GEORGE BARBASTATHIS: So today, we'll start with a demo, so Pepe will be on stage. And then we'll continue with the lectures on the gratings. Pepe, whenever you're ready, you can--

JOSE DOMINGUEZ-CABALLERO: OK. Can we-- so today, the goal today is to show you a couple of very interesting experiments. And again, the idea of this is for you to see all these derivations that we've been doing actually applied in a real experiment and see the results that you would expect. And for instance, these last [? pieces, ?] you were supposed to compute the interference between a couple-- two plane waves propagating at a different angle, so you were supposed to also calculate analytical solution of that. Now, we're going to see the real evidence of that solution.

So for that, we actually built, in this case, a little more elaborate setup than the ones that we've been showing before. This actually a Mach-Zehnder interferometer that Professor Sheppard described last time, and I will explain it very briefly. Can we zoom out, please? Controls? I guess maybe that's the maximum.

All right, so we start here with a laser. So the laser comes from this side, and up to this section here is where we just collimate the light, so basically, what I'm going to focus right now is on this other section. So after this part here, we have a nice plane wave. This component here is a polarizer that, right now, just take it as an element that allows me to control the intensity such that the CCD doesn't receive too much light.

Now, these two pieces of glass here are the beam splitters. So we've seen that these beam splitters-- basically what they do is that they will split a section of the beam to one side and a section of the beam to the other side. In this case, it's 50/50 percent.

Now, then, we have two mirrors. So as you can see, there's one path here, and the other path here. The two paths that now are going to be combined by this second beam splitter here. Sometimes, this is called a beam combiner. And then they co-propagate here, pass through this iris here, and then in this case, they're reflecting-- again, these mirror are going to the CCD.

So essentially, what we have here is these two plane waves that first are splitted and then are combined back again together. And if the two plane waves are exactly in line-- we see that we're not supposed to see fringes because they are exactly in line, but we learned also that if we introduce a little angle here, now the planar wavefronts of each plane wave will interfere, and then we're going to see some sinusoidal pattern. So this is exactly the same as in the [? pieces. ?] One of the problems that had what would be the interference pattern in an xy plane in this case. In an xy plane, [INAUDIBLE] two plane waves propagating with a slightly different k vectors.

So now, let's look at the interference pattern. Now, this is here in my computer screen. So this is a video, real time, of this interferometer. So as you can see, you get very nice fringes of very bright and very dark, so we know that the peaks are the proof of constructive interference, the null destructive interference.

And you also see some vibration here, because we said that these type of instruments are very, very, very sensitive to environmental conditions, and I'm actually going to tap the table. Even if I just blow some air, but I want to get some moisture in there, in the optics, but you can see variations of the fringes. So then these type of interferometers are used for different measurements to calculate, for instance, phases or measure different profiles of reflective optics, et cetera. So there are many applications of these Mach-Zehnder interferometers, and the reason being the sensitivity.

Now, we also talked about another thing-- the period of this sinusoidal pattern that we see is proportional to the angle of the plane waves. So in this case, I'm going to move one of the mirrors-- say, this mirror here-- to change one of the beam paths. The other one is going to remain fixed. So let me tilt it a little bit and see what happens first.

So we see that the period of the plane wave is changing. In this case, I'm moving it in in the y direction. So now the period is larger. Does anyone want to tell me why do you think it's happening? Are the plane waves more close to be parallel, or further away? Yep?

STUDENT: Appears inversely proportional to theta, so your period's smaller data.

JOSE DOMINGUEZ-CABALLERO: Yes, exactly. So essentially, here, by doing this, I'm making the plane waves to be as in line as I can. And of course, since I'm increasing the period, also the oscillations will start increasing, and now, we get these artifacts due to the frame rate of the camera that are going to be very noticeable, I suppose, in Singapore. That's why I started with a small fringe. I can also change the tilting of the fringe to do nice things like this by basically changing the orientation of the mirrors.

All right, so now what I'm going to do is I'm going to block one of these beam paths. So we have a nice plane wave here, and now, we are going to repeat the aperture, this lead experiment that I showed last time. But today, we're going to do it in different L regime. So basically, we're going to try to mimic physically the experiment that Professor Barbastathis showed in the video that he showed in, I think, last lecture.

All right, so we have this slate here. Let me get it down a little bit. And I need to control the intensity, so I'm going to move this closer to the CCD. This is as close as I can get.

So as you can see, there is a plane wave hitting-- let me just reduce the exposure. There is a plane wave hitting this slate, and you can see the diffraction here happening in the discontinuity when basically it hits the metal piece. And I can tune the size here, and of course, if I make this larger, the diffraction effects appear less noticeable. But for a smaller slate-- let's do that-- it's actually pretty nice. We see the nice diffraction, but I'm going to increase intensity, so you can see the fringes that happen there.

Now, let's see what happens if I have it-- I fixed position, so the size of this slate, and now I'm going to move it along the optical axis back. So I'm propagating back. Let me just reduce the intensity here, so we are somewhere here. So now, I'm going to go back, and you can see how the diffraction pattern changes. And this is essentially the video.

