GEORGE BARBASTATHIS: The key point of the previous lecture was to simplify the equations for refraction from a spherical surface, such that we can write them in a simple form. OK, what I just said was redundancy. But we want to simplify them. I should have said we want to simplify them so that we can write them in a linearlized form.

And the linearlized form is actually very useful because with this form, as I'm about to show, we're going to use a very familiar form of math, namely linear algebra matrices. And we can retrace through almost arbitrarily complex optical systems.

Of course, there's a downside that if write trace this way, we do not really get very accurate results. Our results, as we discussed last time, are only accurate within the paraxial approximation. On the other hand, if we do this analysis even approximately, we can get an idea of how the optical system behaves.

And then if we get this approximate design, this approximate idea, then we can plug the system into a more sophisticated numerical tool, for example, Zemax or CODE V. There's a number of different optical design and software suites available. And then we can actually get a more accurate analysis of how the optical systems would behave.

So the other reason we write these matrices is that they give us actually quite a bit of information about the basic properties of optical systems. So it is really worthwhile to do this first, every time we face an optics problem, before we jump into the numerical analysis. And for those of you who really have a real-life problems to solve and you really need to go to software, we will tell you later during the class where we'll give you some information about what types of software are available and what you can do with those. But generally, this idea of using paraxial optics is very powerful in order to understand optical system.

So the summary then of what paraxial optics standard is summarized. These are shown on this slide over here. The most important is the results on the right that allows you to write the angle of departure of a ray to the right-hand side of the card interface and the elevation of the ray, with respect to the right-hand side interface. It allows you to write them as functions of the corresponding quantities on the left-hand side of the interface.

And as you can see there, it actually looks like it two by two set of vectors, which are related by a two by two matrix. And the matrix are not very interesting. As you can see, the most relevant, the most interesting element is the element on the first row, second column, which equals the index of refraction to the right of the interface, minus the index of refraction to the left, divided by the radius of curvature of the spherical interface. And, of course, the radius of curvature is simply the radius of the sphere. We're talking about the sphere.

And then the other result that we put here and looks like a propagation through free space, it looks kind of too much trouble, really, for what it's worth. But you will see in a second that this is actually a very useful tool to have in mind. One of the thing that says is that the ray, as it propagates from the left over a distance d, if it propagates through free space, then what happens is the elevation of the ray changes, but the angle of propagation does not change. So this verifies what we know from Fermat's principle that a ray propagating in uniform space, it minimizes its path. Therefore, it must propagate in a straight line because the straight line is the minimum path between two points.

Now, you might be wondering. We talked last time about ellipsoidal surfaces, hyperboloid surfaces, and spherical surfaces. You might wonder, why did we do this analysis in the context of the sphere? Well, that says it doesn't matter very much because all of these three possibilities, ellipsoids, hyperboloids, and spheres, near the axis, they actually all look kind of the same.

If you compare a sphere-- so I should specify now what I mean by axis. So in the case of a sphere, as Professor Sheppard pointed out last time, in the case of a sphere, really, any lines going through the center of the sphere acts as the optical axis. But for the hyperboloid and the ellipsoid, that's not true. There's a very well-defined major axis and minor axis for the surfaces.

And it turns out you can match the curvature of these surfaces with the curvature of a sphere so that [INAUDIBLE] a common axis, they all look the same, as I've done in this diagram. It's a little bit of an algebraic mess to actually work out how to do this. So I didn't do it in this slide. The idea is basically to do a Taylor series expansion on the explanations that describe all those surfaces. And then when you do that, each one of those is described by a different set of coefficients.

So what you can do then is you can match the coefficients in order to match the curvature. So I will let the graduate version of the class, those of you who are taking 2710, as a homework, I will let you work out one of these, a simple case for matching-- I forget. I think it is matching an ellipsoid to a sphere. So let you do that.

