PROFESSOR: Right, OK. So I think this was where we were up to. I wasn't here last time, so I don't really know. But George told me that this was where you're up to, so that's where I'm going to carry on from. And so this section is all about imaging and resolution, and it starts off here looking in some strange American dictionary at the definition of a "resolution." Of course, myself, coming from Oxford, I would always use the Oxford Dictionary. I don't know what that says.

But anyway, this is quite interesting. The verb here, resolve, "to break up its constituent parts, to analyze, to find an answer to, to solve, determine, and decide," blah, blah, blah. But anyway, I think this first part is-- the very first thing is a good one, isn't it? So break up into its constituent parts. So you look at an image, and you've got to actually determine what the different parts of the image really mean. And if they sort of merge together, you won't be able to resolve what the image is trying to say. So that's what it means.

And yeah, now, there are lots of different ways of determining resolution, different definitions that people apply. The most used one, perhaps, is the Rayleigh resolution limit, which was proposed by Lord Rayleigh. Lord Rayleigh was very interested in astronomy, and so he was interested in trying to quantify the imaging of stars. So if you look up in the telescope at the sky and you've got two stars very close together, then you'll see, like the two images of the stars, this shows to two different separations of those two points.

And now, it's important to note that stars, of course, two independent stars would be completely incoherent with respect to each other because they obviously don't know-- the one doesn't know what the other's doing. So they emit light incoherently with respect to each other. So to calculate the image of these two points, you have to add together the intensities of the two images, and that's what this shows here.

And so this shows two different spaces. And this one here, this is what I think is really normally taken as the Rayleigh resolution limit. I don't know about this other one. I don't where Georgia has got this from, but he talks about these two different limits in these notes. So in this one here, you'll see that this peak here is placed exactly over the 0 of the other, the first 0 of the other image. So that's what defines this separation shown in this picture.

And we find that if they're separated by that amount, the total image is given by this blue line. We get that just by adding together the two intensities of the two points together. And you can see that this drops a bit at the center. What is it? I think the intensity there is 0.888-- I can't remember now. No, it doesn't matter. But anyway, it's about 0.8, isn't it? 0.835, so that figure sounds--

But anyway, the other interesting thing, of course, is that because this one is 0 at this point, it means that this maximum is 1. It's exactly 1. It's just the one point with no contribution from the other one. Anyway, so that is arbitrarily taken as being the definition of the point being just resolved. So we say that if there are any closer together than that, you can't resolve them. You would see a little bit of a dip still, but we say that the dips not really big enough for you to really be confident that there are two objects there.

And if you make them further apart, then, of course, they'll become more distinguished. And this shows an example here where you can see now, this is twice the separation. So this is where the PSF diameter equals the point source spacing. So the-- what does he mean by that? Sorry. This one, the PSF radius equals the point source spacing. Yeah, OK. So that is the distance between the center of the image And the first dark ring.

In this one, the PSF diameter equals the point source spacing. Has he got this right? No, it's not the second zero. It's not at the second zero exactly, anymore, is it? So the spacing here is twice the spacing there. Anyway, I think that's what I'm taking at this meaning.

But as Charlene just pointed out, that means this is not actually at a zero. This is not placed at a zero of this other one because, if you remember, the zeros of the Airy disk are irregularly spaced, aren't they? So they're not equally spaced, like the light for a sink. And so because this isn't 0 at this point here, this maximum here will be not exactly 1. It will be slightly more than 1.

OK. And if this condition is satisfied, then, in this case, the delta R, the-- where does he define that? That's the-- I guess it's this distance here, isn't it?

AUDIENCE: [INAUDIBLE]

PROFESSOR: It's the separation of the peaks. Yeah, OK, it's the separation of the peaks. It's 0.61 lambda over NA. Oh, yeah. OK. So here he's-- R and R dashed are obviously coordinates in the object played in the image plane, effectively. Yeah. So, of course, it depends on-- the numerical aperture in those two spaces won't be the same, of course. But there will be magnification as well, which is going to mean that you get the same answer for the resolution in both planes.

