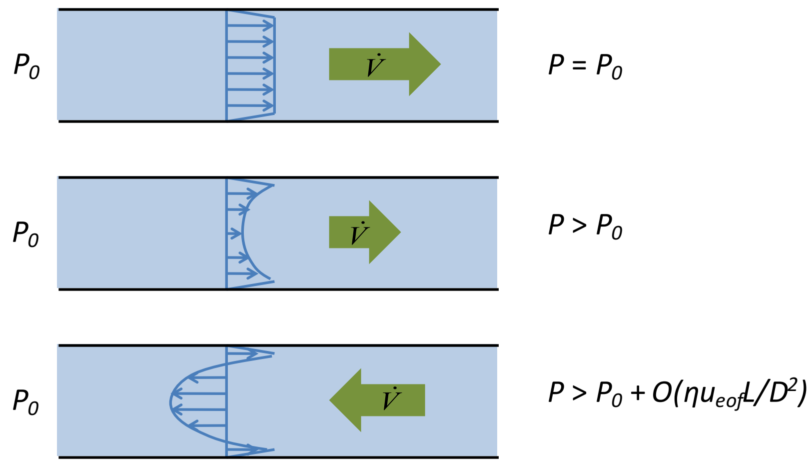
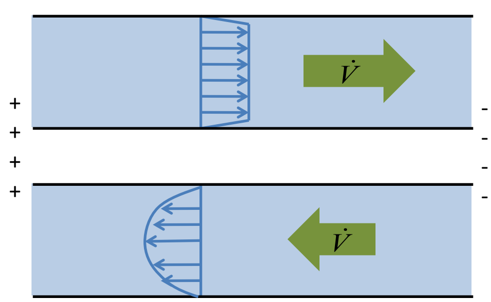
Electroosmotic pump



Electroosmotic pump



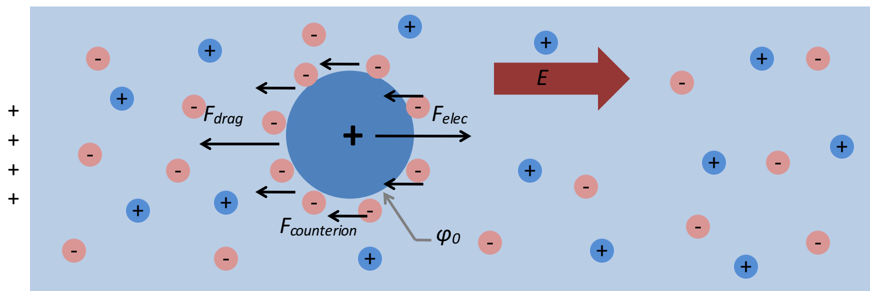
- u_{eof} $(= -\varepsilon\varphi E_x/\eta)$ is independent of diameter
- Velocity due to pressure difference $u_{\Delta P} \sim D^2$

$$\overline{u_z} = \left(-\frac{\partial p}{\partial z}\right)\frac{R^2}{8\eta}$$

$$\Delta P = -\frac{8\varepsilon\zeta}{R^2}L$$

→ Electroosmotic pumping is more effective at small length scales!

Electrophoretic mobility

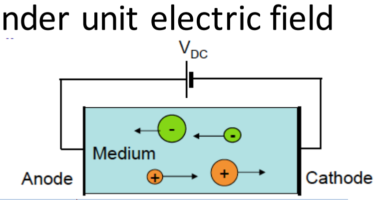


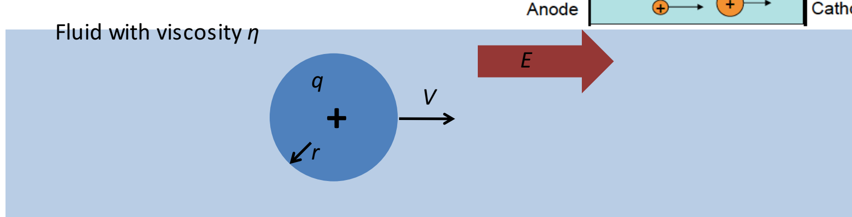
- Presence of counterions affects mobility

