SE BAEK OH: Good morning. My name's Se Baek, and I'm a postdoc working with Professor Barbastathis. And he has a personal emergency today, so he couldn't make it. So actually Pepe and I will give a lecture today. And just for a reminder, you can turn in your first homework today before class or after class.

OK, so yesterday, we talk about a bunch of different things. The first one we derived-- by the way, excuse me, we are going to do the same thing, same format. So if you have any questions, just feel free to interrupt and ask questions, OK?

So the last time, we actually derived the ray transfer matrix for a thin lens, which is the spherical lens has two curvatures, the first one and second one, which is here, the 1 and negative 1 over f. f is focal length and 1, 0. And the focal length 1 over f, this actually we derived this equation, the [INAUDIBLE] equation. So if you know the reflective index and the radius curvature, the left side and right side, then you can just easily compute the focal length of your thin lens.

Then, also, we talked about the object at infinity and the image at infinity. So for example, in this case, we have the object at infinity. So we have this ray is coming from the infinity. And we are going to have the focus point at the focal plane.

That's the definition of the focal length and focal plane. And the other way, if we had the point object at focal plane, then we are going to have an image at infinity. So that's the same thing.

And if you have the negative length, I mean, the focal length is negative, then we have the same thing. We have parallel coming in. In this case, it bent outside. But if you extract and extend, then we have the focal point here.

So as you know the object and image they are conjugate. And in this case, [INAUDIBLE] object or image is at infinity. So we call it, I mean this configuration, as infinite conjugate.

[INAUDIBLE], please. Yeah. Because, you know, one of them is at infinity. So yeah, that's what we talked about yesterday.

And today, we're going to be talking about interesting topic. So what if we have our object or an image is at finite distance? It's more realistic case. Just like when you take a picture of your person, I mean, of a friend, then your friend is not going to be at infinity, right? So it's more realistic case.

And then we'll talk about thick lens. Because so far, we just assume our lens is infinitely thin. So thick lens is more like the realistic case. And then the paper we'll talk about is human visual system.

So I guess this slide is probably one of the most important slides of this whole course. Because, you know, we are dealing with the object and image at finite distance. So the situation is like this. We have a lens with focal length F. And in object space, we are given our object as some distance.

You know, the situation's like this. You have a camera. And you take a picture of a person. Then where should I put my screen or detector?

Or in the other way around, there must be some condition that makes the perfect image, right? So if you take a picture at some distance, then the other object which is not at that distance will look blurry in your final image, right? So we are going to find what condition should be satisfied here in terms of the distance in object space and focal length and distancing image space. And we're going to see what kind of image we will get or how big it is, OK?

So the first one is, what is image. So what it image? So image is just replicate of an object, right?

So in terms of this ray picture, we have infinite number of rays-- oh, sorry-- emanating from the object point. And if this configuration is an image configuration, which means you make a perfect image, then all these rays, at least most of them, they should be converge at a point in object space somewhere, right? So that's the image in question. You have bunch of ray coming from this point, and they should meet at one point.

So you can draw many, many ways, but of your choice is just two rays here, this first ray and the second way. And since every ray should meet at one point in image space, these two ways also should meet at the same point. And the reason why we chose only two rays is kind of obvious, because we know the infinite conjugate configuration, right?

So the first ray, the upper ray here, this ray, it looks like coming from the object at infinity, right? It's parallel to the optical axis. So after the lens, it should pass the focal point like this. And we call this point as back focal plane, since it is behind the lens.

And what about the second ray here? So if you choose, that ray is passing through the focal point which is in front of the lens, we call the front focal plane, then at the lens it looks like coming from the focal point, right? So it should be going to the infinity. So it's collimated and parallel to the optical axis.

And these two rays meet at this point. So that point is where you get the image. And the other rays, the infinite number rays, should meet at that point.

So this is how we draw the configuration image. So we have bunch of ray and bunch of ray in image space. Any questions so far? OK.

So to find the relation between the object distance and image distance, let's define two distance here. So first one is x0, the distance from the object to the front focal plane. And this second one is xi, so from that focal point to the image. So we are going to see how they relate to the focal length here.

So first thing is we can compare it to a big triangle here. So the first one is-- oops-- this triangle OPFf and this one, BCFf. Since they are similar, so if you see this height of PO and this BC is same as h-- actually, they are h object over xo, so ho and xo.

And it should be same as negative h i. The negative sign is because h i is negative. So to make the positive, you have to put the negative sign here. And this [INAUDIBLE], right? So you get this first relation.

And you can compare the other triangles at image space, so this ACFb and IQF, this one. So if you do the same thing, then we have this relation. So QI, which is the image height, and FPQ, this is xi. And we have the h, the object height, over focal length. So we have this relation and the second relation.

So if you combine them, we have two interesting equation here. The first one is h, the image of height, over x, or the object height, is xi over h or equal to negative f over xo. And it's also same as negative xi over f. Can anyone guess what this quantity means, So h i over ho, so image height or object height? Yes.

AUDIENCE: [INAUDIBLE]

SE BAEK OH: Right, exactly.

AUDIENCE: [INAUDIBLE]

SE BAEK OH: Yeah, for he said magnification. Yeah. Course, you know, this is a ratio of the size of the image and object. So actually, this term indicate the magnification of your imaging system, right?

So magnification actually is defined by this, just h i over ho. If the absolute value of magnification is larger than 1, then you're going to get the magnified image. Course, the image height is bigger than object height, right? And if it is smaller than 1, we are going to get the smaller image, right?

And if you combine the rest, these two equations, so f over x and xo and xi over f, then you can get this equation. xo times xi should be f squared. So xo is this distance, so object to the focal plane, the front focal plane. And xi is from that focal point to the image plane.

So this tells you where should you put the detector or object to make an image condition, right? So they are related like this. So this is one of the form of the imaging condition. And we it Newton's form, OK?

And sometimes it is more convenient to define the distance from object to the lens, not the focal plane. So we define two other distance, so s o, object distant from object to the lens, and image distance, si, from lens to image. And actually in this picture in this figure, we can draw another way, which is from object to the image. But it passed through the center of the lens, and it looks like just a path straight through the lens. And can anyone guess why?

Because at the center of the lens, at the very center, the radius curvature is kind of infinite, right? So it just looks like just piece of flat glass. So if you have this piece of glass, then you know from the [INAUDIBLE] law the incident ray and the outgoing ray, they are parallel, right, if you have the same median here and here.

And also, we are talking about the very thin lens here. So we don't really consider the thickness of the lens. So that's why it looks like just passing straight through the lens. So you can draw actually three rays here, so the first one, this one, and this one, and this one.

And then you can compare two, actually, another big triangles, so OCA this one, and IQC, this one. So if you compare that triangle, then we're going to get-- so CA over CB, so CA [INAUDIBLE] CB, which is the, actually, magnification here, ho over negative h i should be same as this object distance and image distance. So this tells you, if you have the object distance and image distance, they are related to the magnification.