What I really want you to live with out of this lecture, for now, is that the paraxial analysis remains valid for all three surfaces. OK? As long as we match the curvature of the sphere, ellipse, and hyperboloid, and we constrain ourselves to the paraxial regime, then our results remain approximately correct. So this is kind of another useful property of the paraxial approximation that it is not only valid for a sphere, but it is also valid for other surfaces of revolution, provided the curvature near the center of the surfaces matches the curvature of the corresponding sphere. OK.

Yeah? Push the button, please. Yeah, so that's a good question.

AUDIENCE: Could you repeat the question? We didn't hear the question.

GEORGE BARBASTATHIS: The question was if there's an angle that we'll consider as a cutoff for the breakdown of the paraxial approximation. So the accurate answer is zero. Any angle other than zero violates the paraxial approximation. In actuality, you can calculate, with the help from this Taylor series, actually, that we saw in the previous slide, you can calculate the accuracy of the approximation.

So treat it as a first order of a Taylor series expansion. And then you can calculate what is the next order accuracy. So this is actually algebraically quite complex. And typically, we resort to software to do it for us, numerical software. As a rule of thumb, anything in the range below 30 degrees, you can kind of trust the paraxial approximation. But that's a rule of thumb.

In some cases, it's not true. In some cases, you need more accuracy. So the guideline is something like that. But, for example, for sure, if you have a ray propagating at 80 degrees, you know for sure that approximation breaks. If it goes to 1 to 3 degrees, then most likely, you're OK. And then there's a gray area that you have to be a little bit more careful. If the accuracy demanded by the application is much higher, than you have to be more careful.

There's also surfaces that are not very well-described by this at all, for example, a cubic. If you make a refractive interface that looks like a cubic surface, would look kind of like this. And then this totally breaks down, right? So you have to go to then to non-paraxial [? methods. ?]

Another thing to discuss is in one the previous equations, I wrote down ray elevations and ray angles. But I was not very careful about defining the signs of these angles, namely, whether they go up, or down, or left, or right, and so on. I was deliberately left this little bit vague.

Again, this may appear like too much trouble for nothing. But you will see, as we go later on with the lecture today, you will see that this is a very useful and highly nontrivial aspect of geometrical optics, namely, the conventions of the signs, for when the quantities that we're dealing with are positive or negative. So bear with me for a second, and I will show examples of how this works later.

So I want to read this because I don't know of a better way to do it. I will read it. And then as I read, please look down at the slide. And you will see on the left-hand side I have examples of all these qualities being positive. And on the right-hand side, we have example of all of these quantities being negative.

OK, so the convention number one is that the light always travels from left to right. So you know immediately when someone draws an optical system or attempts to draw an optical system, you know immediately if this person knows their optics or not by the way they pick the light propagation. If they show light rays going from right to left, it means they don't know optics. So if they learned optics, they learned it very deficiently.

AUDIENCE: [CHUCKLES]

GEORGE BARBASTATHIS: So always light goes from left to right in this diagram. There's one exception, of course. Can anybody guess? One exception that I'm not dealing with here. Mirror. Push the button.

AUDIENCE: Mirror.

GEORGE BARBASTATHIS: Mirrors, yes. So in this diagram over here, I do not consider mirrors yet. We will deal with mirrors later, in about a week. And then we'll see a revised set of sign convention. But as long as there's no mirrors in your optical system, if you have what is called a purely dioptric optical system, then the light is always going from left to right.

Taking this as a given, then we'll define the positive and negative radii of curvature as solved over here. So if the surface is convex towards the left, then we'll call it positive, OK? Then this sounds a little bit weird. It says that longitudinal distances are positive if pointing to the right. I'll show you an example a little bit later where distance can become negative by pointing to the left. But for now, take this definition for granted.

The same is for lateral distances that this measures perpendicular to the optical axis. If they point up, then they're called positive. If they point down, they're called negative.

And finally, angles are positive if they are acute in the counterclockwise sense, with respect to the optical axis. So clockwise, we have to be very careful when we define it, so we don't get any mirror effects. So hope that the cameras that we're using here do not make any mirrors. But counter-clockwise is basically like this, OK? If in this direction you get an acute angle, then the angle is positive. If you have to go all the way around to get your angle, or another way to put it is if you can make it acute by going clockwise, then you call the angle negative. OK.

