GEORGE BARBASTATHIS: So while we're waiting for the phone-- I mean, for the microphone, a little bit of catching up from the previous lecture-- on the slide over there is a telescope. So I don't want to spend too much time on it. You wanted to see the telescope at least in one homework problem of the previous set, so you're familiar with it. Then, once you alluded to it earlier.

So it is kind of opposite than the microscope in the sense that, OK, both instruments are similar because they magnify, but they magnify what? In the case of a microscope, we see very small detail. Thank you. We've got a new battery now, so we're back in business. So in the microscope, the object appears small because it is physically very small, and it is available nearby. That is a definition of a microscope.

In the case of a telescope, the object appears small, not because it is physically small, but it is-- but because it is very far away. So for example, a star-- a star is humongous, and it may be several thousand times bigger than the Earth, but it appears very small because it is also maybe thousands of light years away. So this is the situation where we typically-- where we use a telescope.

So you see there the function of the telescope here. So if this is the case of a telescope with an object with a finite location, so in this case, the object is physically big. It appears small because it is far away. So the telescope, in this case, it is actually very similar to the microscope, to the way it works. It creates an image of this object in the following sense, that each point in the object becomes a parallel ray bundle at the output of the telescope.

So therefore, the observer-- again, recall that the final, final image is produced by the lens of the observer's eye on the retina. So the eye of the observer picks up this parallel ray bundle and focuses it on the retina. Now, if you extend the rays backwards through the final eyepiece of the telescope, you can see that the telescope actually is creating a virtual image that is magnified with its back to the original object. And that's why-- that's how the telescope magnifies this remote object.

It is more common to use a telescope as shown in the second case, where the object is actually at infinity. And that is the classical case of a telescope, where now, instead of having spherical waves arriving from points in the object, now I have plane waves. So to a good approximation, to an excellent approximation, star is located at infinity. But not necessarily-- even terrestrial objects that are located, say, more than 100 meters, so a kilometer away, they satisfy this approximation quite well. So they produce these almost parallel ray bundles.

So the telescope, in this case, it is afocal in the sense that the output of the scalar telescope, again you produce a parallel ray bundle. And the reason, of course, is that you want the observer to form an image with unaccommodated eye. That means that still the image must be at infinity so that the eye-- without any strain, the eye can focus it to the retina.

And since that is the case, it means that the total-- now, let's be a little bit careful with the terminology-- the total power of the telescope is zero. That is, the telescope has no optical power. Its focal length is infinity. However, it does have magnifying power. What a telescope does in this case, the angle between a chief ray and the axis, the telescope magnifies this angle. And this is now by definition the magnification of the telescope, and you can see that by virtue of magnifying this angle, the telescope also magnifies the final real image that will be produced on the retina.

And in order to be afocal-- this was the topic of the homework that I gave you guys a few-- I don't know-- I think it was two weeks ago. In order to be afocal, the distance between the two lenses must equal the sum of the focal length. It is very easy to convince yourselves in this case that an instrument such as this one, then in the imaging, in the matrix of the instrument, the matrix will look like this. And it will have a 0 in the 1, 2 element. In the first row, second column, it will have a 0. If that is the case-- whoops-- if that is the case, then it is called afocal. So by satisfying this condition, d equals f objective plus f eyepiece, we ensure that this is the case.

So there's a number of different types of telescopes. I just picked one here. The textbook lists all of them. There is the Keplerian, Galilean, astronomical telescope. They all have different combinations of lenses. I picked one here that is composed exclusively of mirrors, and it is very common actually in observation-- in observatories. It is called the Cassegrain.

It is also the same type that, if you go to the Discovery Store, for example, and buy a telescope for your nephew or your niece, this is the type. You look at it, and, you know, the tube is oriented horizontally, and you look at it from the top. And this is the schematic here. The object, or the sky, is on this end, and you place your eye this way.

So the telescope itself consists of a large first mirror that functions as the primary, and then there's a second mirror. So if you look at the-- from this side, if you look into the telescope, you will see a small block in the front face of the instrument. So that is the secondary mirror.