So if your object is very far away from the lens, then you're going to get very small image, right? Because you have [INAUDIBLE]. So you have the very big one here, so you're going to get a small one. But if you move your object to the lens, then there are big change. You're going to get the magnified one, magnified image.

And if you combine this equation and this relation and these two, then we are going to have the si over s o. And it's the same as f over xo. So xo is actually s o minus f, right, so s o minus f.

Yeah, let me do the [INAUDIBLE], please. So let me rewrite that equation. I just flip the numerator and denominator. So s o over si is same as s o minus f and f. And it should be same as s o over f. And it's minus 1, right? And I divide by s o here. So I get 1 over si should be 1 over f--

AUDIENCE: [INAUDIBLE] s i or [INAUDIBLE] s o, or so over [INAUDIBLE]?

SE BAEK OH: No, I just flipped the numerator, denominator. So it was si over s o, but I wrote s o over si. So this one also flips, right, s o minus f over f.

And I end up with 1 over si equal to 1 over f minus 1 over s o, which is the final equation. So we call this the lens law. So 1 over object distance versus 1 over image distance should be 1 over f.

So, yeah. So actually, that's the equation in most of time we are going to use. OK, so we just finished probably one of the most important slides. Any questions?

So by using this equation, we can analyze different situation. So let's first see the four different situation here. We have positive lens and negative lens. And the upper left situation is what we just described.

So the object distance is longer than front focal length. So you draw the two rays, the upper one and right one. And you just trace them, and they meet at one point.

And you're going to get the image. And we call this one as real image, because all this ray MRI is departing at that point. They actually meet at this point.

So if you put a screen there, then you can actually see those image of the object. So that's why we call the real image. And it's inverted, right? Because it was going up right. It's down right.

And magnification is definitely small. It's negative, because inverted. And also, if you see the distance, object distance, this one and this one, then you can easily see you're going to have-- actually, it depends on the object distance, but yeah.

And in terms of equation, so this s o is bigger than focal length, right? So this 1 over s o is actually smaller than 1 over f. So si should be positive. You need to add something, right? Because this one is smaller than this one. So that's why we have the si, which is positive distance here.

But if you think about this situation, so object distance now is smaller than focal length. It's closer than the front focal plane. Then you can still draw these two rays.

One is parallel to the optical axis. And the other one is passing through the front focal plane. But after the lens, actually they don't meet, because they are diverging.

So if you back trace, then they actually meet in front of the lens. So we call this as a virtual image, because it looks like coming from that point. But if you put a screen there, the-- just a second. So [INAUDIBLE], I'm not using the [INAUDIBLE] right now. So OK, thanks.

So if you put the screen there, then you don't really see the actual image. It just looks like these two rays behind the lens, they look coming from this point. So that's the definition of the virtual image.

And as you can see here, we have the erect image. And it's bigger than the original object, which is green. So magnification is bigger than 1.

In terms of the equation, so now s o is smaller than f. So 1 over s o is actually bigger than 1 over f, right? So actually this guy should be negative, which means si also should be negative, which indicate your image distance is negative. So you are going to have the image in front of your lens, right?

And the negative lens, we have the negative focal length here. So that's why the front focal plane is actually behind the lens, not in front of the lens. So you can still draw these two rays, but they don't meet.

They make a virtual image in front of the lens. And we have the virtual and erect image and magnification. We get actually the smaller image. And if you do the same thing here, we are going to get the virtual, erect. And it's also smaller image.

OK, so, so far, we just talk about the thin lens and how they make the image-- sorry, Pepe will pass around the convex lens, which was the upper case. So if you move your screen or eyes, then you can see actually it was inverted, but it's erect. So you have to see the flipped image, right? So he's going to pass around.

GUEST SPEAKER: So one way to do it is just look at the lens, look at your notebook. And you can have it just at the right distance, the image would look inverted and floating on top of the lens, which is the real image. And if you put it closer, which means that now we have the second case there, the image will be erect, so not inverted anymore, and would look behind the lens, right? So it's actually pretty dramatic, the change.

SE BAEK OH: Yeah. Actually, the upper right configuration is the exact same from as magnifier. So you know, magnifier that detective are using? Yeah, So the next question is, what if we have multiple lenses like this, this case?

So here we have two lenses. Each one has a focal length of 10. So let's say unit is a millimeter. So the focal length, is just 10 millimeter, 10 millimeter. And the gap between the lens is 5 millimeter.

And we are given an object, which is the distance from the object to the first lens is just 5 millimeter. And we want to find what kind of image we get and where it is at somewhere around here. So can anyone guess how to do it? Because we just talk about a single lens, but what if we have the multiple lens?

No one? So actually the answer is pretty straightforward. So you just cascade, which means your first [INAUDIBLE] the first lens. So you just find the image of the object through the first lens.

And that image becomes the object to the second lens. And do the same thing to find the image. And if you have more lens, you just repeat the same process, OK?

So let's first try it. So here I just neglect the second lens. I only have the first lens, which has the object distance 5 millimeter. And the focal length is 10.

So I can draw the two or three rays to find the image. But to be precise, I just plug into equation. So 1 over 5, which is this distance, the 5 is this distance.

We want to find the image distance. If it is positive, then we are going to get real image. But if it is negative, then we are going to get the virtual image here. And it should be 1 over 10, which is the focal length, right?

And it turns out that this s prime, this image distance, is actually negative 10, which means we have the negative image, which is in front of the lens. This distance is 10 millimeter. And if you compare the magnification, which is defined by the object distance and image distance, so it's negative, there is a negative side.

So negative, negative, negative 10 over 5 is actually 2. So we have a erect virtual image which is twice larger than original image. This is the first lens.

And this image is now an object to the second lens. So we had the virtual image here, which was the 10 millimeter in front of the lens. But if you only think about second lens, then this object distance should be 15 millimeter, right?

So if you do the same thing. so now we have the 1 over 15 from here to here. And we want to find the image distance. 1 over s prime should be 1 over 10, which is just positive 30 millimeter.

So after 30 millimeter, we are going to have the real image. And if you compare the magnification of the second lens, then it's negative 30 over 15. So we have the negative 2.

So we have the real image here, which is inverted one. And that is twice larger than this virtual-- I mean, the object of the second image. Because, actually, we initially had two lens like this. So the overall magnification is just multiplication of individual magnification.

So first one was positive 2. And this is negative 2. So the final magnification is minus 4. So the answer to the question is we are going to have the inverted real image behind the second lens, which is 30 millimeter behind the second lens, and which is 4 times larger than the original object.

So if you have fewer lens, like 2 or 3, then this approach is pretty straightforward, right? We just consider one by one. And just apply the lens law, and you're going to get the right answer.

OK. So the previous slide we just derived the imaging condition, which was the xo times xi should be x squared or 1 over s o plus 1 over si is 1 over f. We just derived those equations from the geometry, right? We just compared the few triangles and get the final equations. Well, I can do the same thing with the ray matrix transform. So let me do [INAUDIBLE], please.