So this set of conventions is self-contained and consistent. To add to the confusion, set and optical text books, they use the opposite sign of conventions. It is also possible, by the way. The set of conventions I use here is consistent with a Hecht text book and the majority of text books. There's a minority of textbooks that use the exact opposite conventions. But we will not deal with those.

OK, using these set of conventions, we can define the power of an optical system based on whether rays that propagate outwards-- by outwards, I mean a ray bundle that expands-- whether a surface, a spherical surface, causes the expansion to become slower or faster. If the surface causes the expansion to become slower, as in the top diagram over here, then the surface is said to have positive power. If the surface causes the expansion to become faster-- that is, it takes the expanding bundle and spreads it further outwards-- then the surface is said to have negative power.

So now, let me proceed to define a specific optical system. And then we'll see more examples of this thing happening. So the system I would like to consider is a lens. And this is actually the first example of a lens that we see in the class. And I will take a very special case with a lens that we refer to as thin. What's a lens? A lens is simply a piece of glass that has been shaped to have spherical surfaces on one or both sides. Typically both, but we'll see examples of lenses that are actually flat on one surface and curved on the other surface.

So when we call the lens thin, well, we call it thin. Consider an expanding spherical wave that propagates through the lens. And consider the maximum angle that can be subtended through the lens. In other words, angles larger than this one would actually miss the lens. So therefore, this alpha maximum is sort of the biggest angle that we have to worry about.

Well, what does the paraxial approximation say? It says that this angle has to be relatively small. So assume that this is true, then. The lens also has a thickness. So this thickness, I denoted as T in this diagram over here. So provided this thickness, if we do a little bit of an analysis here, we can calculate this thickness relative to the radius of curvature of the sphere. And the angle phi that is defined as shown, from the center of the sphere relative to the edge of the ray, that subtends the maximum angle alpha maximum. OK?

So we can see the definition phi for this angle and the radius r. So if, given this angle of arrival is alpha maximum, if it is true that the angle phi is also very small, then we can consider the lens to be thin. So we can basically pretend that the space between these two points on the lens, the space between the entrance to the optical axis and the exit of the optical axis through the lens, we can neglect the space. This is what the thin lens approximation says.

That's very convenient because we can now analyze the lens. As you can see, if I neglect this space, then the ray elevation before and after the lens remains the same. And also can neglect a refraction that might happen within the lens. So basically, we have the system where the ray arrives then is bent in one step and then propagates outwards at a slightly different angle.

So in order to analyze the system, now, what I will do is I will break it into two components. One of them is the curved surface on the left-hand side. And the second is the curved surface on the right-hand side. OK. And now, for each one of those, I can define an angle on the right and then angle on the left, as I have done in the simple [INAUDIBLE] of the refractive surface.

Now, each one of those is a simple refractive surface interface that we know how to deal with from the previous lecture. And I can add equations. Remember the equation for the refraction from the spherical surface. I will add it here on my pad, just to remember. So it goes like this. Hopefully, someone will notice and they will project it. So n to the right, alpha to the right, x to the right equals-- can anybody see? OK, we're having technical difficulties here. Oh, there we go.

AUDIENCE: Can you lower the piece of paper a little? Yeah, got it. Yeah.

GEORGE BARBASTATHIS: So this equals 1, 0, 1. This is easy. And then we have apply our mnemonic, minus n to the right, minus n to the left, over the radius of curvature, times the same property for the left. And left alpha left, x left. OK.

Now, what we're have to do is we have to apply this rule successively for the two interfaces. So here's the question that we get for the second interface. So this case, n to the right is 1 because to the right of this interface, I have air. And n to the left is n because to the left of this interface, I have glass. OK.

Now, what I will do is I will apply the same rule. But I will now apply it on the first interface. And if I do to the first interface, I get this equation. Now, notice that in the numerator of the element in the first row, second column, the quantities are reversed because now n to the right is the index of refraction of glass, whereas n to the left is the index of refraction in air, that is 1. So therefore, I get this matrix.