So this 0.61 is this magic figure that comes about basically because of the-- it's related to the first zero of the Bessel function, j1. And the first zero of the Bessel function j1 is 3.83, and this is 3.83 divided by 2 pi. Does that sound right? Yeah. Yeah, I think that sounds right.

This one here is twice the spacing, so 1.22 lambda over NA. So this is 3.83 divided by pi. Yeah. So here, actually, these are obviously extremely well resolved. You wouldn't have any problem in seeing that there's two points there.

And so here he calls these two definitions the safe definition and the pushy definition. My experience is that the pushy definition is the one that is normally quoted in places, and so this is this one where you can only just see a bit of a dip in between them.

Note that if we did the same thing for the 1D system, i.e. for a slit aperture rather than a circular aperture, then you'd be looking at the first zero of a sink rather than a Bessel function. And so you get these expressions rather than these, so 1.22 rather than 1 or 0.61 rather than 0.5. So in this case you actually you can see you can resolve slightly smaller spacing with a slit aperture than with a circular aperture. And this is a true effect that sometimes people try to take advantage of.

You will see that different authors give different definitions. So I'm not going to go into all that because it's not really important at this stage. Rayleigh, in his original paper, noted the issue of noise and warned that the definition of just resolvable points is system or application dependent. Actually, what you find-- well, a lot of that is true, of course. Here we're-- it very much depends on the actual system and what you're trying to measure all these sorts of things.

And you can come up with lots of different resolution criteria for different shaped objects, and the comparison of different systems doesn't always give the same answers according to what you chose. So beware. There is not one answer to this question of resolution.

The other thing is, as he says here, is noise is an important thing, of course. If you add noise to these image-- we've got some examples later where there's noise added to the images so that you can see what it does to it. But one thing that is perhaps not quite what George is saying there-- actually, you do find-- if you look at the image of these two points, as you bring them together, you actually do find that the degradation in the image is really rather sudden.

So although, OK, it's not a specific point, it is quite so sudden. And so you'll find that if you change the separation around slightly more or slightly less than that, you'd find that this central bit would go up and down quite a lot, actually. So it is quite a sensitive change around that spacing. So that gives us some idea that maybe this resolution limit as it's defined does mean something, anyway.

Now, that's for a perfect aberration-free system. But very often, you've got systems where you've got aberrations, and that's obviously going to mean that the resolution is not going to be as good. So this picture shows an example with coma. So this is with an off-axis ray, off-axis beam going through a lens, and you get this coma spot here. So now, of course, this spot is bigger than it would be if there were no aberrations, and therefore you'd expect that the resolution would be worse.

So this is another thing you have to take into account. I don't think there's going to be much more about that. But did that do something there? Or perhaps this didn't push it fast enough.

Now, the other way of looking at the effects-- resolution and the effects of aberrations is looking at the transfer function approach. And so this is giving results for a one-dimensional system. And it you see it's an MTF, Modulation Transfer Function. So that means it's an incoherent system. The MTF is the Fourier transform of the point spread function.

So this is for the aberration-free case. This is a sink squared, the point spread function for the 1D case. And if you take the Fourier transform, you get the modulation transfer function, as he calls it, and vise versa, of course. These are Fourier transform pairs. This is just a triangle function.

And this shows what would happen if you got some aberrations there. In general, what would happen is that this would be broader, and this would be deformed so that you've got a lower spatial frequency response. Here, actually, he seems to be showing something which has got an improved response at the very high frequencies, which is quite interesting because what that would mean, actually, is that if your object only had those high spatial frequencies, the system with aberrations would actually give a better image than the one without aberrations. So maybe that's a bit anomalous.

So now some common misinterpretations, attempting to resolve object feature smaller than the resolution limit, i.e. 1.22 lambda over NA, is hopeless. So he's saying that that's not strictly true because this is not, for a start, it's not a sudden cutoff. And it depends on the presence of noise and other features. So there is a very big no to say that, yes, you can sometimes see things which are smaller than the resolution limit.