GEORGE BARBASTATHIS: Can you hear me?

JOSE DOMINGUEZ-CABALLERO: Yes.

GEORGE BARBASTATHIS: Can you go back where the fringe in the center was dark? Yeah, something like that. This is the equivalent of the Poisson blinking spot that we mentioned. Go a little bit closer to the camera.

JOSE DOMINGUEZ-CABALLERO: Go a little bit what? I'm sorry.

GEORGE BARBASTATHIS: A little bit closer to the camera. Yeah. Try to have the center dark.

JOSE DOMINGUEZ-CABALLERO: Oh, I'm going to show the Poisson spot with a circular aperture a bit later--

GEORGE BARBASTATHIS: But I want to make a point about this case. In this case, you can see, again, the Poisson spot, but it's actually an entire dark line in the center. That's because the scattering of the two edges is actually out of phase, so that the interference destructively at the center, and you see an entire dark band. And then of course, in this case, with a circular aperture, you see the Poisson spot as a spot. In this case, it is a vertical bar.

JOSE DOMINGUEZ-CABALLERO: There you go.

GEORGE BARBASTATHIS: Yeah, that's perfect. Perfect, yeah.

JOSE DOMINGUEZ-CABALLERO: So here, you see the blinking. So this is dark, and this would be bright. This would be the blinking line, I suppose.

Now, let me change it to the other video that we saw, that was the circular aperture. So this is a circular aperture of a fixed size. In this case, you can see, when it's close to the camera, again, as close as I can get it to, you see refraction patterns. So now, this is very nice, because what we're supposed to see here, again, is going to be that famous Poisson spot or blinking spot. And before we see the results that we, again, saw in the movie, there's a nice historical story about these.

In [INAUDIBLE], Fresnel spent a very nice paper to a contest about the refraction of light. So at the time, remember that there was all this controversy of is light a wave or a particle. So in the committee judging these papers, Poisson, a very respected scientist, was actually in the committee. And he saw Fresnel's paper, and he was like, no, this has to be wrong.

And he wanted to disprove it, so he actually went and did some calculations and then proved that, if Fresnel was right, what this means is that if you have, say, an opening like this, somewhere behind that opening, after shining it with a plane wave, you're supposed to see a dark spot. And that dark spot is very counterintuitive because, basically, you're supposed to see, from the geometrical optics, all the light going straight through. So how could there be a very dark spot in the center? So then basically, he said, no, this is the proof that Fresnel's theory is wrong.

But then another member in the committee, Dominique Arago, he actually went ahead and did the experiment, and he saw this Poisson spot experimentally. And then thanks to that, of course, Fresnel won the contest. And now this, I guess, as opposed to-- like as occurs to Poisson-- it's called a Poisson spot.

All right, let's see the Poisson spot in action. So I'm going to go-- here, you see it in bright, dark. I'm going to go even further away. So I'm moving it back, and you see the blinking spot. Trying to do it slowly.

Going to go back. Can you see it in Singapore? I guess that was a yes. Cool.

GEORGE BARBASTATHIS: They're very beautiful actually.

JOSE DOMINGUEZ-CABALLERO: Yes, they're very nice. So can we switch to the front camera, please? Control? Can we switch to the front camera?

So now, the next thing that I'm going to show-- it's a diffraction grating. So diffraction gratings-- it's a very interesting optical component. The front view, please. I'm going to show something with a piece of paper. Or I guess if not, I'm going to show it here.

STUDENT: [INAUDIBLE]

JOSE DOMINGUEZ-CABALLERO: I'm sorry?

STUDENT: [INAUDIBLE] explained [INAUDIBLE].

JOSE DOMINGUEZ-CABALLERO: OK. So the next thing that I'm going to show is an optical component, that is very used in practicing different optical setups, that is called a grating. So grating-- you can think of it as a material, in this case, that-- the one that I'm going to show is called a phase grating. Basically, the index or refraction within the material varies in a given way-- in this case, sinusoidally. So it's a periodic variation of the index of refraction, and then when the light hits this element, looks like a flat piece of glass, but in reality, it has this index varying, it basically decomposes in multiple angles.

And what I'm going to show here, in this case, is a normal laser pointer, but it actually has two set of gratings in this little end. So what we are going to see, again, now, is we're going to see some beautiful array of patterns. You see them? And actually, I'm going to project them here. This, I'm not sure if the Singapore side is going to see it or not.

So now, I have to cross linear gratings. I actually can put them in things, so I can see very nice spots. I can open them, and you can do all sorts of fun things here.

So the optical component actually looks something like this. This is the second time that I showed a grating. The first time was in the very first demo, when I used the grating to disperse light, similar to a prism. So if you remember, I had a prism dispersing in what is called a normal dispersion, and a grating with what is called the anomalous dispersion. So this also, if you shine it with white light, will form a very, very nice rainbow.

So in this case, it's a bit hard to show this guy with a camera, but I'm going to try, if we can put-- the overhead camear I think is on. So what I'm going to do is going to increase the intensity. And use the aperture, and I'm going to try to show it in a piece of paper here.