Now, what it can do, I can cascade the two matrices because I can substitute the vector n over 1 x1 that is on the top equation. I can substitute it from the second equation. And if I do that, of course, I will get a cascade of the two matrices. And notice that in the cascade, it is very important to remember that in the cascade, the matrix of the last element to the right appears left.

And that is very important to remember and not confuse because as you know, matrix multiplication is not commutative. If you mess up the order of these matrices, you will get the wrong result. So you have to remember that you start from the right. That is, you start from the end of the optical system where the rays depart, and you start cascading the matrices going backwards towards the beginning of the optical system.

So this is what I've done here. I've put the matrix corresponding to the second interface. This matrix appears first. And the matrix corresponding to the first interface appears second. And then I cascade them. I do the matrix multiplication, which is a relatively straightforward task to do. And I get the second, the last equation that appears on the slide. OK.

Now, let's go ahead and interpret this equation. It looks very nice. But let's see what it really means. Also, before I move on, let me notice one more thing. Notice that in the vectors of the ray, angles of the ray elevations, the index of refraction has disappeared. It has not really disappeared. It is simply because the angles alpha right and alpha left are in air. And therefore, the index of refraction is 1. That's why there's no index of refraction in that location. OK.

Let's look at this equation again. The top quantity that appears relatively complicated, it has a name. It is called 1 over the focal length. Now, you might wonder, why did they call this the focal length? I will justify that in a second. But if you believe me that the focal length follows from this equation, then this equation is known as the lens maker's equation.

And it is very useful because it gives you the focal length of the thin lens as a function of the three quantities that define the thin lens, that radii of curvature to the left and to the right and the index of refraction of the lens. And what I want to emphasize, again, is that this equation is valid if the lens is surrounded by air. If the lens is surrounded by another material, for example, water, or oil or something with index of refraction different than 1, then this equation does not hold anymore.

And as a bonus, in the next homework, which has already been posted, you will actually derive the corrected equation if the lens is immersed in a material that is different than air. But for now, this equation is correct because we put our lens in air. So therefore, everything is fine. And this is a lens maker formula.

So then let me justify that this quantity that appeared over there and has units 1 over distance, why on over distance? Because this quantity has 1 over the radius of curvature in the denominator. So therefore, it is 1 over distance. So let me justify why this quantity is actually the focal length of the lens.

So to do that, let me consider a ray that is arriving from infinity horizontally at an elevation x1 with respect to the optical axis. So this ray, I can retrace through the system. Basically to retrace through the system means that I have to propagate this ray by a distance z. And then I have to find out what is the ray angle alpha 2 with respect to the optical axis and the ray elevation x2, also with respect to the optical axis, as function of this propagation distance z.

So now you can see why it was convenient to define the matrix for propagation through free space. Because now, in order to figure out where this ray lands, what is its elevation and propagation angle, all I have to do is cascade the matrix that describes the lens with a matrix that describes propagation through free space. And we have seen this matrix of propagation through free space. It is simply 1, 0 in the first row. And then the distance and 1 in the second row.

And there's actually, if you look back at your equation, it is distance divided by the index of refraction. But, again, because we are propagating in air here, the index of refraction is 1. So we don't have to worry about this at the moment. And because the free space is following the lens, the matrix corresponding to free space will actually appear first before the matrix corresponding to the lens. So this is the cascade [INAUDIBLE] correctly here. And we can solve this. And we could find that is given by this equation over here.

So what we observe now is that if we set the propagation distance to-- first of all, I did this in the general case. So as the ray entrance angle, I left a general symbol alpha 1. But I said already that the ray is arriving horizontal. That means alpha 1 equals 0. So therefore, the elevation of the ray, with respect to the optical axis, is given by this equation. It is simply x2 equals x1, 1 minus z over f, OK?

And we can see very easily over here that if z equals f then x2 will equal to 0. So this means that if I let the ray propagate by a quantity equal to this magical amount that appeared in the lens maker's formula, then the ray will hit the axis. So the ray will land on the optical axis. And moreover, we can very easily verify that this happens independently of x1, because as you can see, x1 is outside the parenthesis here.