So the way you do it, in the ray transfer matrix, you first define the angle and height in object space and image, right? So here I have alpha i and xi, which is this angle and the height of the image, h i actually. And the input was xo and ho. It's just this angle and this height.

This ray is propagating from left to right, but I write the metrics from right to left, right? So first thing is the propagation from here to here, which is the distance by s o. So the first matrix should be 1 0, s 1, am I right?

Oh, as oh Yes so we just first consider from here to here. And then you have the lens, which we just derived last time, which was 1 negative 1 over f, 0 and 1. And the next one is from here to here, the same propagation.

So we have 1 0, si 1. So this is the matrix formulation in this case. So if I compute these matrices, then actually I get this equation.

So let me write s o f si plus s o minus si s o f 1 over f, 1 minus i over f ho. So can you explain the imaging condition from this matrix?

So we just derived the relation between angle and height in object, for the angle and height in image. And we have the transfer matrix here. So what's the imaging condition in this case?

So let's go back to the first [INAUDIBLE]. So what is the image? So we have a bunch of rays starting from the object point.

And then all they arrive at the same point in the image, right? So if you just think about the h i and ho here, so at that point all these rays have the same height, ho. But they have different alpha o, these guys, right?

But the image aside, the same thing, all these rays they have the same height, h i. But they have a different angle, alpha i. So if you just compare the h i and ho, you know, those rays, even though they have different angles, but they have the same height, which means actually this h i should be independent from alpha 0. Because no matter how they depart at that point, they have the same height here, right?

So the answer is actually this term, because this is alpha i and h i. So this term should be 0. Because as I just described, h i should be independent on alpha object, right? So if I do the math, then this is 1 over-- I just divide by s o and si.

So this one is 1 over s o plus 1 over si minus 1 over f. It should be 0, which is, again, the lens law. Yeah, so that's the [INAUDIBLE] I described.

And if I plug in this equation in this matrix, finally I get these matrices So if I continue at this occasion then minus x over F1 of them zero minus x I over f OK so the upper right on is we still have a negative on about f which is our last power or optical power

and what is this time the upper limit Yeah this the last time negative x I over f anyone and there It's easy one. Press the button, please.

AUDIENCE: Magnification?

SE BAEK OH: Yes. Since this time is 0, so if you think about the second row here, then you have the h i is just this guy multiplied by ho, right? So if you just think about the second element here, then h i is negative xi over f, which is ho or object height. So this time actually tells you the relation between the object height and image height, which is the magnification.

So I should say the lateral magnification, because we are talking about the size. So this term is magnification. And it turns out that the upper left term, this term, we call the angular magnification. Because there is related with the alpha i and alpha o.

To be more precise, actually angular magnification is defined by the ratio of change in the image angle and object angle. And if you do the math, then we finally get this term, negative xo over f. And it turns out that the angular magnification is 1 over lateral magnification. Yes.

AUDIENCE: Yeah, what's the idea of angular magnification? We understand that lateral magnification means how much the object is scaled. Yes. But in the case of imaging, what does it mean?

SE BAEK OH: That's actually a good question. So actually, if you see the definition of the angular magnification, it's actually there is the delta, right? So this delta alpha change in image angle over change in object angle, which means let's think about us those rays here and here.

So you have the ray angle in object space and another ray in image space. If you change this angle in object space, then you're going to also change the angle in ray, I mean in this side, the image space. So actually, angular magnification tells you how they are related.

So if you change this much, then what do you get in this side? How much change you get in image space? So I guess that's the proper interpretation of angular magnification. It's the answer to your question?

AUDIENCE: Yeah, thanks.

SE BAEK OH: Yeah.

AUDIENCE: And does it relate to numerical aperture in some sense?

SE BAEK OH: Sure. Actually, the angular magnification is xo over f, right?

AUDIENCE: Yeah.

SE BAEK OH: Not really. Yeah, it just depend on the-- you know, in this case we don't have the notion of the size of the lens here.

AUDIENCE: Yeah.

SE BAEK OH: So it's not related. Yeah.

AUDIENCE: Yeah, thanks.

SE BAEK OH: Any more question? I'm supposed to go very slowly, but there's no questions. Then let's talk about the next topic, so thick lens.

So, so far, we just think about just thin lens. All these were ray transfer matrices or imaging condition. We just consider the thin lens, which has the two curvature.

But we didn't really account for the thickness of the lens. But if you think about the real lens we've just seen, the biconvex lens, actually they have finite thickness, like this. So you have the first surface and second surface, but it has the finite thickness.

So what is the more accurate or, I should say, more rigorous way to model, to make a model for this lens? This answer is obvious, right? So you just first take the refraction at first surface and just propagate the array inside the glass, right?

And then take the second refraction at this second surface. So that's, you know, the proper way to describe. So I can do the ray transfer matrix here.

Yeah. So we start with the alpha 1 and x1, which is the angle and height in left side and alpha 2 and x2. So it's alpha 2 and x2 and alpha 1 and x1. And the first matrix should be the refraction at this first surface.

So it should be one negative left to right and R1 1 0. By the way, is R1 positive or negative? So R1 is the curvature, radius of curvature this first surface.

We talked about that last time. If the center of the radius curvature is that way, I mean, which is the positive way, then this radius curvature is positive. So actually R1 is positive.

And next matrix should be propagation from first surface to the second surface, which is 1 0 d over n. So don't forget n in denominator, because it's not [INAUDIBLE]. The ray is propagating inside the glass.

And the last matrix should be 1 and minus negative R left to right and radius curvature 1 0. So these three matrices describe this thick lens, right? So we just consider the two reflection and the propagation through the glass.

And if I compute these matrices, then I get then actually M21 M22. So I'll get the four different elements, right, which is--

AUDIENCE: [INAUDIBLE]

SE BAEK OH: So actually if you compute those three matrices, I get this one. But I just symbolize M11, and M12, and--

AUDIENCE: [INAUDIBLE]

SE BAEK OH: Which one?

AUDIENCE: [INAUDIBLE]

SE BAEK OH: d over n.

AUDIENCE: d over n.

SE BAEK OH: Yeah. Because distance is d, but the reflective [INAUDIBLE] is n. So you need to divide by n.

Because, previously, we had the thin lens, which is just thin lens like this and incoming ray. They converge at point like this. And this was focal length, right? And the ray transfer matrix for this thin lens was 1 negative 1 over f, 0 and 1, right? So we get the-- yes?

AUDIENCE: [INAUDIBLE]

SE BAEK OH: Oh, where? Oh. [INAUDIBLE]. I'm sorry, yes. All right, so it should be n minus 1 and 1 minus n. Yeah, it's confusing.

Actually, the way I remember is the reflective index at left side minus the reflective index right side. But there was negative sign, so yeah. I got [INAUDIBLE]. Yeah. Thank you.