So you see here, the three spots, in this case, of this element being, in this case, more orders. So each of these spot we call a diffraction order. The one that goes straight through would be the 0 order. Then we go to plus/minus 1, plus/minus 2, so on and so forth, and I guess after these, we're going to do the actual derivation, the mathematical derivation of this. You want to add something else, George, for the grating side?

GEORGE BARBASTATHIS: I think this only had the plus/minus 1, plus/minus 3, and a very weak 0?

JOSE DOMINGUEZ-CABALLERO: Let's see.

GEORGE BARBASTATHIS: Because I only saw four spots, so it must have been--

JOSE DOMINGUEZ-CABALLERO: There's more, I think.

GEORGE BARBASTATHIS: Oh, OK. Not very visible. Oh, yeah, that's right. There's more.

JOSE DOMINGUEZ-CABALLERO: Yeah.

GEORGE BARBASTATHIS: 0 is also present, but you can see that the plus/minus 2, plus/minus 4 are missing. We'll talk about this in class later. The point to see here is that you see the pattern is not quite regular. It looks like there is one missing order between the last-- if you label the central one 0, then it goes 1, and then it keeps 2, and then it goes to 3. So that's the point I wanted to make.

JOSE DOMINGUEZ-CABALLERO: Good. So I think we switch back to you, George.

GEORGE BARBASTATHIS: Thank you, Pepe.

JOSE DOMINGUEZ-CABALLERO: You're welcome.

GEORGE BARBASTATHIS: I feel jealous actually because this is much better to see it in real life than it is to see it in math. But hopefully, this will motivate you to tolerate the math, because if the physical phenomena are so pretty, maybe it is worthwhile to go to the effort of understanding them, I suppose. So today, I will talk-- I will basically pick up where Pepe left, and I will talk about diffraction gratings.

Oh, before I forget, some of you asked if you could have the movies from-- the animations that they played in class the other day. So if you go to the website, I have created a new section called Animations. And I've put the movies as AVI files, so you can download them from there.

And also I have revised the notes. Last time, if you recall, we caught a couple of factors of 2 that were missing, and so on and so forth. So I fixed those. If you find any more errors in the notes, please let me know so that I can fix them as well.

So today, we'll talk about-- so basically, I will describe, mathematically, what Pepe showed in real life. So a grating basically, as Pepe mentioned, is a periodic, thin transparency. So we've talked about thin transparencies before. We mentioned that they modulate in general.

Thin transparencies modulate the amplitude and the phase of the optical field, so in that particular case, when the thin transparency's a periodic function of space, then we'll call it a grating. And I guess for clarity of presentation, it is customary to classify gratings as amplitude or phase, depending on whether the grating acts upon the magnitude of the electromagnetic field or the phase of the electromagnetic field. Now, this can cause a little bit of confusion because we have agreed to use the term amplitude to denote the complex amplitude of the optical field.

Confusingly enough, when people talk about amplitude gratings, they should be calling them magnitude gratings, because as you can see in the case on the left, the transmission function of an amplitude grating is a positive real number. So therefore, this kind of grating acts directly onto the magnitude of the optical field. Nevertheless, for historical reasons, people call these are gratings-- I suppose they call them magnitude-- I'm sorry, they call them amplitude gratings. So hopefully, this will not create confusion.

But anyway, the simplest possible modulation is sinusoidal, and you can see here sort of juxtaposed in a sinusoidal amplitude grating and the sinusoidal phase grating. So starting with the amplitude grating, the complex transmission function of the grating is actually not complex at all. It is a real positive number, and what I've done is, on top of it, I've plot its magnitude, which is, of course, itself since this is a real positive function, and its phase. And of course, the phase is 0 because the phase of a complex number, which happens to be real and positive, is 0.

The quantities to mention here are the period of the grating. We use the upper case Greek symbol lambda, which looks like a hat, I guess. That symbol is called lambda.

So this is the period of the grating, so in this case, you can see the grating seems to fulfill one undulation between approximately minus 5 units and plus 5 units. Therefore, the period here is 10 units. And actually, the unit I've picked on the horizontal axis is the wavelength of the light, so this grating has a period of 10 wavelengths. So if the light were a visible-- green light has a wavelength of approximately half a micrometer, so in real units for the green light, the period here would have been approximately 5 micrometers or 10 wavelengths.

Then m-- we saw the symbol m also when we talk about interference. We called m the contrast between the brightest and the darkest part of a fringe in the case of an interference pattern. In this case, we still call it contrast, but it is between the most transmissive part of the grating and the darkest part of the grating. So this case, you can see that it goes between approximately 0.9 and 0.1.

The maximum transmissivity is 0.9, and the minimum transmissivity is 0.1. So therefore, of the value of m for the grating shown here is approximately 0.8. If the grating was swinging between maximum 100% transmission, totally bright and totally dark, that would have been 0. Then the contrast would've been 1. So this is the function of this m parameter here.