And then also another point is that there are ways you can actually improve the performance using, for example, the CLEAN algorithm. The CLEAN algorithm is a thing they use in astronomy for doing deconvolution. It works particularly well-- it would be very good, actually, for some of these microscope things that people do nowadays with these very sparse molecules because it's designed to work with you know in astronomy with isolated point images. So if it looks at a continuous object like a motorcar or something like that, it wouldn't work. It would actually only work on point-type images.

Then vena filtering-- we'll say something more about that-- expectation, maximization, et cetera. So there are various digital ways you can improve the image and get information out of the image, even if you can't see it by [INAUDIBLE].

And then there's this other word, super-resolution. Super-resolution, it means, basically, getting better resolution than you are suppose to, i.e. beating the Rayleigh diffraction limit. And various ways have been proposed for doing this. And until recently, I guess, most of them were pretty unsuccessful. But more recently, nowadays, some of these techniques are doing really well. I think the most noticeable one probably is STED, Stimulated Emission Depletion, microscopy, where Stefan Hell is getting resolutions which, I don't know, more than a factor of 10 better than you should according to the Rayleigh diffraction limit.

But anyway, here he's talking about what are called super-resolving filters. What you do is you alter the pupil function of the lens by some clever trick in order to sharpen up the point spread function. And I think that maybe George did mention that earlier on at some point. But the problem is what always happens is that it doesn't actually-- you never get anything for nothing. That's the problem. You make a central lobe narrower, but you find that the side lobes become stronger.

And the other thing is that very often the power transmitted through the system is much reduced. So you can imagine if you've got some filter that absorbs part of the power, then you're wasting power. So both of these can be bad. And so, consequently, this idea of super-resolving filters in practice has never really, up until now, been very successful from a practical point of view.

But anyway, this shows one example. This shows a circular pupil. And yeah, and then this one is an annual pupil, which you can model as being one circular function minus another one. And of course, once you write it in that simple way, as a circ function, as the difference between two circ functions, you can do the Fourier transform with that very easily and get the point spread function of these two cases very easily.

And so he's working it out for a focal length of 20 centimeters and a wavelength of 0.5 microns. And the width of that aperture is-- what is it? It looks about 4 millimeters diameter, something like that. Yeah.

Anyway, so this is-- and he works out what it looks like. So this is the PSF. This is supposed to be an Airy disk. It's been sampled a bit coarsely, so it looks a bit ragged. And so what you see is when you look at the annular case, you put this central obscuration in the center there, what it does is it narrows up that central lobe.

And you can see, though, it also increases the strength of these side lobes, that these are much higher than these ones. So although in some cases this can give a good an improved image compared with this, for example, for point objects it would do well, but if you've got an extended object like an image of a car or something, it might not work so well.

The other way of thinking on it is in terms of the MTF. And this is our normal MTF for a circular pupil. It's this so-called Chinese hat function. And then for the annular pupil, this is what you get. You remember, you calculate these. This is the convolution, the autocorrelation of a circle. So you have to-- you get two circles and you displace it one relative to the other. You calculate the area of overlap, and you plot the area of overlap as a function of the distance between the two center. So that's how you actually calculate this.

And you do the same with this one, and you can see what you get is you see that it drops more quickly, first of all. This is normalized to 1 because you normally normalize transfer functions to 1 for 0 spatial frequencies. That's the way we normally do it. But I guess maybe it's not the only way of doing it.

This drops down quickly because you can see, if you're working out the autocorrelation at this angular aperture, you can see that [INAUDIBLE] you move it a small amount, the one the one bright region is not going to overlap with the other one anymore. So that's why it drops off quickly there.

And then it has another blip out here, which basically comes about from when the toe annularly-- where they both got their bright bits on top of each other. And so if you compare this one with this one, you can see that here the spatial frequency response is much lower. Here, actually, it can be higher. The cutoff of course, is still the same. So you could calculate the image of any object you fancied from knowing that.