Sorry. Yeah. So what I was going to talk about is if we have the thin lens, then the ray transfer matrix it was the 1 negative 1 over f and 1 0. But now, we have the thick lens. That's why we have complicated these four terms.

But we get the same analogy of distance, because distance describe how much ray bend, right, the optical power or lens power. So we get the same thing here, which is actually this complicated term, this term. And since it is a thick lens, we define a new quantity which is effective focal length.

So we consider that is a single quantity which was negative 1 over effective focal length. So effective focal length is actually the quantity inside the bracket, so n minus 1 and 1 over R1 minus 1 over R2. And we have extra term. So what is this term, the n minus 1 and 1 over R1 minus 1 over R2?

Yeah. Actually, that's what I described at the very first slide, right? So that's the lens maker's equation. So focal length of thin lens and you can compute the focal length from the reflective index and radius curvature, right?

But now we have thick lens, so we have extra term, this function of the d, the distance from the first and second surface. So if you consider a thick lens, then the effective focal length is not same as the focal length, right? So we just find the--

AUDIENCE: Where is the focal length measured from? Is it from the middle of the lens?

SE BAEK OH: That's what I was going to talk about, yeah.

AUDIENCE: OK.

SE BAEK OH: So we just find the effective focal length of the thick lens. But, yeah, the exactly, where the focus will be? That's the next question, right? Because we didn't define any planes yet.

So let's think about it. So first one is back focal plane. So we have an object at infinity. And so we have the ray parallel to the optical axis. And they bend twice, here and here. And finally, they want to make our focus at here. So this is focal plane. And we can define the focal length from this point, OK?

To find that point-- actually, this distance, right? So actually, we defined this distance as back focal plane, so from the second surface of the lens to this back focal plane. And then what I'm going to do is find the distance by ray transfer matrices.

So what I do is, so same as before, alpha 2 and x2, which is here. But you know, the x2 is 0, because it is on the optical axis, right? So it is actually 0.

And input is alpha 1 and x1, this one, so this x1 and angle. But since it is parallel to the optical axis, the alpha 1 is 0. And I just derived the ray transfer matrix of the thick lens, this guy. I mean this whole guy.

So I have M11, M12, M21, M22. And it's not the end, right? So we have to propagate again from here to here by distance zb. So it's going to be 1 0 zb 1.

So if you solve this equation, then-- so this is just what I've written. So we have the 1 0, zb 1. And this is our M matrices. And if you solve this equation, then we get two [INAUDIBLE], right? We had the two equations.

So one is alpha 2 is negative x1 and effective focal length. So this sign means this alpha 2 is heading downward. So the amplitude means x1 over effective focal length. And zb is actually effective focal length times 1 minus this guy. So in this case, actually the zb is bigger than effective focal length or smaller than effective focal length.

You know, in this case, n is bigger than 1. So this guy is positive. So zb is actually smaller than effective focal length, right? Wake up, guys.

So we define this distance zb as a back focal length. And we just talk about the effective focal length, the quantity in the bracket. And the 1 over effective focal length is power, the lens power or optical power of this thick lens.

So this distance is the effective focal length. This is the answer to your question. And we can do the same thing in the other side.

So we start from the point. We don't know where it is yet, but the focal point in front of the lens, which is the front focal plane, front focal point. And they bend twice. And finally, they get collimated, because it's going to the image at infinity.

So the ray matrices, in this case, are alpha 2 and x2 and alpha 1 and x1. But now, we start from the point on the optical axis. So x1 should be 0. And at the end, we are going to have the x2, but alpha 2 equals 0.

So this should be 0. And first matrices we should consider is this propagation by the zf, right? So it's going to be 1 0 zf 1. And then we have the thick lens, so M11, M12, M21, M22.

And if you solve this equation, then we also get the 2 [INAUDIBLE], which is the front focal length, which is zf from here to here. And this is what you get. And the second in x2 is alpha 1 times effective focal length. Question?

I guess it's time to break. Yeah. Let's take a break here. So next time, I'm going to continue to explain these front focal length and back focal length and how they are related. And meantime, we had the demo set up here. So you're welcome to come and see.

And what we have here is actually the imaging set up. We have a single lens here and object, which is the resolution target. And we have the screen here.

So if you move this screen back and forth or lens and the image-- we cannot move the object. So basically we're going to demonstrate the imaging condition. So if the distance is properly located, then we are going to see the very nice image. But if not, then the image looks blurry.

GUEST SPEAKER: So before we continue with all the math, some of you came to see the demo. And for the ones that didn't come, I suggest you to come after class. It's easier to see the image we try to form, because we don't have the dynamic range problem that any camera has.

But anyways, the main idea of this demo is to show what we've been learning about, an imaging of a single lens, right? So if you can summarize this points so far what we've learned, we've learned the thin lenses, thick lenses, right? So it's just a refractive element that is capable of forming an image from a given point, either at infinity or a close distance, to another plane. That's what we learned.

Another important point that we've learned today is the imaging condition, right? Imaging condition is just depending on three variables, the distance from the lens to the image plane, si, the distance from the lens to the object plane, s o, and the focal length, right? This is one of the most important things. Because basically this tells you, if I want to have an in-focus very sharp, nice image, I can either play with two of variables, keep one constant.

So for example, in these demo what I'm going to show you is how a single lens, a positive lens, performs imaging of a thin transparency. So in this case, I'm going to start showing here how I'm illuminating this transparency, which is here. It's just a piece of glass that has some dark coded lines of different sizes. It's called a resolution target.

And we're illuminating these with white light from the back. So it's just like transparency for an overhead projector you can think of. Then this is a positive lens here that will basically form an image at another screen here, right?

So now, let's look at these quantities physically that we've learned in the math here. This distance from here to here is the object distance. That's the s o from the equation.

This lens has a fixed focal length, which I cannot change. It's just a piece of glass curved in a given way. So this distance here is the distance to the image plane.

So these two distances I really can change, A, to get a sharp focus to make sure that my image is exactly on my screen, and/or if I want to play with a magnification. Because remember from the math, if you can see the notes before, that the lateral magnification depends on the ratio of minus si, which is these distance, over s o, which is this distance, right? So let's lower the exposure here just to see the image that we have here, please.

All right, so we lowered the exposure again for the same reasons of dynamic range. So I'm going to try to focus here on the image. So this is the image that we are forming on the screen.

And as you can see, so there is a bunch of lines, stripes, and squares with numbers. And it's very sharply in focus. So now what I'm going to do, and hopefully we can see it with a camera, is that I'm going to change the ratio between these two distances.

So what if, for example, I want to demagnify my image? So the magnification quantities I use describe si over s o. Forget about the minus sign. The minus sign just tells you that it's an inverted image. si over s o, I want that quantity smaller. What should I do?

I'm going to repeat the question one more time. Magnification equal to 1, I want magnification, say, less than 1. How can I reduce that M number? M equals minus si over s o.