And finally, we allow a phase shift, which basically says, if the maximum-- since the grating is written as a cosine function here, phi tells me if the maximum transmission, the maximum value of the cosine, is centered with x equals 0 on the horizontal axis-- so you can see that, in this case, it is indeed center. So therefore, the phase phi would equal 0 in the grating shown in this case. So that's what I had to say about that. Under the grating, I should go to the phase grating. It is actually kind of the opposite.

So the phase grating is expressed as a complex exponential with a purely-- with a pure phase without any amplitude modulation. So it's magnitude then is 1. So you see that the magnitude is a flat function, so this means that this grating transmits all of the light. What it does do, though, is it modulates the phase, and you can see that it modulates the phase in a sinusoidal fashion.

So this is what you see now in the bottom plot, which is that the phase, the angle-- by the way, the phase, sometimes we denote the phase as this angular symbol of a complex number. So the phase is simply the argument of the exponent here, and is, itself, sinusoidal. And it also has a period and a contrast. The contrast in this case is called the phase contrast because we're talking about the phase, and you can see that, in this case, it is swinging also approximately between plus 0.4 and minus 0.4. So the contrast is also 0.8 in this case, but it could be anything, really, between minus pi and pi to get a complete phase contrast.

The other two parameters are the same. The period is also about 10 wavelengths, and the phase shift is 0. We have to be a little bit careful here not to confuse the phase of the grating with the phase shift. So again, the phase shift is telling us whether that transmission is aligned or no with the origin of the coordinates.

So this case, since I have a sine in the phase of the grating, you see that it assumes the value 0 at x equals 0, so again, there's no phase shift. So this phase shift is not to be confused with the phase delay that the grating imposes upon the complex amplitude of the electromagnetic field. So that's another important point, and I would like to point it out.

So how do these things behave now? So what I will do is I will play two movies, which I guess are not as convincing as the movies that Pepe showed because Pepe is doing an experiment. In this case, these are simply Matlab calculations.

But basically, the two movies will show you two great things. One is an amplitude grating, and one is a phase grating. And it will show you how the field propagates after grating. And these movies are also posted to the website so you can play them yourselves again later.

Now, I should say, the grating grooves are oriented horizontal in this case, and you cannot see them on the camera or on the video because the pixel size is not big enough to allow you to see them, unfortunately. This is an artifact of the calculation. So what you see here is some variation of the grating that is actually not the grating itself. As I said it, is called a moire pattern, if you are familiar with the term. But anyway, I don't want to go into moire theory here, but the reason you can see this pattern is because the pixel size of the camera in the display is not small enough to show you the period of the grating.

So the grating is actually expressed by this function. X is the same in the equation as in the axis, and the period of the grating is 2 wavelengths. So this grating has a very, very small period. The contrast I picked to be 1 in this case, and there's no phase shift. So I will play the movie now, and you will see what happens.

So the field will propagate along the z-axis, and you can see that the light that originally illuminated the grating actually now splits into three parts. These are, also Pepe mentioned, called diffraction orders. One of them propagates straight through. That is called the 0 diffraction order, and then you get a plus 1 above and a minus 1 diffraction order below. So this is the typical behavior of a sinusoidal amplitude grating.

Now, if I play the same thing for the first grating, there's two things to notice. First of all, there's no moire at all here. That's because the phase grating does not modulate the magnitude of the electric field at all. So in other words, at the entrance toward the grating itself, the grating looks just like an empty transparency. It has not changed the magnitude of the field.

What it has done, it has changed the phase, and because the phase of the field got modulated inside this slit over here, what you will see when I play the grating is, again, that it splits. But now, it splits into more than one. It splits into, in this case, you can see very clearly five diffraction orders.

You can see very clearly, there is one in the center, then another one and another one. So these are the plus 1, plus 2, and then at the bottom, you have minus 1, minus 2 diffraction orders. So it is, in some ways, similar to the amplitude case, but in this case, we'll get more diffraction orders.

And so I guess what I would like to do now is I will try to explain what is the difference, and why the phase grating gives us more diffraction order. I should say, by the way, that the sinusoidal-- I want to emphasize the sinusoidal amplitude grating gives us only three-- the 0 order, the plus 1, and the minus 1. In general, other types of gratings, for example binary or phase gratings and so on, they give us more than three diffraction orders. So first of all, let's look at the physical picture.

So I'll start with the sinusoidal amplitude because this is the simplest possible grating, and let's look at the physical picture of what's going on here and why we get this diffraction order. So if I have a plane wave impinging-- oh, so again, this symbol, being Greek, I have an advantage, I guess. I can recognize it.

It is the uppercase lambda, so we'll use the symbol lowercase-- control, may I have the paper? Actually, I already have the paper, do I? Yeah.

So this symbol that we use for the wavelength is the lowercase lambda. This symbol that we use generally for a special period is the uppercase. It may sound a little bit ridiculous that I emphasize it so much, but believe me, in previous years, all the generations of students have been endlessly confused by the lambda and the lambda. So I would like to emphasize it-- these are two different symbols. We use the lower case for the wavelength and the upper case for the spatial period.