Electrophoretic mobility

- Mobility (μ): Particle velocity under unit electric field

$$\vec{V}_{ep} = \mu\vec{E}$$





$$F_{drag} = 6\pi\eta r V$$

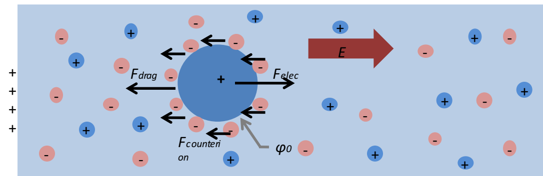
$$F_{electric} = qE$$

$$\vec{V}_{ep} = \frac{q\vec{E}}{6\pi\eta r}$$

$$\mu_{EF} = \left(\frac{q}{6\pi\eta r}\right)$$

(Hückel equation)

Capillary Electrophoresis (CE)



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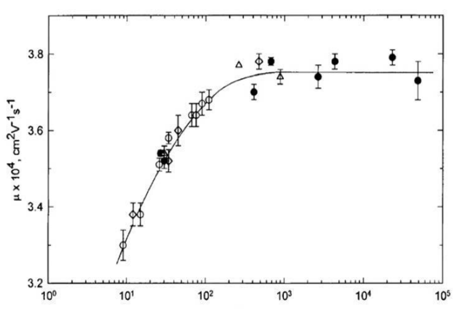
Electrophoretic/electroosmotic mobilities

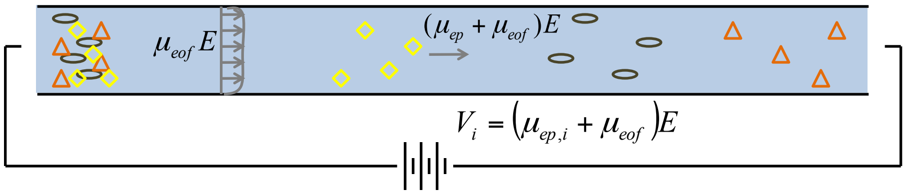
- Small monovalent ions (Na^+, K^+, Cl^-, etc.): ~10^{-7} m^2/v.s
- Cells: ~10^{-8} m^2/v.s
- In PDMS, glass: ~10^{-7} to 10^{-8} m^2/V.s
- DNA: 4 x 10^{-8} m^2/V.s or lower (in gels)



DNA mobility in TAE buffer
Salieb-Beugelaar et al., *Lab Chip* 9, 2508 (2009)

Image courtesy of U.S. National Library of Medicine.

Electrophoretic separation



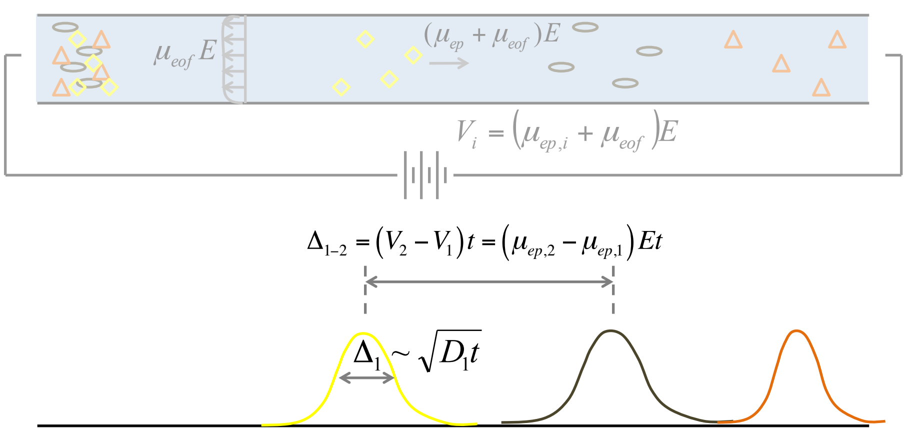
Separation based on differential migration velocities