And this is another example. This is the case of a Gaussian apodization. So what we're now doing is instead of-- the annulus is effectively boosting up the edges of the pupil, which is why you get this improved high spatial frequency response. Here we're doing the opposite of that. apodization-- if George was here, he would tell us-- in fact, he already did it one of the previous lectures-- apodization means to cut off the feet in Greek, I believe. He'll probably be listening to my lecture and tell me I've got my Greek all wrong.

But anyway, it's to do-- it means to cut off the feet. And what it means, why it's given that name, is because it cuts off the side lobes. So you see the side lobes of the Airy disk have disappeared now. So by making this shaded aperture-- in this case, is a Gaussian function-- you make something which is a bit broader.

Actually, in this case, doesn't look very much broader at all. But the side lobes have gone, and this is what it's done to the transfer function. You see now it's slower to fall off here than this one. But the high spatial frequencies are now reduced relative to this other one.

So overview on this, then. This is what we call-- or what George is calling pupil engineering. And so if you try to make the main, the central lobe of the PSF smaller, then you're going to get bigger side lobes. And that seems to be something that you can't avoid. Nobody's come up with a way of doing something clever that avoids that.

And the opposite, of course, if you make the main side lobe bigger, the side lobes get smaller. No doubt, that is maybe not so necessarily going to happen. I suspect that if you really wanted, you could probably make something that had a big central lobe and big side lobes. But nobody would really want that anyway.

And then also this power loss problem, that there is going to be some worse signal-to-noise sort of performance. So annular-type pupils typically narrow the main lobe at the point spread function at the expense of high side lobes. Gaussian-type pupil functions typically suppress the side lobes but broaden the main lobe. And so, as it says here, different sorts of filters like this might possibly be worth considering for certain particular applications.

Yeah, and then he gives another couple of problems with these things. Making a filter which is of varying amplitude is not easy. And I guess nowadays you might do it with some liquid crystal device, hopefully. We might be doing that so, won't we, Charlene? But until recently, this would have to have been done by some sort of photolithographic process. And actually getting controlling the absorption of the filter to accurately fit what you designed is not trivial.

And there's also this energy loss problem, then. Yeah, nowadays people are very interested in phase filters. You can do a lot of these clever things also by using phase masks rather than amplitude masks. And of course, a phase apodizer is going to be lossless. So that might make it sound attractive. And so George says that maybe it is attractive.

Actually, it turns out, I think, it's not quite as attractive as it might sound because, actually, very often what happens is if you use an absorbing filter, the absorption that you can introduce actually can reduce the size of the side lobes. So if you do it cleverly, you might be able to make the side lobes lower and also still get as much energy into the central lobe with it with an absorbing mask as with a pure phase mask.

Yeah, and then we get on to all these terminologies that is obviously annoyed by as much as me. This is one. This super cool digital camera has a resolution of eight megapixels. And so what's he going to say about this? Big no again.

So this is using the word wrongly. This has nothing to do with resolution. And so what they are actually referring to is the space bandwidth product of the camera, how many pixels there are, which is not really measured as resolution.

The other one, as all the people in my group would know that I always get very cross about, is these interferometry people who are always talking about inerferometers being able to resolve one Angstrom, which is also wrong because it's nothing about resolving. So resolving, resolution, we've said what resolution means.

So this question of the number of pixels, is there a connection between the two? Well, I guess the answer is, there is and there isn't. There is a relationship if you design the optical system properly.

But what George is pointing out here is that, actually, this is an example where the pixels are very much smaller than the point spread function of the optical system. So actually, here you're very much over sampling this image. So all you're measuring-- you're measuring very accurately a blurred picture. So that's not really giving you very good resolution and not really very-- the number of resolution elements in your image is actually much smaller then than the number of pixels.

Sometimes they use this word resels, don't they, resolution elements. Have you seen that? They spell it R-E-S-E-L. It's an abbreviation for resolution elements, so it's the number of resolved spots you've got in your device rather than the number of pixels.

Some more misstatements-- it is pointless to attempt to resolve beyond the Rayleigh criterion, however defined. No, the difficulty increases gradually as feature sizes shrink, and difficulty is noise dependent. So it's not a sharp hard and fast line. But that said, I think you I think it probably would be pointless to attempt to resolve 10 times the Rayleigh limit with an ordinary optical system because you know you're not going to be able to do that. But STED, that we were talking about earlier, does do that. So this is [INAUDIBLE] well, OK, so maybe it is worth attempting to do it because people have come up with ways of doing it now.