So that's it then. That's the period, and the inverse of that-- the inverse of a period generally is a frequency. So since now we have a spatial period, the inverse [? u0, ?] we'll call this special frequency.

And let's illuminate this grating now with an incident plane wave. So throughout this lecture, I will be illuminating gratings with plane waves on axis. In the homework that I've posted today, and actually just became visible a few minutes ago, in the homework, you will see what happens if you illuminate the grating with a tilted plane wave or with a spherical wave. This is also possible. You can illuminate a grate in any way you like.

Now, we'll start with the simplest case, which is on axis plane wave. So we don't know yet what's going to happen, but we know one thing. We know Huygen's principal, and Huygen's says that, at each point where the grating is transmissive, we'll get point sources.

So of course, I do not only get only these two, I get the point source here, which is very strong because this is the location where the grating has maximum transmission. Next to it, I will get another point-- another Huygen's point source that is slightly attenuated. Then next to it, another one that is slightly more attenuated, another one, another one.

There comes a point here where the grating is totally black, that it is totally absorbing. Therefore, there's is a missing Huygen's source over here, and then as I move on along the optical axis, I will get more Huygen's point sources that are progressively less attenuated. And then finally, I get over here where I have another strong, as strong as possible, Huygen's point source.

So we derived an integral. Professor Sheppard last week [INAUDIBLE] of an integral that allows us to basically sum the contributions from all these Huygen's point sources and derive different L diffracted field after the grating. So of course, this integral, we call the Fresnel integral, and here is another linguistic lesson-- this time in French, not in Greek.

This is pronounced "Freh-nell." The S is silent. So if you go to a conference, and you say "Fres-nell," people will conclude that you don't know optics. So part of knowing optics is part of knowing the French and the Greek language, I suppose.

I'm joking, of course, but it's still actually, professionals are like children, who are vicious. If we see someone make a silly mistake like this, say "Fres-nell" instead of "Freh-nell," we make fun of them. And you don't want to be made fun of in a professional setting. So always remember, this is pronounced "Freh-nell," not "Fres-nell."

So the Fresnel integral, therefore, allows us to compute the field, the diffracted field, after the grating. But we will not do it here because the Fresnel integral, in this case, is awful. It is very difficult to compute. However, I will show you that, by using very simple physical arguments, we can nevertheless derive the Fresnel diffracted field anyway without making use of this nasty Fresnel integral.

So let's go back to our Huygen's picture here. I would just consider this two point sources of Huygen's which are the strongest. And this is, of course, an approximation, a simplification, but it will still give us the correct physical picture. That's why I'm doing it. So basically, over here, you can see that these are point sources, so they basically emit light in all possible directions. So I will pick one arbitrary direction, and I will call it theta.

So here is a ray, if you wish, a ray emanating from this Huygen's point source at angle theta, and here's another ray emanating from the next Huygen's point source at the same angle theta. Now, these rays-- are they the same or different? Geometrical optics says they're the same, but now, we know better. We know that these are not really bullets that propagate like particles.

These are actually waves. And if you compare the wave emitted here and the wave emitted there, if you draw a line perpendicular to these waves, this would be the wavefront. So you can see, easily now, that if you compare the wave on the lower ray with a wave with the upper ray, you can see that the wave in the lower ray is actually delayed. It has traveled the longer distance. Therefore, it is delayed by a certain amount with respect to the ray that is below.

And the way, of course, to compute this delay is to draw an orthogonal triangle here. This side of the triangle is actually the wavefront. This side of the triangle is the phase delay, and the hypotenuse of the triangle is the period-- is the special period of the grating that we denoted as lambda. So by using the Pythagoras theorem, then, we can actually compute the phase delay, and we can compute-- there's, of course, a typo here. Let me fix it right away, and I will post a fix in the notes.

Anyway, I think it's better if I don't fix it now, but the correct equation over there should say the optical path length difference-- that's what OPLD means-- equals lambda, the grating period, times sine theta. Sorry about that. These notes came from another old [? convention ?] where the period was denoted as d.

So with this little fix, that OPLD equals the period of the grating times sine theta, we can see now that, if this quantity of OPLD equals a wavelength, then the two rays are actually in phase. And not only one wavelength-- it might equal two wavelengths, three wavelengths, any integral number of wavelengths. Then these two waves are in phase. Therefore, they interfere constructively.

And this is the key, now-- the constructive interference of waves in these specific, now, directions is what we call diffraction orders, that Pepe very beautifully showed you earlier. And you also saw them in my simulation in Matlab. And so q here, the quantity q, is an integer. It is meant to denote an integer, and we can derive this now.

So we can derive the sine of the angle of the q, over the qth, I guess, diffraction order where qth might be first, second, third, and so on. So q runs 1, 2, 3, and so on. And of course, because typically, we operate in the paraxial approximation, we can also drop the sign here and say simply that the plus/minus 1 diffraction order is propagating at an angle that is given by this ratio over there, the ratio of the small Greek lambda to the big Greek lambda, that is the ratio of the wavelength over the period.