If I set this quantity to equal 0, then independent of what x1 was, we will actually get all of the rays to go through the same point of the optical axis. So this is a central focus. You can see that the ray bundle that arrived parallel from infinity actually now comes to focus at this location of the optical axis because of this simple equations that we just described. And this property over here justifies the name focal length. It is the length at which the rays come to focus. OK.

The inverse of the focal length, just on its own, as a sort of standalone quantity, has a name, also. It is called the optical power. And that is a little bit confusing because in our minds, usually, power is measured in what? Well, this power is not the usual power that we measure as energy per time. It is a different power. And for historical reasons, we use the same name in optics, but it is measured in inverse meters. And the inverse meters, they're also known as diopters in optics. And they define the optical power as the inverse of the focal length.

So, for example, if the focal length is 1 meter, which is a pretty long focal length, you call it one diopter. If the focal length is 10 centimeters, it is actually 10 diopters. If it is 1 centimeter, it is 100 diopters, and so on and so forth. Very often, people like myself who are myopic-- is anybody in the class, is any of your myopic? Anybody use corrective lenses because you have myopia? OK. Do you know what is the power of your prescription?

AUDIENCE: 0.75.

GEORGE BARBASTATHIS: 0.75, you're pretty likely. You have very small correction.

AUDIENCE: [INAUDIBLE]

GEORGE BARBASTATHIS: That's a pretty small correction. Can you say it in the microphone?

AUDIENCE: 0.75 in the one eye and in the other, 1.25.

GEORGE BARBASTATHIS: OK, 0.25. Is it 0.25 or minus 0.25.

AUDIENCE: Minus 1.25.

GEORGE BARBASTATHIS: Minus 1.25, OK. So that is also in diopters.

AUDIENCE: Yeah.

GEORGE BARBASTATHIS: It means that the focal length of your lenses is approximately 80 centimeters, right?

AUDIENCE: Right, mhm.

GEORGE BARBASTATHIS: Yeah. Do you have a--? Yeah.

AUDIENCE: Mhm.

GEORGE BARBASTATHIS: OK.

AUDIENCE: Yes.

GEORGE BARBASTATHIS: So we will see later why then correction for myopic people is negative. I'm also myopic. And my correction, I actually have pretty high correction. Mine minus 7 and minus 5 diopters in my left and right eye, respectively. So it means, again, that the focal length of my glasses is approximately minus 1 over 7, which is-- I don't know, what-- 17 centimeters or something. OK.

So that is the power. Now, why we use the term optical power? Actually, it is best explained if you look at the final animation on this slide, which is what happens if the ray bundle comes from infinity again, but it is not horizontal. It is coming at an angle alpha 1. So now, the angle alpha 1 is not 0 anymore. But I let it have a finite non-zero value.

And if you do that, then you find relatively easily from the equation over here. Let me write the equation. We have that x2 equals alpha 1 z plus x1, 1 minus z over f. OK. So if I set z equals f to the above equation, then I get x2 because alpha 1 times f.

This term will disappear, of course. This will go to 0. And z will become f. So get x2 equals alpha 1 times f. So this is the equation for the elevation of the focus for a parallel ray bundle that is arriving non-horizontal from infinity at some angle alpha 1.

So you can also write this equation-- it is not on the slide-- I will just add it on the white board. x2 equals alpha 1 over the power, the optical power of the system. So the power is kind of like a lever. It tells you as you change the angle alpha 1, it tells you how does the elevation of the focus change, with respect to the optical axis. So it is basically, again, if your thinking is if you order transduction amplification or something like that, it tells you as you change the angle that the rays arrive into the optical system, how does the focus move with respect to the optical axis. OK.

Now, basically, these equations that we just described, let's see some different cases of what would happen to the rays. So what I've done here is I put the different types of lenses together with a simple spherical refractor. And the different types of lenses are plain or convex where you have one convex surface followed by a planar surface, Then biconvex where you have two planar surfaces, then planar concave and biconcave.