Apodization can be used to beat the resolution limit imposed by the numerical aperture. And there's a big no there again. Watch the side lobe growth and poor efficiency loss. I don't know if you've noticed, but I spoke to one of-- maybe Charlene-- I can't remember now. There was a paper in *Nature Methods* recently by Nikolai [? Segilev, ?] where he gave an example of a super-resolving mask, and he gave the pic this picture of a very narrow point spread function, improving on the Rayleigh rate resolution limit by a factor of, I don't know, a few, I think.

But I did some calculations on the design he gave. It turned out that this little central peak was surrounded by side lobes that went up to something like 10 to the power of 40-something or something really ridiculous. And the amount of energy that went into this central spot was, like, nothing. And what made it even worse was that if you actually calculated what happened as you went out of focus, you found that you also got these big walls around it, very high all around it, actually. So it was this like this very small bright spot completely surrounded by something which was, like, 10 to the 40 times higher. So how you would ever use that in practice, you can think it really obviously wouldn't be a very practical device.

Then the number of the rest of the pixels in your camera is not the resolution. So what is resolution? Our ability to resolve to point objects based on the image. However, this may be difficult to quantify. Resolution is related to the NA, i.e. it's proportional to the NA. The distance is inversely proportional to the NA.

That's another thing that people sometimes get into knots about of course because resolution means that the bigger the number, the more is the resolution. But actually, the smaller the size, the more the resolution, so beware. So sometimes, actually, people use the term "resolving power" to get around that problem because otherwise-- people say things which they don't really mean sometimes.

Yeah, so other factors that can affect resolution-- aberrations, apodization, and noise. So is there an easy answer? When in doubt, quote, 0.61 lambda over NA. And then you get the marks in the exam. So that's that one done.

So how are we going? Got a bit more time yet. That was last week's lecture I just gave then, anyway. So I think it must be this one. So I don't know how far we're going to get with this one, but we can at least start it.

So what we're supposed to be doing today, more applications of the transfer function, a bit about depth of focus, and a bit about deconvolution. And there's some nice simulations that George is done to show that. And then he's got down for Wednesday, which I'm also going to be giving. I think George is not back. Do you know when George comes back?

AUDIENCE: No.

PROFESSOR: No. Anyway, so I shall be giving the lecture on Wednesday. And so polarization-- what we don't finish from today will obviously also be on Wednesday, and then there's polarization, intensity distribution near the focus of high-NA imaging systems. And yeah, I don't know what else we're going to do, actually, because George keeps changing his mind about what we ought to put in that last lecture.

So defocus. So this is an example of defocus. Anyone know what this movie is? I don't. I'm out of touch with cinema. Recognize who the other stars are? No? No. I'm only asking because I just did--

AUDIENCE: It's *Fight Club.*

PROFESSOR: Is that right?

AUDIENCE: Yeah.

PROFESSOR: Right. Anyway, what you'll notice is this guy here-- is it a guy? I can't really recognize what it is, actually. But it's out of focus.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah, it could be. OK, so anyway, so that's an example of defocus. Then this is what happens when you get a lens in the proximal approximation and look at the intensity in the focal region. You get something that looks a bit like that. It doesn't look quite the same as [INAUDIBLE]. I don't know how George did this calculation.

But you'll notice that this is the focal plane, so this is a cross section through the Airy disk. So you see the central-- ah, well, I can tell you why it's different, then. You can see these have got its regularly spaced zeros. So he's done this through a slit aperture, hasn't he? And so that might explain why it looks a bit different.

But what you'll see-- excuse me-- is that as you go out of focus-- well, note one thing. It's symmetrical about the focal plan. And this is-- I think we probably had that somewhere before, actually, that it's symmetrical. If it's calculated using the usual Debye approximation, you get a result like that. So if you've got, for example, the aperture stop is in the front focal plane of the lens and you look at what happens in the back focal plane, then that would be the case. You get this symmetrical behavior like this.