And you can see immediately, for example, that if I make the grating period smaller than the wavelength, I will get in trouble because this sine over there will attempt to become bigger than 1. Anybody knows what happens in this case if I make the grating period smaller than lambda, than the wavelength? Mr. [INAUDIBLE]?

Yeah, actually, I can do-- I can make a period smaller than a wavelength. What will happen then is I will get what is called an evanescent wave, so I will not get a diffraction order anymore. I'll get an evanescent wave from the grating, and this is a term that we've seen before. We haven't defined yet, mathematically, but I want to alert you that, basically, an evanescent wave is an exponential decay of a diffracted field. So you do not quite get a diffracted order anymore.

For the purposes of the class, in the next few lectures at least, definitely for the next three or four weeks, we will assume that the grating period is much larger than the wavelength. So therefore, this quantity is always less than 1, and the sine is always properly less than 1. And we can solve it, so we can get the real and not evanescent diffraction order.

I think my computer-- so as I said before, the plus 1-- this is actually arbitrary. I can call plus 1-- by convention, the plus 1 is the one that goes-- if we denote the light-- as in geometrical optics, the light always goes from left to right. And the angle is positive if it is acute measured counterclockwise from the optical axis. So that is why this is the plus first diffraction order. It has a positive propagation angle, and then the one that has negative propagation angle, we'll call it minus first diffraction order.

The 0th diffraction order is the one that has theta equals 0 that is propagating on axis. That is also known as a DC term. Now, for those of you who are electrical engineers, the term DC already alerts you to some kind of a Fourier transform or some kind of a frequency representation. We will see later that, indeed, light propagating on axes can be thought of as the equivalent of a DC component in Fourier series decomposition. For now, let's take it as a term, as a sort of-- what do you call this-- as a jargon, and we'll see later why we call this the DC term.

This is a mathematical derivation that we can do in order to actually become a little bit more quantitative and a little bit more correct. So the mathematical definition is very simple. We start with the expression for the grating. Then we decompose the cosine into the two complex exponentials that we can [? write ?] it. And then what we do is we simply make a substitution according to the equation that I saw before already.

We say that 1 over lambda equals sine theta upon lambda. This may seem arbitrary, but I can always do it because lambda-- I'm sorry [INAUDIBLE]. Uh-oh, we're out of focus. Anyway, you cannot see the question, but you can certainly see this substitution here. And I cannot-- thank you.

So we can always do the substitution by defining sine theta to be the ratio of the wavelength over the period, and we can also make a substitution of [? u0, ?] the special frequency as I defined it before. The point is not that. The point is that each one of these terms actually is recognizable as a plane wave. And now, we'll really need my transparency here.

So if you recall, when we derived plane waves-- we did this some time ago. Actually, I think Professor Sheppard did it in the context of electromagnetics. The way we write a plane wave is like this.

These are what you will see if you go back into your notes by about two or three lectures. If you have a plane wave propagating in the direction of this arrow here, then the expression that I put above is actually the phasor for the electromagnetic field describing that plane wave. And if, in addition, we say that the grating is at z equals 0-- which I can always do. I can define my axis so that the grating is located at z equals 0-- then this term will actually disappear. And you can recognize now this expression that is left for the plane wave is the same that you have already here with the complex exponent, provided you make the substitution that we had before, provided you make the substitution-- this one.

With a substitution, the grating itself becomes like the expression for not just one, but actually three plane waves. And what are the three plane waves? Well, here is one, which is the plus first diffraction order. There's a plane wave propagating at angle theta.

Then the next one, the constant is actually the DC term that propagates on axis, because if you substitute theta equals 0 here, then this entire thing disappears, so you just have the constant. That is a plane wave propagating on axis. And finally, this term is propagating at the negative angle, and this is what you call the minus 1 diffraction order. So this may seem a little bit like mathematical trickery, but has a very nice physical interpretation, if only you recall our convention for the phasor expression of plane waves.

The rest is relatively straightforward. A sinusoidal phase grating, as Pepe mentioned, is basically a grating which does not modulate the amplitude, but the phase of the optical field. And you can do it in two ways.

One is what I saw here. That is known as a surface relief grating, which has been processed. Typically, this is done by chemical-- by optical exposure and then chemical development, that causes the surface of, typically, a polymer or a properly prepared colloidal, colloidal glass. It causes the surface to pick up a sinusoidal modulation like this.

So because the material has a refractive index n, you can see that light, depending if you take a ray, depending on where it hits this material, it might suffer a different phase delay. Another way that Pepe mentioned is you might actually have a flat piece of material and modulate the index of refraction inside. Again, that would have a similar effect.

The complex transmission function would be of this form. It would be an exponential whose value depends upon the optical thickness of the material that the light went through. You can see the period here, and you can see the phase delay. So the phase delay, of course, depends on the difference between the index of refraction inside the material and the index of refraction in the surrounding middle that is air.