So you can see from these that if you have a ray bundle that is arriving horizontal, now, horizontal from infinity, you can see that what will happen at the output is actually different, depending on the type of refraction that you get in the different surfaces. So I will let you work out, based on the equation, the lens maker's equation, I will let you work out why these cases are all different. But what I would like to do is actually work out the location of the image, depending on the different cases, and also, see how to ask with respect to the sign convention.

So starting, for example, with a biconvex lens, we can see that the biconvex will focus the parallel bundle to the right-hand side of the lens. This is the case that we have implicitly done so far. In my drawing so far, I have assumed that this is the case. However, you can see that if I have a biconcave lens, like the one shown at the bottom of the slide, you can see that each one of these rays will actually expand outwards after it passes through the lens.

So in a sense that we described so far, this lens does not really focus the rays. It creates a divergent ray bundle. You can get a sense of focus if you extend the divergent rays, if you extend them backwards towards the optical axis. If you do that, you will actually see that the rays meet. They do meet. But they meat on the left-hand side of the lens.

So now, this is an example of focusing that happens not on the right-hand side of the lens, as our equations have shown, but on the left-hand side. And we can now justify this fact if you actually define z to be negative and f to be negative. So this is what we call a negative lens, or a lens with negative focal length, or a lens with negative power.

And if you do that now, then you will see that the focus will appear at the left-hand side, consistent with our sign conventions. And this is what we call a virtual image, as opposed to the case of the biconvex lens that we'll call a real image. So this is an example of a sign conventions, not convections, the sign conventions in action.

Are there any questions about this? This is a point when I often get a lot of questions. Let me move on to a different case. And if you think of a question. Please interrupt me, or say something aloud. Push on your microphone button first.

Let me do a slightly different case, which is what happens if you try to-- so in the previous example, we saw how a lens can take an object at infinity and focus it to a finite distance equal to the focal length. What I will do now is I will argue that the positive lens actually can take a point object located at one focal distance to the left and image it at infinity.

Now, why is that true? I will do this in a second. But you can very easily justify it to yourself if you simply reverse the orientation of the rays in the previous case. What we did before is we had the ray bundle that was starting from infinity. And the lens was focusing it. We can very easily reverse the radiation of the rays. And if we do that, then we can see that if I have a point source over here, it will actually get imaged at infinity.

Now, of course, according to my sign conventions, this is not the proper way to write it. We have to make sure that the light propagates from left to right. So therefore, the correct to write, it is not what is showing over here, simply by convention, not because this is physically wrong. But it does not obey the convention. So we never actually draw something like this.

What we draw is what is shown on the slide where the rays are propagating from left to right. They have a focus that is a point object on the left-hand side of the lens. And then the lens, of course, will collimate these rays. They will convert them from a divergence spherical wave to a parallel plane wave. And therefore, we limit at infinity.

The same thing can be said about the negative lens, except in this case, the ray bundle arriving at the lens has to be convergent and has to come to a focus on the right-hand side of the lens. So therefore, in the same way that this type of lens formed the virtual image in the previous case, in this case, we talk about the virtual object for this type of lens. OK.

So let me now try to put this all together so it can make some sort of sense. Again, recall the first sign convention, which sees that the light propagates from left to right. If it propagates from the left to right, it means that the object should be to the left of anything that is of interest. And the image should be to the right of anything that is of interest to the optical system. So that is why if an object is to the left of the optical element, as it is in this case, then we say that the distance from the object to the element is positive.

If, on the other hand, an object happens to be on the right of an optical element, as it happens in this case with a negative lens, then we say that the object is virtual. And the image from the object to the element is negative, because it has to be on the right-hand side, whereas in the proper case, it had to be on the left. That's why we put a negative sign.

Now, similar things can be can be said about an image. Except, now, the image is properly positioned or the right-hand side of the optical element. So everything is really on the right-hand side, as it happens with a positive biconvex lens. Then the distance from the lens to the image will be referred to as positive, whereas on the other hand, if it happens to be on the left, as it is for the case of the negative lens, then this image will be referred to as virtual, and the distance will be referred to as negative.