And you can see also these sort of bright bands that seem to go off at diagonal lines here. I guess, yeah, so that corresponds roughly to the edge of the aperture then, of course. And yeah, so the width of the central spot in this direction, then, we've said is this 0.61 lambda over NA, and this distance from the focal plane where you get to this first zero is given by some expression like this. It goes as if-- this is really only true for the paraxial case. But for the paraxial case, this NA here, NA squared here, if it's the high aperture case, which we might consider on Wednesday, you'll find that this is not quite right anymore.

Yeah, so delta x is the radial resolution. Delta z is the depth of focus, today's topics, depth of focus or depth of field. And both of those he calls DoF. Note the very high numerical apertures. The scalar approximation is no longer good. The vectorial nature of the electromagnetic field becomes important. That is true.

There's other things as well, though. When you do all this paraxial stuff, if you remember, all the time you're replacing sine theta by theta and things like that, making approximations like that. So there are actually lots of approximations you make. One is the fact that the angles are small. The other is that you neglect the polarization effects. When the apertures become big, then the polarization effects become very important.

So we're going to look at this sort of system. And so this is a 4F system with magnification, so the two the focal length of the two lenses are different in this case. And so this shows what happens if you're looking at an object which is placed a distance F in front of this lens and your screen is placed the difference F2 behind this lens, and your aperture is F1 from this one and F2 from this one. So everything there is set up properly as defined for a 4F system. And this is the case where you'd expect, as I was just describing, the defocus point spread function is going to be symmetric.

So we place our mask there, and we've got our pupil. So this is our object function. This is the angular spectrum, the spectrum of the object in this plane here, which is then filtered by the aperture stop. And then that gives some sort of amplitude in this plane here, which is going to be inverted and also magnified or otherwise according to the ratio of F1 over F2. And there's the rays going through it.

And this is showing how-- yeah, this plane, this is his terminology. This is x. This is x prime. This is x double prime. And you can see here x double prime maximum is equal to the and the radius of the aperture. So this is his terminology. We are going to get that in a minute coming [INAUDIBLE].

This is a function of x This is a function of x double prime. This is a function of x prime. This is the numerical aperture of the lens that's looking at it. And yeah, so the numerical aperture is equal to x double prime maximum, which is equal to A over F1, again assuming that the sine theta or tan theta are both theta.

Now, what we're going to look at now is what happens if we defocus this system. So now our object has been displaced the distance delta away, and we want to calculate what the image of that looks like now. And so what we can say is that after we've eliminated this object, we've got this GT of x, and then that light is going to propagate from here to here. And in order to calculate that, we have to convolve it with the propagation kernel. So you convolve it with this thing.

For some reason, when George converted this from his whatever his other thing's called, keynotes-- it's a PowerPoint. All the equations are like as though someone's had their martinis before the lecture. So we then look at the Fourier transform of that in this plane here. So this thing convolves-- Fourier transforms to this. The convolution transforms to a product, and this Gaussian transforms to another Gaussian.

And notice that, as usual, of course, you remember that the scaling thing for Fourier transforms, the bigger a function is, the smaller it's Fourier transform and vice versa. So this lambda delta at the bottom appears now at the top. So the bigger delta is, the bigger this thing's going to be here. Yeah?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah.

AUDIENCE: Is E to the imaginary x squared a Gaussian? Or only the E to the real x squared a Gaussian?

PROFESSOR: OK. Yeah, the real Gaussian is E to the minus x squared, isn't it? E to the minus x squared does that. This is what you might call an imaginary Gaussian. But it all comes out in the wash, actually. So just the same as the Fourier transform of the normal Gaussian is another Gaussian, so the Fourier transform of this imaginary Gaussian is another imaginary Gaussian. So what's really converting is a parabolic phase front here to a parabolic phase front here.

So you can get it just by getting the normal expression. I mean, it sometimes amazes me having how much liberties you can take with these things. But you just take the expressions for the Gaussian, and you put in this complex number, and it gives the right answer. So it's very nice. I guess occasionally there might be occasions when it doesn't work.