So the combination of primary and secondary-- of course, you use two of them to get even higher magnification, and the combination is the objective. And then, there's an eyepiece, which is where you stick your eye, and you see the-- you see the image of the sky. So this is composed purely of mirrors. So last time, we're going to call this kind of instrument catoptric.

Then, there's another type of telescope. Actually, it's not a telescope. It's a kind of a method for correcting optical systems called Schmidt-- Schmidt telescope or Schmidt optical system. So let me say why you use this. So you recall that the ideal mirror surface to give perfect focusing of an object at infinity is actually parabolic.

Now, suppose that we cannot make a parabola, so we have to make a sphere instead of a parabola. Well, in that case, a little bit of thought will convince yourself that the sphere, because it is more highly curved than the parabola near the edges, away from the paraxial region, it will actually bend the rays that are far from the axis. They will bend more than they should. So the result is the failure to reach perfect focus. This is what we have already seen, and we'll call it spherical aberration.

So Schmidt basically came up with an idea to correct this spherical aberration by sticking a slightly-- a glass surface at the front pupil here. He put a glass surface which is slightly shaped. It has toroidal shape, and the purpose-- I don't know if you can see it in the image. It was a-- perhaps I should have exaggerated more. It is too subtle.

But what it does is it takes this ray and sends it slightly higher, so this kind of functions like a negative lens, actually. You can think of it this way, with a very small power. And then, because this hits the surface at a different point with a different angle, then, basically, by engineering the shape of the surface here, you can make sure that most-- that all of the rays will actually meet ideally at the focal point.

So this is a very simple way to correct for this. I don't know if it is simple, but, anyway, it is a way to correct for spherical aberration. So this is called catadioptric because it contains a reflector, as well as a refractive element. So according to our terminology, that's what we call it.

So that's all I had to say about telescopes. Are there any questions or-- either about telescopes or microscopes or any of the optical systems that we saw?

So next, I'm going to shift gears a little bit, and I'm going to expand on the very last topic that we saw, the topic of aberration. So we saw quite extensively-- we mentioned several times this spherical aberration. It turns out it is not the only one, and we'll see a number of different types of aberrations today.

So to do a little bit of organization and classification, we start by classifying them as two types. One is chromatic, and this we have also seen in the past, in the context of a prism primarily. And it is due to the fact that the index of refraction has a variation with wavelength.

So if you apply the lens maker's formula, or any of those, in order to design an element with a given focal length at the given wavelength, you have no guarantee that the focal length will be the same at the different wavelength. That fact is called chromatic aberration.

Again, this comes from a Greek word. I had to do this, and I sound like this movie, *My Big Fat Greek Wedding,* but since I'm Greek, I guess it is natural. So chroma-- in Latin, this would spell the chroma, with the accent here. It means color in Greek. So that's where the term chromatic comes from.

The second time-- or for the second type of aberration is called geometrical. And there's a long definition here, but I will go back to it, and perhaps this long definition will make more sense. Let me start with chromatic because this is easy to deal with-- OK, not easy to deal with, but it is easy to understand.

So as I said, we have seen this type of plot before, where the horizontal axis is the wavelength. The vertical axis is the index of refraction. And forget this dashed line for a moment. Just look at the solid lines. So these are the index of refraction, this function of wavelength, for different glasses. You have seen this before in the context of a prism.

This is, of course, a disaster. It means that if you have a lens like this one, which you designed for a certain focal length, say, somewhere in the middle of the spectrum, say, near yellow, then this means that-- well, as you can see, the index in the red wavelength for sort of normal materials-- this is actually called normal dispersion-- the index is smaller at the red wavelengths.

So if you remember the lens maker's formula, smaller index means that the focal length is what? OK, the lens maker's formula says that 1 over f equals n minus 1, the difference in radii. So if n goes down, it means that 1 over f goes down. It means that the f goes up. So it means that the focal length is longer. And this is also what you see in this picture over here. You'll see that if you have a white light coming from infinity, you'll see that the red rays focus at the longer distance than the blue rays.

So that is the evidence of chromatic aberration, and it turns out there's a way of correcting it, which is to stick two lenses, one of them with a-- that is a positive lens and the other a negative lens. And first of all, they have to-- the two lenses have to have a slightly different indices. Otherwise, it wouldn't even make sense, right? Then this would be just one element.