Finally, and I will conclude with this, and I will let Pepe do his demo, is how this applies to off-axis objects. So remember we derived this equation, x2 equals alpha 1 times f, for an object that is at infinity. Now, for a positive lens, because the focal length is positive, if the angle alpha 1 is also positive, as shown here, it means that x2 must also be positive. And this is indeed what we can verify. If we apply Snell's law repeatedly for each one of these rays, we will see that indeed this is the case.

On the other hand, if I use a negative lens, where the quantity f, the focal distance, is negative, but the angle of arrival is still positive, then I will get x2 is negative. That is, x2 will also appear below the optical axis, OK? So this virtual image not only appears on the left of the negative lens, but it is also below the optical axis. OK?

Now, what about an off-axis image at infinity? I will skip this derivation. It is very similar to the derivation that we did before for the case of an object at infinity. Basically, what I've done here is I have reversed it. I have used an object at a distance f to the left of the lens. I have cascaded the propagation matrix for this free space propagation to the propagation method corresponding to the lens. And I have proceeded to solve.

If I do this, I will let you do it at home and then come back tomorrow if something is unclear about this. But what I've done is I have solved it. And when I find an equation that looks like this, I find that the angle of propagation of the ray bundle to the right-hand side of the lens actually equals minus the elevation of the object divided by the focal length. Or another way to write this equation is alpha 2 equals minus x1 times the power of the optical element, according to the definition of power that I remind you by definition equals 1 over the focal length. OK.

So there's a minus sign now, which means that if we apply this equation to our positive lens, the ray bundle that will emerge from the lens will actually now propagate downwards, OK? Because in this case, x1 is positive and the focal length is positive, alpha 2 has to be negative. And negative alpha 2 means that ray bundle has to propagate downwards. OK?

If I do the same exercise for the negative lens, then we can see, again, very easily that because x1, If I pick it to be positive, f is negative and I have a spare negative sign from the equation. Then I can see that the propagation angle, alpha 2, will be positive. That is, the image at infinity will be on the positive side, above the optical axis, OK?

I hope the positive and negative signs are not confusing. Let me restate. When I learned optics, I found this to be also horribly confusing. So I realize and I sympathize how at the beginning, this can be a little bit confusing. I've done my best here to present them better than the professional who taught them to me solve them. So I hope, because I sort of used my own confusion as a guide, I hope that I managed to make it a little bit clearer for you if it is the first time you see it.

But, of course, it will become even more clearer as you practice, OK? So the homeworks and the examples that we see later, they will make it progressively clearer. For now, what I want you to do between now and tomorrow's lecture is to go back and make sure that these pictures that I've drawn, that they make sense with Snell's law.

In other words, what you do is you go to the spherical surface. You imagine a normal to the surface at each point. You apply Snell's law. And you convince yourselves that the rays bend the correct way. And this is consistent with the way they image is formed, OK? And then you apply all the methods, all the equations, and so on and so forth.

I broke my promise and I did not leave Pepe with his 5 minutes. But that's because it took us 5 minutes to fix the figures, the display, or whatever was happening at the beginning. So hopefully, you guys can stay for a little bit longer, so we can do the demo. Is that OK?

PROFESSOR: Sure. I'll try to do it. We'll try to do it fast.

GEORGE BARBASTATHIS: Also, and look at the--

PROFESSOR: So now--

GEORGE BARBASTATHIS: Yeah.

PROFESSOR: We can switch to their cameras, please. OK. So here we go. OK so we tried to set up these webcams because last time it was a bit hard to see the demo. So this demo has actually two parts. But today, we're just going to show one part, namely, the refraction. And we're going to basically witness how Fermat's principle that we've been learning in the past couple of classes, the Snell's law, applies in these systems, like lenses or prisms that we've been learning so far.