Migration velocities depend on

- Minimize elelctroosmotic flow, PVP (polyvinylpyrrolidone)
- Size and charge of particles
- Interaction with gels/pores, if any

$$\mu_{EF} = \left(\frac{q}{6\pi\eta r}\right)$$

Electrophoretic separation

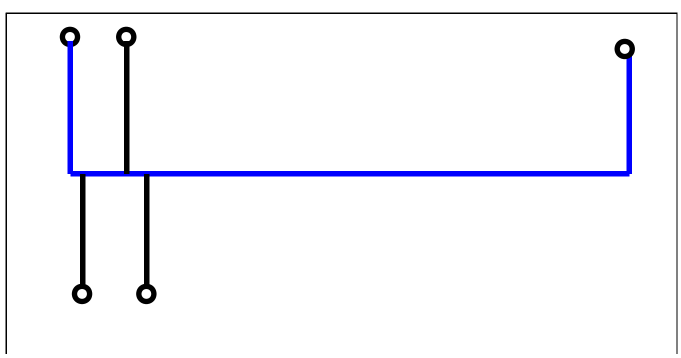


$$\frac{\Delta_{1-2}}{\Delta_1} = \frac{(\mu_{ep,2} - \mu_{ep,1})Et}{\sqrt{D_1 t}} \propto E\sqrt{t}$$

Electrophoresis in microfluidics

- Advantages:
  - Higher electric fields easily possible
  - Shorter time-to-result
  - Integration with other processing steps
  - Low sample requirements

Step 1: Injection of buffer solution



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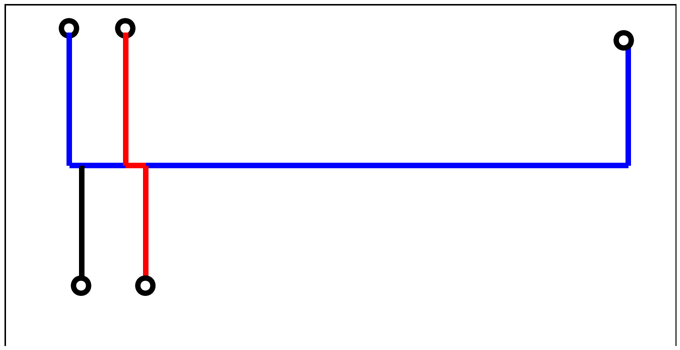
Step 2: Injection of sample solutions



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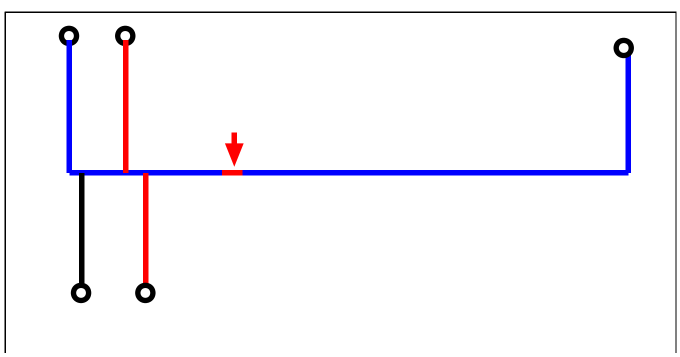
Step 3: Start of electrophoresis



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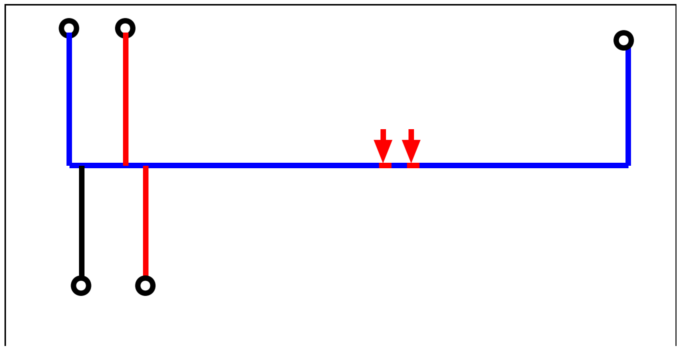
Step 4: EP Separation



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Capillary Electrophoresis (CE)

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Please see the image on Page 9 of http://nanoparticles.org/pdf/noh.pdf.

This week

- Examine the motion of microparticles under electric fields
- Study the effect of surface treatment on electrokinetic flow
- Separate and analyze molecules using gel electrophoresis

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