Yeah, so what he's then saying is that you can think of this-- this is the object for our-- this is the spectrum for our defocused objects, which is multiplied by the pupil function of the system. But that is exactly equivalent of course to thinking of it as the ordinary object spectrum multiplied by a modified pupil function. So it doesn't matter whether you associate this with this or with the pupil function.

So that's what we can think of. We can think of the objects as being the same as where it is if it wasn't defocused, and we can think of the pupil function of the thing being modified to take into account of defocus. We sometimes called that the defocus transfer function.

And so he's now going to calculate that. Yeah, so the defocus-- he calls it here the defocus amplitude transfer function. So remember, here we're dealing-- this is dealing now with coherent system. Before, when we were doing the resolution, we were looking at incoherent systems. So now we've gone back to coherent systems. So you have to be wary of that because sometimes there's some differences.

So all this is saying then-- yeah, you remember, to go from the front the pupil to the transfer function, all you have to do is a coordinate scaling. So the amplitude transfer function is just the scaled version of this pupil function. And again, it comes out to be now, of course-- it's going to be-- it's multiplied, of course, still by the circular aperture. But inside that circular aperture where it's got some value, its value is given by this complex exponential-- a complex Gaussian, like we were just saying before.

And that's what this looks like. Now, notice, so this is cosine. This is looking at just the real part. So actually, it was a complex-- it was an E to the minus I something. So you can break that up into cos plus I sine. And the cos part is the real part, and the sine part is the imaginary part.

And as when we looked at imaging earlier, the real part images real objects. So an amplitude object will be imaged by the real part. A phase object would be imaged by the imaginary part. But we're not going to say anything about that. We'll just keep with the real part.

So this is cos, but it's not a normal cos. It's cos of an x squared. So that's why it looks a rather strange shape compared with the normal cosine. You can see it's become very flat there, and then it becomes more oscillating later on.

And so this is what it looks like. Of course, the actual scaling of this is going to depend on the value of this delta. So here he looks at where this crosses the axis. So this parameter, x double prime over lambda F1, is equal to u, the spatial frequency. So this is just u squared here.

And he says that this crosses the axis where this argument is equal to pi over 2. And if you solve that, you'll get this. And so that's where it crosses the axis.

And then you have to multiply this, of course, by the circular aperture. So this is the circular aperture. This is the radium where the object spectrum is non-zero. And so you can see that only the parts of this blue curve up to here are going to have any meaning. The bits out here are not going to do anything.

So this is obviously looking at a case where this is almost flat. If you made it a bit smaller, even, it would look even flatter. So this is a not a lot of defocus.

And so here he's got, then-- yeah, so here now is labeling where that cutoff is. So this is equal to NA over lambda, as we know. And in terms of the x prime coordinate, it's given right by this thing here. So this is talking about what he calls mild defocus, and the condition for that to occur is that this has got to be much smaller than this.

And then-- yeah, and that corresponds, then, in terms of delta. So we've now got a condition for this to happen. The value of delta has to be small compared with lambda over 2 NA squared. So you remember, this is what we said was the depth of field or depth of focus of the system.

Depth of field is measured in the object space. It doesn't say anything about the defocus on that one. I thought I was going to [INAUDIBLE]. He says a bit about that later. This is showing another case, then. This is with much more defocus now. So now it's oscillating a lot more wildly, so now this cutoff is much smaller than this. And so this is where you've got a lot of defocus.

And the fact that this goes negative is probably the worst thing. It means that some spatial frequencies are going to be imaged with the wrong contrast. So when you try doing your Fourier synthesis to add up to make the image, it's not going to work right.

And the other point he makes is that the regions around here, of course, these spatial frequencies around these zeros are not imaged at all. So that's another reason why it doesn't give a very good image. So this is the case of strongly focus where delta is much bigger than this thing, not necessarily much bigger, I suppose.

I think we ought to stop there. We've run over time anyway. So let's stop at this point and we'll pick that up next time.