AUDIENCE: [INAUDIBLE]

GUEST SPEAKER: What?

AUDIENCE: [INAUDIBLE]

GUEST SPEAKER: OK. Just push the button and repeat what you just said.

AUDIENCE: Increase the distance s o, so you move it closer to the screen.

GUEST SPEAKER: So if I move the lens closer to the screen, effectively I reduce si, the imaging distance. So let's try it. So I'm going to move these lens closer to this side.

So s o increases, of course. And I can just look at the right position here for focus. So now, this distance is smaller.

And it's hard to see with a webcam, but the image is still in focus. It's smaller, demagnified respect to the one that I showed you before. But yeah, that basically will accomplish this, right?

So for this part, if you can come and play with these distances later after class, you will see how this relationship works, right? So in this case, we have these two variables, the s o and si that we're playing at. But as we'll see in a second, for the eye, it's a very different story.

Because in the eye, the lens of the eye and the retina, so in this case would be your si, is fixed. You cannot change it. It's the shape of your eye.

So how can the eye focus far distance objects or when you read nearby objects? So it has to do similar tricks playing in the imaging conditions, such that it always has a sharp image. The way the eye does it, and we'll see it in a second, is by changing the focal length. So now, the two variables become f and s o. And you can do the same trick.

So this was a positive lens. We will learn about negative power lenses, so the ones are curved like this, right? So in these ones, we've learned that these generate these weird non-intuitive virtual images, right?

So I was just asking the students that came here, what did they expect to see if I put this lens here? Anyone wants to guess other than the students that already answered the question? What do you think will-- let me put it in this way.

Would I see an image and where? Or should I not see anything if I put this negative lens? Image or no image? Singapore?

AUDIENCE: Unless you form the image with another lens, you don't see it.

GUEST SPEAKER: OK, let's try it. I put the negative lens. Actually, I'm going to put it in this. Oh. We lost the video. Give me just one second.

All right, so I put the negative lens. Let me move it back and forth. And yeah, you can see here I have just a bright patch of light, no image, right?

So you said that, if we use another lens, we can make that image into a real image. What type of lens do you think we need to add and where? OK, I'll answer that one. OK. You can answer?

AUDIENCE: A positive lens after the negative lens, before the screen?

GUEST SPEAKER: Exactly. So essentially, physically what this is doing is that the light going out from the object gets even bent outwards more, right? So it basically essentially forms a virtual object behind the lens, right? So the positive lens, what it's going to do is grab these outward rays and focus them back into the screen to form an image. And let's just try it.

Now, I have the two lenses. Just make sure that the exposure is-- We're reducing the exposure.

Oh, it's hard to see here, but there's lines again, right? So then we have an image. So essentially, as I was mentioning before, this is what our eye would do for us when we see virtual images.

We have a visual image, and our eye is the lens that focuses this light and converts this virtual image into a real image, right? We'll understand more how the eye works in a second. So I think we're going to switch back to the normal class mode. Thanks for the presentation.

SE BAEK OH: OK, so we were talking about the thick lens. So by using ray transfer matrices, we just derived the effective focal length of a thick lens. And we defined the back focal plane, back focal length, and the front focal length, which is function of effective focal length.

And let's see how they are related. So first one we have the infinite conjugate configuration. So we have an object at infinity. And at back focal plane, we have focus.

And we just derived the two equations, which is alpha 2 is x1 over effective focal length. And that focal length is effective focal length times some extra factor. And what I'm going to do here is I just measure the effective focal length from the back focal plane.

And since the back focal length it means the focal plane to the second surface is shorter than the effective focal length, so actually in this case we are going to have the plane, the virtual plane, inside of the glass. And from that equation, which is alpha 2 and alpha 2 equal x1 over effective focal length, it turns out that if you extend this ray over here and this ray over here they actually meet at that plane. So it's essentially you can think, even though you have thick lens, but you have thin lens there. So ray just-- propagating from straight and at this plane and bend and make a focus there.

So we call this plane a second principal plane, because it's associated with a back focal plane. So it is a second principal plane. And you can do the same thing in the other side, of course.

So we had different focal plane. And I measure from here to here by effective focal length. And effective focal length is actually longer than the front focal plane, zf, from here to there.

So it's still inside the glass. And if you extend out those rays, actually they meet at this plane. So we call this first principal plane.

So I just put these two figures together. So I have thick lens. And we have different focal point. And this distance [INAUDIBLE] the surface of the lens is front focal length.

And we defined the same thing here. So we have the back focal point. And we defined the back focal length from here to here.

And from the ray transfer matrix, we defined the effective focal length of this thick lens. So we defined the first principal plane and second principal plane. And that's the effective focal length.

And so conceptually, these thick lens can be assumed. We already computed ray transfer matrices, right? So we can conceptually think we have the thin lens, but it's the first surface in a infinite thin lens. The first surface and second surface is actually the first principal plane and the second principal plane.

And if you remember the infinite conjugate configuration, if ray from the object at infinity, so it's propagating parallel to the optical axis, then it bends at the second principal plane and make focus at back focal plane. And same thing here if you have the point object at the different focal plane. And it bends at the first principal plane, and collinate it, and goes to the infinity.

So actually reason why we introduced this concept, the principal plane and effective focal length is actually pretty simple. Because we start with the thin lens, but now we had the thick lens. So what if we have multiple lenses, like thick lenses?

So no matter how complicated the optical system is, we can always find the effective focal length and this first and second principal plane. So once we have this first and the second principal plane and effective focal length, f here, then we can treat this whole system as one thin lens. So for example-- come on.

So we have the object in object space. And ray, it's exactly the same as a thin lens. We choose two rays, right?

So the first one is bent at the second principal plane. And it's passed through the back focal plane-- back focal point, sorry. And the second ray goes through the front focal point and it bend at the first principal plane. And it's collimated.

And they meet at some point. So you can easily find the image even though your system is pretty complicated. But once you find these two principal plane and effective focal length, then you can still use the same equation as a thin lens.

Come on. Yeah, all these equations. So you can define the same thing, xo and s o, the image distance and object distance like this. And you can use the same equation, so Newton's form, xo times xi equals to f squared. And the lens law, 1 over object distance plus 1 over image distance, should be 1 over f.

So f is effective focal length in this case. And the lateral magnification and angular magnification, they're defined the same way. And I just want to mention that the-- yes?

AUDIENCE: Where is [INAUDIBLE] on that picture? Because the effective focal length you talked about depends on the [INAUDIBLE] between the first [INAUDIBLE].

SE BAEK OH: So actually, the effective focal length depend on the stuff inside of the neuro-optical system. So the thick lens we just consider one thick lens-- so one, the first surface, and lens, and second surface. But I'm going to actually talk about that later.

But what if we have two lens? So we first consider first lens and propagation and the second lens. So you can define the effective focal length the same way, OK?

So in this case, the effective focal length is not related to the aperture or whatever. It just depend on the reflective index and distance between the optical element.

