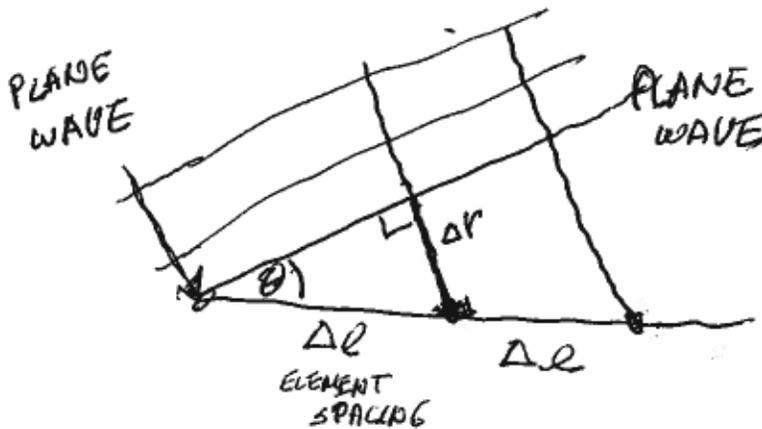


Lecture #11 Lecture Notes

Some simple beamformer equations

Again, at this level, the simple concepts of beamforming should be well known, but at the risk of a little triviality (never a great risk....people are good at skipping over material they already know!), here are a few small concepts and equations that are useful. They are: 1) the plane wave beamformer, 2) the focused beamformer, and 3) the grating lobe equation.

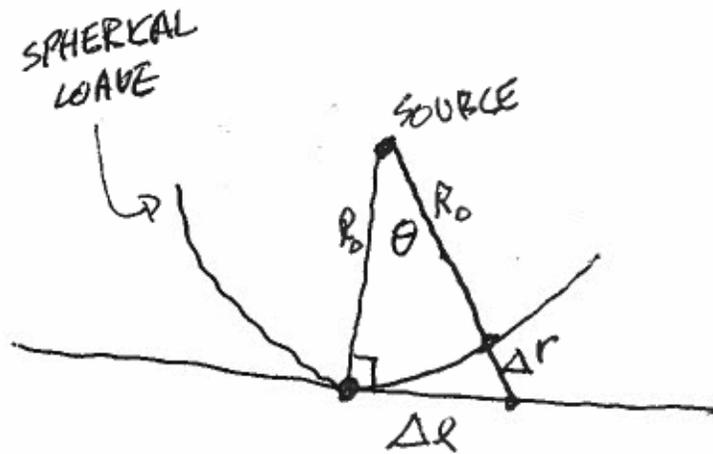
1) Plane wave beamformer



Consider the diagram above, showing a plane wave impinging at angle θ on a line array whose receiving elements are spaced uniformly by a distance Δl . Using the law of sines, we easily see that the added distance from one element to another is $\Delta r = \Delta l \sin\theta$. This distance results in a delay time for the wave hitting one element to the next, which is simply $\Delta r/c = \Delta t$. If we counter this delay electronically (called a “time delay beamformer”), we get the beamformer to steer to the angle θ , with (approximately) the same shape as the broadside beam. This “time delay beamformer” also works the same for all frequencies, i.e. can do broadband beamforming. There is an alternative way to do beamforming, where one looks at the phase shift (at a given frequency) at each element, using the simple relation $\Delta\phi = \omega\Delta t$. One then adjusts the phases for a given frequency. This is an inherently narrowband beamformer, as the phases for one frequency do not work for another.

2) Focused beamformer

For a point source that is near enough to a receiving array that wavefront curvature effects need to be considered, i.e. greater than a nominal $\pi/4$ phase shift across the array due to the curvature, the array can be focused to deal with that curvature. Again, the geometrical considerations are very simple, as seen in the figure below:



The distance one has to correct for wavefront curvature is simply given by Pythagoras' Theorem, i.e. $R_0^2 + (\Delta l)^2 = (R_0 + \Delta r)^2$. If we expand the RHS, and ignore terms of order $(\Delta r)^2$, we get

$$\Delta r \approx (\Delta l)^2 / 2R_0$$

Converting this to a time delay or a phase, as above, lets us correct for the curvature and again get (approximately) a broadside beam.

3) Grating lobe equation

A final "simple but useful" short equation is the grating lobe equation, which tells you where a "repeat major lobe" (a duplicate of the main lobe with the same amplitude, as opposed to a sidelobe) will show up IF one does not spatially sample the array at the Nyquist criterion of half a wavelength or smaller. It is also the "grating equation" from spectroscopy and the "Bragg equation" for crystals. It is

$$n\lambda = (\Delta l) \sin \theta$$

In the above equation, n is the order of the repeat major lobe and θ is the angle it will be found at. Generally, repeat major lobes are an undesirable feature for beamformers. (However, they can be useful in spectroscopy, as the higher order spectra are more dispersed.)

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