So before actually changing the exposure, let me just show the set up. So we are going to be focusing on this half of the set up. So hopefully, you can see it fine here. Let me just move this window. So to avoid any problems about chromatic dispersion or wavelength-dependent behavior, we're using a laser, a green laser. So this component here is the laser.

Then this is a new component that we're going to call-- it's called spatial filter. This, for now, it has a very interesting property that we'll learn in the second half of the course when we learn about Fourier optics. For this first part, just think about this produces a very nice point source object. So therefore, after this lens, there's going to be a spherical wave emanating, propagating.

So what we want to do after that is to convert that spherical wave into a plane wave, right? So this is what this lens here is doing. And that's what we call collimation. So after this lens, as you can see-- well, you'll see it in a second. OK. So now, I reduce the exposure of the camera, and I can see the rays. Now, I see parallel rays. Hopefully, you can see them clearly. Could you zoom in a little bit? OK. And we have basically parallel rays coming out of the collimated lens.

So now, this brings another interesting. As you can see, our eyes, and we'll see it next lecture, how they are really robust to focusing in different conditions, to change illumination. And more particularly, they're adapted to high-contrast in light, like very high intensity in the light and low intensity. Your eyes can see it fine, whereas this camera, we just need it to change exposure, so you could be able to see that. So this is a problem of dynamic range that occurs in optical systems.

So, all right, let's look at the first component. It's this lens here. I'm going to put cylindrical lens. It's a plano curve lens here that it's cylindrical. Basically, it means that the surface only depends on the x-coordinate. So we'll put it here. And [INAUDIBLE] wavelength.

GEORGE BARBASTATHIS: If I may interject here. Pepe, if I may interject something?

PROFESSOR: Sure, sure, sure.

GEORGE BARBASTATHIS: That equations that we derived, strictly speaking, they're for cylindrical lenses, right, because we only use x in our equations.

PROFESSOR: Yeah.

GEORGE BARBASTATHIS: If you have an x and y, then you get the proper spherical lens. So strictly speaking, this is--

PROFESSOR: Yeah.

GEORGE BARBASTATHIS: --what that equations--

PROFESSOR: Exactly.

GEORGE BARBASTATHIS: OK.

PROFESSOR: So now, you can see now the ray tracing, but now done optically. You can see how the rays started focusing in to this point here. Well, in this case, now, since this is cylindrical lens, we see that it's a line. And I can put this card here and we see that we actually form a line instead of a point.

Other nice things to see about this lens is that so far, we've been ignoring reflections that occur with this lens. But let me just show you some of the reflections in this cardboard here. So we'll see in the second part of the course that these reflections are occurring, transitioning from the air and glass interface and back again. So in these interfaces are very sensitive to the angle of incidence and to the refractive indexes of both the lens and the surrounding media.

So in this case, you can see that quite a lot of power-- so this is the incidence wave. And you can see quite a lot of power is actually going to this side because this lens might not be AR-coated, or coated with an anti-reflection coating as a better lens would be.

So now, let's witness Snell's law with this prism here. So you see the little rays here that are going straight. So now, I introduce this prism shown here. It looks very dark. But believe me, there's a prism here. And then as soon as I started putting it into the system, you see how the light starts bending to one side more. And then you can see how basically, the tilted surface is bending the light, and I can just change the angle.

And let's now see the other phenomena that we learned in the previous class, which is TIR. So I still rotate this more. I make the angle to be larger than the critical angle, and boom. All the light gets TIRed and we can actually see, now that it's getting to this side. But in this side of the prism, there is a diffuser. And that's why you see the diffused light here very bright.

So I'm going to do it one more time. Without TIR. All the light goes, escapes out straight through. And now, I start rotating this more. And then you see TIR here. There are any questions?

So I also brought a parabolic reflector for you guys, like the people here in MIT if you guys want to see the reflection, how you see your face. Look at your face with this parabolic reflector. It's actually quite funny to see the virtual and the real images. And in this case, you'll see, and we'll learn in next class, how the virtual image generated by such a reflector, it's actually not inverted. It's erected, as it's called. So I'll leave it here, so you can come after class.