AUDIENCE: But in the effective focal length you've got the parameter b, [INAUDIBLE].

SE BAEK OH: Yeah, distance between the elements, right?

AUDIENCE: OK, so that's width of the lens?

SE BAEK OH: Yes.

AUDIENCE: So--

SE BAEK OH: Yeah? Yeah.

AUDIENCE: Over here we have this duality between x object and x image, but we always have 1f. Is that only because we're assuming that the lens has the same radius of curvature on either side for now?

SE BAEK OH: No, no.

AUDIENCE: Or is it always just 1f?

SE BAEK OH: No, no. So effective focal length is just function of-- yeah, maybe it's confusing, but the-- so in this thick lens, the effective focal length is just function of the radius curvature and reflective index and d, right? So the concept of this effective focal length and principal plane, we treat the whole system as one thing lens.

And one thing lens has the same focal length in front side and rear side, right? So effective focal length is already same in front side and the back side of the lens. But it depends on the system.

Because right now, I just talk about this thick lens. So that's why we have only, 1, 2, 3, 4 parameter here. But if I have another thick lens as [INAUDIBLE] here, then effective focal length is more complicated, right?

So it's also a function of the reflective index, and distance, and the curvature of the second lens, OK? Now, we'll talk about-- I mean, actually I'll show you [INAUDIBLE].

AUDIENCE: So for one lens, you always have one effective focal length to characterize the entire lens?

SE BAEK OH: Yeah.

AUDIENCE: No matter what the sides are?

SE BAEK OH: Yeah. So let's say if you have the 100 length. But you can always find the one effective focal length. Yeah.

So actually, if you have a digital camera, then typically you have multiple lens, or three or four or five. But you only have the one focal length in it. I mean, they only specify one focal length, right? So that is actually effective focal length. So they already compute this whole thing, OK?

So that's the reason why we introduced this concept. And I just want to mention the center ray which is starting from the object point, but is incident on the center at the first principal plane. And if you remind the thin lens case, then at the thin lens that ray actually just pass through the center of the ray, right? So the same here, so that ray is starting again at the second principal plane, at the center of the second principal plane. And it goes to the image.

And of course, if you have the same medium in front of the principal plane and behind the principal plane, then this angle and this angle are same, right? Because you know, at the center you will just have a piece of glass. So by the [INAUDIBLE] law, they should be parallel.

OK, actually, that's about it. So the take home message is we just talk about the imaging condition. Anyway, by the way, he's going to talk about the human visual system.

So take home message of my part is we just talk about imaging condition, which is 1 over s o and plus 1 over si is 1 over f. And there are some mathematical way to find or to write the condition, OK, which is the ray transfer matrices.

And we just introduced the front focal length and back focal length and principal planes. But I didn't really talk about how to find this principal place OK so let me finish we are hard to find the principal place if you have multiple elements which he was the of course we have the two example of two lenses

so that completes so initially we had the example that two lens which has focal length is 10, 10. And distance is 5 millimeter. And the object was also 5 [INAUDIBLE].

By the way, Professor Barbastathis, he posted a different way to do it, to find where the image is, or how big it is, and where is the principal plane, and what's the effective length. He summarized all these different method, and he posted on a [INAUDIBLE] website in a supplement material. So please go to the website and review it.

And it looks complicated. But once [INAUDIBLE], then it's pretty straightforward. And you will never forget.

You're not going to forget. So please do it. I strongly suggest you.

So let's continue how to find the principal planes here. So there are a bunch of different ways. So I'm going to do the first one the easy way.

So let's first find the back focal plane first, because we know. The definition of the focal plane, if you remind the thin lens, like this, then these incoming rays, they converged at one point, right? So in this case, let's first assume I have the ray coming from infinity like this.

And I do the cascade method. So I just consider first lens here. And where is my image? Actually, we already computed, like, it's already computed.

So this focal length is distance 10, right, so distance f. So by the first lens, I draw only one ray here. This is parallel to the optical axis. So third ray actually goes to the focal point of the first lens, which is 10, right?

But we do have the second lens here. So if you do the lens equation, then actually it bends more. So finally, what you get is it coming in, bend like this. It's supposed to go like this, but it bend more like this.

So this point is back focal point of these two lens system. And if you do the same thing, so you extend the [INAUDIBLE] ray like this, and if you extend like this, then where they meet, actually that's the position of the second principal plane. Because you have all the numbers, you can easily find the position of this guy.

And so this distance is actually the effective focal length. And the distance from the second lens to the back focal point is actually back focal length. So we just find the second principal plane here, second principal plane.

And you can do the same thing in object side. You start with the point source on optical axis. And at the end, you're going to get the collimated light.

So since it is symmetric, so it's actually you're going to put the ray like this and bend like this. And it's finally collimated. So if you do the same thing, then you're going to get the first principal plane here. And this is effective focal length, which is same as before. And you can find the front focal plane, front focal length.

So that's the just conceptual way to find the principal plane and effective focal length. But if you want to be accurate or precise, then you can always use the ray transfer matrix. So let me do this.

We have two lens, focal length f. So let's start with actually this interface at the end of the second lens. So we have here alpha o and xo and here alpha i and xi. So I have alpha i xi, alpha o and xo.

And I assume this lenses are thin lenses. So first matrices should be 1 over 1 and negative 1 over f and 0 1. And next one is the propagation from here to here. So this should be 1 0. Actually, this is 4 1, because-- that's 5, 5.

And the second lens, it should be same as the first one, 1, 1 over negative f 0 1. So if you could compute this one, then you are going to get this is 1 minus 1 0 and 1 0, 5 1, and 1 minus 1 10, 1 0, 0. And you're getting, it should be, 1 over 2 negative 3 over 20 and 5 1 over 2 and alpha object and x object.

So I should remind the thin lens case, which is 1 minus 1 over f 0 1. So actually this guy is 1 over effective focal length. So in this case, effective focal length should be 20 over 3.

So this is how to find the effective focal length by using ray transfer matrices. And you can find the back focal length and front focal length. Just cascade another propagation, you know, just like what we did in class. So here and here, you can find the back focal plane, back focal plane, and front focal plane, and front focal length, and, you know, first and second principal plane.

So since Professor posted the detailed material, he actually derived four different rays. So please visit the website and do it by yourself. Thank you.

GUEST SPEAKER: OK. So, so far, we've been learning about lenses and some sort of imaging techniques and principles. But more than that, we've learned about some optical principles that regulate how light propagates, namely refraction, reflection so far, total internal reflection. So now the question is, how does nature does it? And how has it evolved over the past several, several years?

And it actually comes down, according to these reference here, to eight different possible configurations of eyes. And just by the way, defining an eye is more complicated than just a simple photoreceptor. So an eye, it's a little bit more evolved like some sort of forming either an image or a shadow based system.

So these type of eyes can be broadly categorized or divided into chamber eyes, the ones that I'm showing here, OK? So let's look at these eyes here and see what principles apply from what we've learned so far. So in C and in D, we can see what? Snell's law, right? So this is refraction.