So they have different indices, different dispersion probably, so one can pick, for example, between these glasses. You could pick one lens out of this one, another out of this one. This is how one designs this kind of element. And then, the way to understand it is because one of them tries to focus the rays. The other, because it is negative, it will try to actually defocus them. You can imagine how one can compensate the action of the positive lens with the action of the negative lens and eventually cancel, or at least approximately cancel, a chromatic aberration over a relatively long range of wavelengths.

I used to give you this problem as a homework. I don't do it anymore. But, anyway, it is a fairly straightforward thing to do. And if you go over the textbook, there's an entire section where he tells you in detail how this kind of thing can be done. Conceptually, it is very simple. All you do is you balance the focal length. So you basically make sure that the composite has a focal length that is the nominal focal length, and then you balance the chromatic by use of the two elements of the composite.

The geometrical aberrations, they're more interesting because, conceptually, they're actually much richer than chromatic, and also, algebraically, they're much more difficult to handle. So people have come up with all kinds of clever tricks in order to understand and express geometric aberrations.

So again, let me remind you that at the exit of an optical instrument, the ideal wavefront should be spherical. So this is the dashed line over here. If you draw normals to this spherical wavefront, of course, there will be the radii of the corresponding sphere, and they will meet at the center of the sphere.

So if I actually produce a spherical wavefront, then I can expect a perfect geometrical focus at the center of that spherical wavefront. So the problem, of course, these are optical instruments, as we saw repeatedly in a number of cases in the past, including the spherical reflectors, spherical refractors, and so on and so forth, they don't produce spherical wavefronts at the action. They produce wavefronts that, in general, may be some complicated surface.

So in this case, you see the eye is-- of course, are exaggerated-- I made it terribly aberrated, and this is the solid line over here. So the difference between the actual aberrated wavefront and the ideal wavefront, which is, of course, spherical, this difference is by definition the aberration.

So I tried to be fairly loyal here. So I-- again, this is a cartoon, but I took the difference between the solid line and the dashed line, and this is what you would get. Typically, you measure it in optical path length, and it doesn't have to be very dramatic. The vertical axis here can be as small as a few wavelengths, and that is often sufficient to destroy the quality of the image formation in an optical system.

In fact, the book has the story of the Hubble telescope, which is kind of famous, or I should say infamous study of a disaster, and heroic, in optical design. The telescope was sent to space with a serious error in the design-- actually, in the assembly, that caused a huge spherical aberration. And then, they had to send another spacecraft to fix it, and so on.

Anyway, they actually succeeded, which is impressive. But, anyway, so what the book says is that the error was-- actually turned out to be-- I think it was half a wavelength or so. Whatever that had was sufficient to destroy the quality of the images that were beamed from the telescope. So one has to be very, very careful in this business.

So as you can imagine, if you tried to express this kind of complicated surface or its difference from a sphere, you will probably get a very nasty trigonometric expression, so for a number of reasons. One of them has to do with the ability to compute analytically, which is kind of an old-fashioned problem because now we have computers.

So that's one reason, but the second reason, because of the desire to get intuition about how these things work. People normally express aberrations in the form of a Taylor series. And, of course, you are dealing now in-- this is a surface, not just a curve, as I show here. The Taylor series is high dimensional. You actually have many terms for each order in the Taylor series expansion.

So these terms are actually grouped into orders, and the aberrations that-- and the terms that we consider as a first step in an optical system are the third-order aberrations, that are also known as Seidel aberrations. And, of course, the second-order aberrations disappear because we assume rotationally symmetric optics. That kills all the second-order terms in the Taylor series by definition.

And the first-order aberration is actually only one, and that is the focus. So the first-order aberration is actually paraxial. It is something we've already considered. And then, it is the third order that we deal with in optical design.

So there's all-- after you go through all the symmetries and all that, it turns out that the further the aberrations, it's five of them. One we've already encountered, it is spherical, and the other four are coma, astigmatism, curvature of field, and distortion. And what you see here is the aberration wavefronts, so this probably doesn't mean very much. What we'll do next is we'll actually see a ray diagram of all of these aberrations and what they look like.