So that's the expression over there, and this is, of course, the phase contrast that we mentioned before, when we defined the grating. And in order, now, to figure out what happens after this grating, I will have to resort to mathematical, again, trickery, I guess. If you go to books of optical tables, you can find very easily that this expression can also be written as a sum, as a sum that involves some kind of nasty Bessel functions.

We don't even have to worry about this, but what we will go ahead and add in a second is that each one of these Bessel functions corresponds to diffraction order for this grating. And the way you can see this is if you rewrite the grating like this. So this is now simply using a mathematical formula, but now, if you actually do the same trick I did here, if you recognize that each one of these complex exponentials basically corresponds to the plane wave, then you can see that what I've written there, it came out of a book of mathematical formulas, but physically, it contains an amplitude that is given by the value of a Bessel function. But then more importantly, it contains a sequence of plane waves that are known as diffraction orders, and the propagation angle of the plane wave indexed by q is given by this explanation here, the same expression that they had before. So these are then the diffraction orders for the sinusoidal phase grating.

Even more generally, if you don't have a sinusoidal grating, but still some kind of a periodic grating with arbitrary shape, and also possibly you might even have absorption, so this grating truly implements a complex amplitude transmission, which is nevertheless periodic. You may recall from your basic math that you have an arbitrary complex periodic function, you can write it as a Fourier series. And in this case, the Fourier series also has the same physical meaning as the one I did before, the Fourier series expansion coefficients. And actually, the amplitudes and the harmonics-- these are called harmonics, if you recall. The harmonics of the Fourier series themselves now correspond to plane waves that were previously called diffraction orders. So then you can see that each Fourier series coefficient actually corresponds to a separate diffraction order.

And in addition, recall, again, Professor Sheppard last week. He mentioned that if you have an optical field, and you take the magnitude squared of its phasor, then what you get is actually the amount of energy that propagates that is carried by this field. So therefore, the magnitude squared of the Fourier series coefficient actually corresponds to the energy that goes into each separate diffraction order. So in this case, so they're called diffraction efficiencies.

And I should have mentioned earlier, actually, but I forgot-- so in the case of the sinusoidal amplitude grating, the diffraction efficiencies are given by the squares of the coefficients that go in front of each plane wave. So you can see that approximately-- how much-- 25% of the energy is going into the 0 order. It is propagating on axis.

And let's assume that the contrast is 1. In that case, approximately 12.5% of the energy is going into each diffraction order. That is 1/8.

If I sum them up, how much do they transmit? 25% plus 12.5% plus [INAUDIBLE] 12.5%-- 50%, right? What happens to the other 50%? [INAUDIBLE]?

STUDENT: I think there possibly maybe had a plus/minus 3 order or something, I mean, other orders?

GEORGE BARBASTATHIS: That is plausible, but in this case, we derived only three orders-- the 0, the plus 1, and the minus 1, so there's no higher order. What happened to the rest of the energy?

STUDENT: Is it absorbed depending on the grating?

GEORGE BARBASTATHIS: That's right. The rest of them got absorbed or actually reflected backwards by the opaque parts of the grating. So remember, this is an amplitude grating. It blocks part of the light. So in this case, it's actually blocked about 50% of the energy.

So as the last example-- which I will not derive here, but I will let you go over it yourselves at home-- I did the case of a binary now. So this is a phase grating very similar to the surface relief that I showed before, but in this case, the phase is a binary function. So why is it binary? Because this a grating which is either-- the light goes either through a tall part or a short part.

The tall part, of course, suffers more phase delay because the light has to go through glass for a little bit longer than the short part. Therefore, if I plot the phase of the complex transmission, expect to see something like this. The part of the grating that are taller, they give me a longer phase delay, and of course, the phase delay can now be tried normalized so that it is 0 at some reference value.

And then what is the phase delay at the long part actually depends on the height of this groove. So even the height of this groove is such that s equals 2 pi n minus 1 over-- let me get this straight. So delta phi equals 2 pi upon lambda times n times s minus 2 pi over lambda times s2, where this is s1. This is s2.

So if I say that s1 equals s2 plus s, then what I will get is 2 pi over lambda times n minus 1 times s. So you can see from here that-- so if s is given by lambda divided by twice the index difference between the glass and the surrounding medium, then the phase delay implemented by this grating is actually pi. So if you have a function like this, then, phase delay pi means that the complex amplitude transmission is minus 1, and phase delay 0 means that it is plus 1.

So there's is another way to write this function. It is 1 at half of the period, and minus 1 the other half of the period. So you can actually-- it's a little bit of a [? grind. ?] Next time I will show you a faster way to derive it that you can derive the Fourier series for this kind of a binary phase grating, and you'll find actually something very interesting.

I don't know if you noticed the animation when I showed it. Let me play it once again. You'll find that even orders of this grating are actually 0, and next time, I will actually say a little bit more about this.

Anyway, you find the expression for the diffracted orders equals to something that looks like sine of 5 times the order. So of course, if the order is integral, if the order is even, then there's no diffraction. It turns out that this grating has a relatively strong diffraction efficiency of about 40% into the first order.