So these type of eyes, again, like our human eye, focus the light and form an image in some sort of photoreceptors located here, right? Then we have also reflection based. These lenses here-- I'm sorry, this mirror base here where the light gets reflected and forms an image in a photoreceptor here.

And now, we also have another one that is an aperture based, or you can think of these has been as a natural pin hole. And these images form by shadows, right? So you see the shadow difference.

So these are the four types of chamber eyes. But even more older type of eyes, we have the compound eye, as we said in the very first lecture. They're made out of a segmented array of many apertures or many elements. You can think about like many lenses for the case of our refractive based system.

Or for our reflective based system, it's several apertures that reflect the light somewhere here and also focus it into some photoreceptive sensor. Or in this case, for instance, here, each one of these segments sees a very tiny field of view and integrates all that field of view into that photoreceptor. So in reality, it's not like the type of images that we're used to, right? It's just sort of like a blurry version of our images in this case.

But of course, of very important interest to us is how the human eye works. Because as we know, we've experienced-- we have eyes-- it's an extremely robust and very elaborate optical system. I just said one of the problems we've had with the demo is the adaptive to dynamic range. Our eyes can do that very well.

We can also go to read a book or just look at something far away. And we can focus pretty well, too, right? So all these things and all these adaptations regulate these optical principles that we presented are actually what made a lot of researchers to try to study the eye and how the eye perceives light. So that's under the field of visual perception.

So this is the structure of the eye. The eye, it's essentially a sphere. And it's all covered by some white opaque layer called the sclera. And that basically blocks straight light going through.

The only transparent section of the eye is the cornea, here. So in the cornea, the light can go through. And the cornea is the first refractive element of the eye.

It's curved. It has a refractive index of something like 1.37, which basically is the largest bending of light. Because it comes from air to this 1.37 element.

So that's why you can think of that when you are actually swimming it's hard to see. Why? Because the index of refraction of water is 1.33, very close to that of the cornea. So then the bending of light that occurs in this first layer is very minimal.

So after the cornea, we have some chamber filled with a solution called aqueous humor, which basically nourishes the eye, especially the cornea, keeps it alive. That is very similar to water. Now, we have then the iris.

The iris is the element that controls-- well, it's functional purpose is to control the amount of light that your eye receives. So as we know, the iris expands and contracts depending on the amount of light present in the environment. And actually, it's very impressive.

It can go from, like, 2 millimeters in diameter to 8 millimeters in diameter depending on the light condition. But it's also responsible for the nice color of the eyes that we see, like blue eyes, green eyes, you know? That's the one to blame. The aperture or the hole that we have is what we call the pupil. So after the iris, we have the second most-- yeah?

AUDIENCE: Does the color of the eye [INAUDIBLE]?

GUEST SPEAKER: No. It's due to-- and actually it's very interesting. So no, it basically just blocks the light. It acts like an aperture, like of the aperture of your camera.

So if you like, photography would control the speed or the f number of your system. But the light reflected from the eye gets the color in a similar principle to why the sky is blue. And I'll just let you investigate, because actually it's very interesting.

And we used to have that problem in the first p set. Why the sky is blue? Why clouds are white?

And I guess the physical phenomena of why that happens is very interesting. So you guys should read. It's in Hecht.

OK, so after the iris, we have these what is called the crystalline lens, this lens here. This is the second optical component most important in our eye. It's basically multilayer fibrous mass element covered by some elastic membrane.

It's transparent. And it can contract and deform itself in such a way that it changes the effective focal length of this lens. But you can think about this element as some transparent quasi-onion.

But in addition, it has very interesting properties. Because these lens is actually a gradient refractive index lens, is, what we learned in some classes, a green lens, this element here. It has a refractive index in the core something like 1.4. And it decays all the way to something like 1.38 to the edges. So the light gets basically-- see the transition. So this is similar to the p set one. And it basically sees multiple layers. So depending indices of refraction, it starts bending accordingly.

OK, so this lens will, in conjunction with this cornea, form what we call or we could model this system as a dual or double lens optical system. And combined, they have an optical power-- and remember the optical power is 1 over the focal length, which it's measure in diopters-- of equivalent of something like 59 diopters, just so you get an idea for an unaccommodated eye or a relaxed eye just looking at infinity.

So then after this lens, we have another chamber with another fluid called vitreous humor. And it's another fluid that basically gives support to the eye. And going back to our question, I think someone in Singapore had in the very first lecture, this fluid has a lot of little particles floating, debris. And actually sometimes you can see them.

And maybe many of you have seen them. If you see to the sky or you squint your eyes and see a bright light source, you see the shadows of these little particles that are freely flowing in the vitreous humor. And you can see the fringes if you look carefully of the light refracting that.

OK, so after that, the light basically gets focused into the retina. We know the retina is composed of photoreceptors, photoreceptive cells, cone and rods. And I'm going to explain those in a couple of slides.

And then we can also identify two other points. We identify here that each of these cells are connected to optical nerves. And they all basically come together into this output that goes to the brain.

And this is what constitutes the blind spot. And I also explained that. And we have a fun exercise in a second.

And we also have this other region, which is like a spot of 3 millimeters in diameter, more or less. It's like a yellow spot. It's called the macula.

And it has the largest concentration of cones. And I'll explain what is its role, but it is very important. Just that section here accounts for 90% to 95% of our visual perception.

So I talked about the ability of these crystalline lens to accommodate, change the focal length. And similar to this demo that we were changing the distance between these guys, well, in this case, as I said before, this isn't fixed, right, unless you have some sort of disease that actually changes the length of your eye. And that actually happens. We can think of as this fixed.

So if we want to focus objects other than infinity, we need to play now with a focal length, right? So now, this lens has to be capable of changing the effective focal length in such a way that it will conserve or preserve the imaging distance here. So here, this lens does it by contracting or expanding the ciliary muscles that are connected via some fibers to these lens, right?

So in a relaxed state, you see this case here. The radius of curvature of this lens, it's very large, like closer to infinity, say, like very flat. And then it focuses nice into the retina, like a plane wave, whereas, for nearby objects, these muscles are stressed.

So then the radius of curvature of the lens changes. The focal length changes. Now, this image is focusing to, again, the retina.

So we can think about some of the conditions that would affect the normal behavior of the lens of the eye. And the two very important conditions that we have is farsighted or nearsighted. So in the farsighted case, for example, our eye lost its ability to focus effectively.

And in a relaxed state, it will actually form a focal point. So the back focal point, it will be far away behind our retina. So actually what you see would be a blurry image.

In their myopia case, or nearsighted, you see the opposite. Now, the point is formed in front of the retina. And again, you see a blurry signal.

And by the way, a far point for a normal healthy eye it starts from something like 5 meters on. So 5 meters, more or less, what you can see there is still like a five point. So for someone that suffers myopia, typically they see close by objects pretty fine. But far objects, let's say further than the far point, something like 5 meters away, they become blurry.