So the spherical, these diagrams are straight out of the book. We have seen actually several times, and the way spherical aberration looks like, it basically happens when the rays that are away from the axis, they receive more curvature than rays that are near the axis, or more or less. So in both of these cases, you'll get the situation where the rays from away from the axis, they focus before or after the paraxial focus.

And it is interesting, the way-- but if you look at this kind of system, you wonder what exactly is the focal spot? And that's not an easy question to answer, so if you look at the marginal ray over here, the size of the beam that you create is, of course, defined by the marginal ray. So you see that the marginal ray kind of focuses here, then starts to focus less. Then it reaches a minimum size, and then starts to expand again because now this aberrated ray takes over.

So there's a minimum width in this optical beam, in this aberrated optical beam to produce, and that is called the least-- the circle of least confusion. It is kind of a compromise to see whether this system is approximately in focus. Another thing that I want to emphasize from this diagram is that, as you can see, near the edge here of this bundle, there is a congruence of rays that approach.

So if you-- this-- you can only see a few rays here, but if you imagine that you fill up the space with rays, then you would see that you would get a very tight intersection of successive rays that trace a curve here. This curve is called a caustic, and, again, that's another Greek word. [GREEK] means something that burns.

And the reason it is called caustic or the reason it alludes-- the word alludes to burning is because this congruence of rays, if you were to put a piece of paper there, if you wanted to put an element, it would create and increase the intensity of light near the edge of this bundle. And therefore, it would cause, presumably, elevated temperature since you have more light energy, and that might actually result in burning.

Caustics are very interesting things. I will say a little bit more perhaps next time about caustics, but they occur often when you deviate from a perfect spherical wavefront. When you pick different wavefronts other than spherical, these caustics have a tendency to occur.

And there's a very beautiful mathematical theory that describes that. It is a little bit advanced, so we're not going to-- it is kind of beyond the scope of this class. But I thought since also the book has mentioned it, I thought it would be interesting to point out the existence of this caustic.

When it comes to spherical aberration, there's only several things one can do in order to control the spherical aberration. One is the actual shape of the lens. If you'll go back to the lens maker formula, this one over here-- I've already written it-- you can see very easily that you can get the same focal length with a number of different combinations of radii for the front and the back surface of the lens.

So from the paraxial point of view, it makes no difference whether you pick-- for example, whether you pick a plano-convex, or whether you pick a biconvex lens, or whether you pick a meniscus, and so on, from the paraxial point of view, they're all the same because-- as long as you have the same focal length.

However, because this theory now-- we've gone beyond the paraxial approximation-- this theory is non-paraxial. Actually, the shape factor of the lens makes a difference. And then, forget the definitions here. This plot is the most interesting one, where you can see different lens shapes. This is a meniscus, kind of that the-- a convex meniscus. Then, this is plano-convex. This is biconvex, again plano-convex, but flipped the other way around, and then a concave meniscus.

And this plot was computed so that all of these different shapes, they have the same paraxial focal length, but on the vertical axis, you kind of see the amount of spherical aberration that you receive. And you can see that it is very different, of course, and it actually has a minimum. It is not quite zero, but it is a minimum, and the minimum occurs typically near the plano-convex shape for an object of infinity.

So why is that? Well, the reason is-- this is kind of nicely illustrated by another diagram that I stole from your textbook. If you place the plano-convex lens such that its flat surface faces the plane wave coming from infinity, then, basically, refraction occurs only at this spherical interface. So you have no hope. You will get very strong spherical aberration because, as we learned earlier, this spherical surface is very far from ideal as a focusing element. So this case is very highly aberrated.

Now, you can do a very simple thing. You can flip the lens so that this spherical wave meets this spherical-- I'm sorry-- so that the plane wave meets this spherical surface. If you do that, then you actually get refraction to occur in two steps. And because the first step produces an aberrated spherical wave, but the second step also incurs some spherical aberration because of the nonlinearity in the Snell's law, it turns out that, in this case, the second surface, not exactly, but closely compensates the spherical aberration. And you get a much better focusing quality over here.

So this is very good, practical advice that we give to the sort of first-year graduate students when they go to the laser lab, is that when you have-- when you try to focus a plane wave, you always orient-- but for A, you use a plano-convex lens, not a biconvex, and B, that you place it in this way, so that the spherical surface of the lens meets the planar wavefront.

And, of course, it goes-- it goes the other way around. If you have a point source, and you want to collimate it, then again, you have to make sure basically you can reverse the rays over here. And you have to orient the lens so that the planar part of the lens faces the point source. So this is an example of doing a very simple trick in order to not quite eliminate, but reduce the spherical aberration. It turns out that-- yes? Push the button, please.

AUDIENCE: [INAUDIBLE]

AUDIENCE: He didn't push the button.

GEORGE BARBASTATHIS: Yes.

AUDIENCE: In that graph, how did they actually measure the spherical aberration in different lens shapes?

GEORGE BARBASTATHIS: Yeah. So the way to-- thank you. I should have mentioned that. So you can characterize a spherical aberration based on one of these parameters here. One is-- so it's called the longitudinal spherical aberration, and it is the distance on axis between the paraxial focus and the focal point of the last-- of the marginal ray. Right?

Another way to do it is to basically measure the size of the least confusion. So any of these-- if you think about it, all of these are proportional to each other, so you can put any of these quantities on the vertical axis here. I'm not sure what he did here. I think it says axial here, so it was the longitudinal distance here.

And I thank you. I should have mentioned that. This way of characterization is common for all of the aberrations that we'll see in the next two slides. Yeah?

AUDIENCE: [INAUDIBLE]

GEORGE BARBASTATHIS: Button.

AUDIENCE: We knew which is the lengths. Do they specify paraxial or focal length, or circle of least confusion as a f?

GEORGE BARBASTATHIS: Yeah. That is a real good question. If it is a sort of high-end lens, it should be well corrected. So then, what it will specify is actually the least confusion focal end, but it is pretty close to paraxial. If it is a poor lens, then most likely what they specify is the paraxial focus, and then it is kind of up to you to position it. Any other questions? The--

AUDIENCE: In the previous--

GEORGE BARBASTATHIS: Yes.

AUDIENCE: Hi, George. In the previous slides, what is the curve in the third and previous at-- hold on. Yeah, what is the curve in the third graph?

GEORGE BARBASTATHIS: The curve in the third graph?

AUDIENCE: Yeah. So the vertical axis is H, and horizontal axis was--

GEORGE BARBASTATHIS: This one?

AUDIENCE: Yeah.

GEORGE BARBASTATHIS: I don't remember, actually. I think it is the-- I suppose it is the amount of spherical aberration as a function of position in the field. I'm not really sure. Maybe you can check that. Actually, I have the book. You can check the book, huh? This came-- is directly from the book. I think it is the amount of spherical aberration. It makes sense because-- here. Any other questions?

So it turns out that what we said before, and, actually, Colin made that point when I was discussing ellipsoid, that the fractals and so on and so forth, and what we said before, that a spherical surface produces spherical aberration, it is not always true. There is one condition where a spherical surface does not produce spherical aberration.

And this condition is given here. That's another diagram I got-- that I got from the book. And you can see here, basically, the condition says that if the radius of this field is R, the index of refraction of this field is n2. The index of refraction outside this sphere is n1, and you place a point source at the distance R n1 over n2. Then, of course, the point source is inside this sphere.

If you think about it, this is a negative lens. It will produce a virtual image. This virtual image is located over here, and it is free of spherical aberration. I will not prove this. This is actually in the book. It is given as a problem in the textbook, and the problems in the textbook are solved at the end of the textbook. That's why I never assign problems from the textbook. And anyway, you can look at it afterwards. It is a very simple geometrical proof of this one. So we'll skip it here.

But the point of this is that there is actually a single combination of image-- I'm sorry-- of object image pairs, for which the spherical aberration is canceled for this spherical surface. And again, I want to emphasize that if you place the object point elsewhere, it will still form an image in the paraxial sense. This lens will remain a lens, but it will not be free of spherical aberration. It will only be free if this condition here is satisfied. And actually, this distance that I mentioned is from the center of this sphere.