And the near point, which is the counterpart of the far point, is how close can we see. That's a point that also varies with age. Normally, teenagers the closer they can see is something like 7 centimeters.

For us, maybe it's around 10 to 12-- you can try later by just trying to focus your notebook, centimeters. And then if you are over 60 years old, it goes up to 100 centimeters. So basically, you can see how the eye starts decaying in its ability to modify this crystalline lens.

So to correct for these conditions, we can use what we've learned so far, positive and negative lenses, right? So let's think about it. In this case, the rays are not focusing enough, are not bending enough, right? Because they are basically focusing behind that.

So I want to help the rays to focus a little bit more. So a positive power lens or a positive lens, like this one, allows me to bend the rays inward more, right? And this essentially what this is happening. The positive power bends these parallel rays more and then assures that this focuses in the retina.

So let me ask a question. What do you think happens with the magnification for this case? Any idea?

For those of you that wear glasses, do you guys see the world larger, magnified? No? There is shaking head, no. Why do you think it's not the case?

OK-- so the power of the optical system, the compound optical system. So if you work out actually the power of the cornea plus crystalline lens-- which I said is something like 59 for a healthy eye. But in reality when it's unhealthy, it changes.

And if you consider the power added by this element here, it basically comes down to the same power as the healthy eye. So therefore, you don't add or subtract any power. So therefore, the magnification stays the same.

For the myopia case, we can use the opposite. We can use a negative lens, as shown here, to actually bend the rays a little bit outwards, right? So essentially this is interesting

because, again the myopia case you cannot you see far away objects blurry right but some objects are closer than the fire point you can see them fine so what the negative lens is doing-- and again, going back to these virtual images-- is bringing a far away object into a near distance closer to the far point.

As you can see, this appears to come from some virtual point here that is closer from a region that you can see naturally with your eye. So therefore, intuitively this is how this ray makes this lens still project the image into the retina. But as a side effect, the near point-- yup?

AUDIENCE: So can people [INAUDIBLE] focal point closer than [INAUDIBLE]?

GUEST SPEAKER: No. And actually, yeah, that was the second point I was going to say. As a consequence of this, your near point, which I said that for like a healthy eye would be something like 12 centimeters, becomes larger. So people that have myopia-- I don't know if there's anyone here.

But probably you know that sometimes, if you want to read a very close by document, you need to remove your glasses, right? And people adjust OK for small print. They get out their glasses. And for far away objects, they put their glasses on and see. All right, so any other questions?

OK, so the retina, the retina has a bunch of cells that are photoreceptive cells that can be classified into two types, rods and cones, right? And we can see here rods and cones. For those of you that like photography, you can think about the rods being equivalent of a high speed black and white film.

What is the meaning of that? It basically means that is very, very sensitive to light, OK? For very low levels of light, you can detect them very fine. But it's insensitive to color and gives you images that are not really good quality, sort of like blurry type of images, right?

The cones, on the other hand, can be thought of as low contrast or low speed color film. So the cones have cells that are sensitive to red, green, blue, are responsible for the colors that we see and are also responsible of the nice, sharp images. So all around the retina, we have sort of a combination of both.

Although as you can see here in the density plot shown here, there is a region that is called the macula. And inside the macula, there is even a smaller region of something like 300 micron. That's called the fovea. That has the highest concentration of cones, again, those responsible for the color vision-- as you can see, the plot just peaks here-- and almost no rods, right?

This is called the fovea centralis. And that's responsible of the central vision. And basically, that's the one that I was saying that's responsible for 90% to 95% of the visual perception that we have.

So it's interesting to note that, in contrast to a normal camera, there sampling of these photoreceptor elements-- so you can think of it as, like, pixels-- it's different. Outside the macula, they are large and course. As you can see, these rods are just responsible for the peripheral vision and black and white.

But in this macula, they are very concentrated and dense. And as a matter of fact, the cones inside this region, so when they are here, they have a different diameter than the ones outside. There is something like a factor of two that are actually thinner. So they are very close packed together. So you can see very sharp images here.

As I mentioned before, each of these photoreceptors is connected to these nerves that all bundle up together and go to the blindspot, shown here. Now, have you guys been bothered by a blind spot all your lives? Not really. Why?

Because as you can see, it's actually just blocking a very small portion of your vision. As I said, a lot of the vision or visual perception happens in this region, right? This is not true if you have a camera.

If I have a camera and we have a bunch of bad pixels like that, it might corrupt our system. And then we're done. Like, if a microscope experiment, for example, forget about it. We lose a lot of pixels, and then the image gets really corrupted.

So I brought a little card. I'm going to pass it around. I have actually multiple of these. So this is a fun experiment.

It has, in the top part, a little cross and a little black circle. In the bottom part-- another cross and a line that is basically truncated, right? So what you're going to do when you get it is that you're going to have the cross to your right side.

And you're going to close your right eye. And then you're going to just play with this distance staring at the cross. So you're going to focus your left eye at the cross.

And you're going to find a position here in which the circle will disappear, right? Because the circle essentially will be imaged into this blind spot, and we're not going to see it. But the second one is more interesting.

Because you do the same thing, and the hole here disappears. But not only disappears, it gets filled with a line. So you'll see a continuous line.

And it's more shocking, because basically our brain fills that missing information. It's actually pretty interesting. So I don't know if you can help me pass this around.

How do I go to the next one? [INAUDIBLE] this one? OK, takes forever. OK.

So this is a simulation that it was done by a group in Caltech. They were trying to simulate what would be the image that our eye actually is projecting at the retina. What is exactly what we see?

And interestingly enough, compared to a normal image of the same scene produced by a CCD camera, as you can see, the image is really blurry in the surroundings, very sharp in the center, as we described due to the sampling, et cetera. So the eye tries to take advantage of the fact that it has a very small, very sharp region in focus as much as it can. And the way it does it is by jittering the eye, moving it at a frequency of something like 30 to 70 Hertz, to scan a given scene and try to collect as much information to create the scene that we perceive in our brain.

And also, there is some interesting image processing to try to account for this blurring. Or you can think of that, for those of you that are in signal processing, deep learning algorithms embedded in our brains. I think it's pretty interesting. So the phenomena of moving of the eye, it's called saccadic motion.

Also, the cones and the rods, as I said, are connected with nerves. But it was first proposed by studying the eye of a cat and then for humans that the eye actually has this type of responds to a light as stimulus, that in some regions it's going to be positive. Some regions it's going to be negative following what is called a Mexican-hat trend. And actually, this response is what is responsible for many optical illusions that we see, for example, this one.

So what do you guys see? You've seen this one probably before, no? So you see this flickering gray dots that are really not there right

and if you try to focus your eye and if you are like as close as I am to the projector and I'm focusing my eye here I don't see anything of the at least at that point I don't see any great does however if I focus in another point right I'm also seeing this flicker.