All right, so the reason for that-- so this now has a number of implications. For example, this is one reason why meniscus lenses are very popular in photography, or at least in very old-fashioned cameras that were very expensive. Now, we're talking about cameras in the '20s and '30s, you know, the type of camera that the cameraman would go under a black hood and then going, and then click. And, you know, the family would be sitting with the-- whatever, with the children, all that stuff.

And so these cameras, they would typically use a single meniscus lens, and the reason was exactly that the meniscus, at least if you designed it for the typical distance-- remember, again, this was a camera for a photography shop. So it has-- you can kind of predict the location of the object because the camera's already always sitting in the same distance from the-- wherever you place the family to take the family picture. And then, in that case, you can engineer the meniscus to minimize this spherical aberration for this particular distance.

It turns out the meniscus also has some other interesting properties, but I will not go into that. So of course, in that case, you cannot really put the family in glass, so you're going to have exactly this situation over here. But, nevertheless, if you think about it, you don't need-- you know, you can satisfy this condition, even if you have two spherical surfaces, provided that, as you go from one surface to the next, you have to satisfy the same condition. It is the same condition that you have to satisfy twice.

And I forgot to mention, this condition is called the aplanatic condition. Actually, that, I don't know why it is called aplanatic, but it sounds like a-- you know, Colin, why-- where that comes from? Aplanatic means non-planar. It makes no sense at all, but, anyway, that's what they call it. And it is also used in oil immersion microscopy.

In this case, we mentioned that. Actually, it was also a topic of one of the homeworks. Oil immersion means that you put the liquid, the droplet, between the objective and the sample in the microscope. And in this case, again, you can kind of make sure that when you apply the aplanatic condition-- because, again, in the microscope, you have very good control over the distance between the exit pupil and the object. And you can make sure that the aplanatic condition is satisfied for this particular distance, and you can use the index of refraction of the oil in order to satisfy the aplanatic condition.

So now, that's the case where you exactly cancel the spherical aberration, so it is very interesting. It is different than the previous case. Here, we exactly cancel it by using this particular condition.

The next aberration I wanted to discuss is coma. So coma is different than spherical in the sense that spherical occurs even if you have on-axis incidence. Coma actually occurs for off-axis incident rays. So this is the typical image, or the typical sort of depiction of coma that you see in books.

Again, you have a plane wave coming from infinity, but it is incident at an angle of focus. So it is likely quite possible that you might have spherical aberration as well in this case. But in this particular case, we're only concerned for the aberration due to the fact that you are off axis, and that is called coma.

So if you work out sort of the non-paraxial imaging for this case, you will discover that at the focal plane, sort of up to here, what you see is a cross-section of the system in the meridional plane. And this is what you would see as a front view on the imaging plane, and it turns out that you can also see it here. This is perhaps a better picture.

It looks like a teardrop, and this is where the term coma comes from. It is not a condition that you enter after a very serious accident, but it is rather from the punctuation symbol, a comma. In old-fashioned books, kind of the comma looks like this. So ah, I don't know how good my drawing is, but this teardrop shape is kind of reminiscent of the punctuation, a comma. So this is what it looks like.

And now, the geometry of this is a little bit complicated. You can basically calculate the shape of the coma in the meridional plane and the sagittal plane, perpendicular to that. So the meridional plane is this one, the one that you see over here that is-- if you take the plane of incidence, it is the plane that cuts through the center of the lens. And then if you rotate by 90 degrees, that would be the sagittal plane.

So according, because we are off axis, the two are not symmetric anymore. On axis, they are symmetric, but off axis, they're not, so that's what gives rise to the strongly asymmetric nature of the coma. And it turns out, if you do some simple geometry, you find out that the length of this comatic shape along the tangential, the meridional plane, it turns out to be three times the length along the sagittal plane.

So this follows from geometry. To be honest, I never quite went through the derivation of this one, but, anyway, someone did it. And these derivations, by the way, they have a way of becoming very complicated, even when they give very simple results, like this case over here.

Whether we'd like to-- there's a few things I would like to say about coma. One is, of course, that it also depends on the shape of the lens. So here you see what this spherical aberration and coma plotted for lenses of the same focal length, but different shape factor.

And you can see that, fortuitously, there's a region over here, where, above the spherical-- actually the coma is completely eliminated, and this spherical is close to zero. So this means that this is a good design point, if you want to simultaneously minimize coma and this spherical aberration.

Now, you might wonder-- let me skip for a moment the discussion of the bottom figure over here. I'll come back to it. But you may wonder how it can be that coma is completely eliminated. It turns out that-- and, again, this is something I will give without proof. It turns out that there is a condition that, indeed, you can cancel coma.

So this is called the sine condition. Sine, now, this is not a Greek word. It is actually a Latin word, [LAUGHS] and it is the same sine as the sine we have in [INAUDIBLE]. And this requires a little bit of explanation, so we'll do that, and then we'll quit for today. So the sine condition follows from another result for the optical sine theorem.

Interestingly, for those of you who are mechanical engineers and know thermodynamics, this theorem was proven by a kind of a hero of thermodynamics, Clausius. There was-- I think there's some kind of something called Clausius Inequality, or something like that. It's a very big, important result in thermodynamics. So he proved it first, and then two other people associated with optics more closely, Abbe and Helmholtz, they derived it independently 10 years later.

So what is it this theorem say? Well, here's again the geometry of an optical system. We use the notation alpha as before to denote the angles, and y-- for some reason, the book uses y, not x, to denote the elevation of rays. So this is not an exact result. It is not quite axial, but it says that if you satisfy the imaging condition, then the index of refraction, the ray elevation, and the sine of the ray angle in the object space are the same as the product of this-- actually, I should've said the product of the three is the same as the product in the image space.

So you can see these quantities over here, all of them, a sub i-- I'm sorry-- this is a sub i, a sub o, and so on and so forth. Why is this significant now? If you take the ratio of y at the image plane-- so this is y sub i over y sub o-- that ratio is the magnification. So you see a problem here. The problem is that if you calculate the ratio from this quantity, the ratio would turn out to be what? I could do-- turn out to be something like n i-- sine alpha i over n object sine alpha object.

And because of the presence of the sines here, you can very easily deduce that the magnification is not the same. The magnification that you had depends on the size of the object, which, of course, very annoying. We did not see that in the paraxial approximation, but, of course, this is non-paraxial. That's why it happens, but it is very annoying. It is-- and it is the reason why coma happens. You can also see it over here.

You can see a very-- you can see sort of the different rays. They attempt to reach-- to focus at different locations in the image plane. So that's the reason. So the sine theorem says, boy, if I could actually make-- I'm sorry-- if I could actually make this ratio, sine alpha in the object over sine alpha in the image, if I could make equal to the corresponding paraxial quantities, then I would be happy because then the magnification would be the same, independent of object elevation. And therefore, I would be free of coma. So that is the sine theorem.

So basically, the reason coma can be eliminated here, it is because, for this particular lens configuration, the sine theorem is actually satisfying. So you basically get a complete elimination of the coma.

So unless there's any questions, I think we should quit here, and we'll continue with the remaining aberrations on Wednesday.

AUDIENCE: George--

GEORGE BARBASTATHIS: Yeah.

AUDIENCE: --can I make a comment about the-- I've been thinking about this aplanatic word that you mentioned. If you go back a couple of slides to the--

GEORGE BARBASTATHIS: Maybe far?

AUDIENCE: Yeah. That-- yeah, that one will do. I think that-- now I'm thinking where that word comes from. I've never really thought about it before, but the-- that, of course, the aplanatic condition will be satisfied not just at one point, but on a complete sphere, with center at the center of the spherical surface.

And the perfect image will be formed on another sphere, also with its center at the center of the spherical surface. So I think maybe that's why it says not plane. It's because the image on a spherical surface is going to be formed perfectly on another spherical surface.

GEORGE BARBASTATHIS: Yeah, because with a virtual surface, as we'll see a little bit later. Yeah. Yeah. Yep, that makes sense.

AUDIENCE: Hmm.