STUDENT: I've got a question.

GEORGE BARBASTATHIS: Yes?

STUDENT: How does the polarization of the incident plane wave change the diffraction efficiencies?

GEORGE BARBASTATHIS: So these guys were actually in the scalar optics approximation, so we neglect the effect of polarization. To include it is actually rather complicated business that we may get to near the end of this class, but it requires something called coupled wave theory, which is relatively advanced. A rule is that if that period is relatively long, a rule of thumb about 10 wavelengths or longer, the effect of the polarization is minor. You have a grating whose period is 2 wavelengths, 3 wavelengths, something like that, then the effect is actually significant, and it cannot be neglected. But in this class, we will actually-- at least at the beginning, we will pretend that the period is large enough that we can neglect polarization effects.

Very good question. Practice actually makes a big difference. Other questions?

STUDENT: Just now, we show one grating, which modifies the amplitude of the wave. So what's a physical picture of such a grating?

GEORGE BARBASTATHIS: So for example, what you can do is you can Just take a piece of glass, and evaporate the aluminum or some other metal into the areas that you want to block the light. So that's one way to make the grating, So we take a piece of glass, then you can evaporate aluminum uniformly here. Then on top of it, you can put photoresist and patterning. OK.

Then you can etch. So why when you etch, you will actually remove these parts. What you will end up is something like this. Then you also remove the rest of the photoresist. So that would be, now, a binary amplitude grating because you have aluminum blocking the light here and then not in here, so the light goes through. Yeah, as I said, this is called a binary amplitude grating, and you will deal with it in the homework.

To make a grayscale, like the one that I showed in my calculations here, it's actually a little bit difficult. To make this kind of thing is rather hard. It can be done with sort of proper evaporation tools, but very hard. So this is a rather-- it's a simple construction that they need in order to describe the physics, but it's not very easy to implement the practice.

STUDENT: And I guess, just to tie it to the demo for the phase counterpart, you can do this by-- you saw the interference pattern produced by the Mach-Zehnder, right, that there were sinusoidal fringes. So instead of the CCD, you could imagine putting there, for the sensitive polymer, for example, like photoresist, that will expose that material with these bright and dark fringes. That basically will cause that refractive index in a sinusoidal pattern, and that will produce the phase version of this grating that were shown before also.

GEORGE BARBASTATHIS: As it turns out, if you do what Pepe just described-- let me see if I get to it here. If you do what Pepe just described, you get two effects. One is that the index of refraction changes in the exposed areas, but also, because there is-- you can understand this is a fluid effect. The index of refraction changing means that the material becomes, actually, slightly denser. So this will result in a surface undulation in order to conserve its-- to conserve matter, the surface will actually have to undulate. So you end up with a grating that is non-surface relief.

So these exposure processes are relatively complicated. So if we have time, I may go into it later during the class.

But another way to do this, to do the binary grating, which is done very often in practice, is using the electron-beam lithography. Where is my-- so this kind of grating is also done with a similar process, but using electron-beam lithography. So what you do is basically-- one way to do it is you start with glass, then you coat it with a material that is called HSQ. And actually, you can pattern it using an electron beam.

You put an electron beam resist on top, which is PMMA. Then, using the electron beam, you remove parts of the PMMA. And then you etch away these parts. Then you end up with a pattern that looks like this.

So HSQ is actually clear. It's very similar to glass. So this would be a physical realization of the binary phase grating. The light is all transparent, but the light propagates a longer distance inside the material here, whereas here, it propagates in the air. So that's why we'll get this effect.

STUDENT: [INAUDIBLE]

GEORGE BARBASTATHIS: With a laser beam, you can actually get very small gratings. If it is completely periodic, then people can make something like 100 nanometers or less easily. So what I mean by 100 nanometers is that this period over here might be 100 nanometers, even less, actually. So of course, for visible light, that would be a subwavelength grating. It wouldn't do much. But it could be useful, for example, for ultraviolet light, or for other applications where you don't use it as a grating.

If it is nonperiodic, very often-- we will see later in the class-- sometimes, we want to make patterns that look like this. OK, so this is, again, a phase grating, but it is nonperiodic. You can see that the period changes. So if the period changes in the quadratic fashion, this is a very important element. It's called a "Fresnel" Here is, again, the French term-- "Fresnel zone plate."

This I will describe in class later. It's a very, very important optical component. But it turns out because of this kind of pattern it is nonperiodic, it is not so easy to make using electron-beam lithography. So in that case, you have to go to great pains to make small features. So in this case, it's a little bit strange. But because of the nonperiodicity, the feature sizes here are limited to bigger values-- so for example, 200 to 300 nanometers.

This has to do with the way electron beams scatter from the photoresist. Actually, some of you are taking, simultaneously, Professor [? Bargrand's ?] class. He goes into great detail into this particular-- into why this is the case.

For visible light, it is actually quite easy to make reasonably sized gratings with lambda bigger than, say, five or 10 wavelengths. It is actually easy to make these kind of gratings either with optical exposure, like Pepe described, or with electron-beam lithography. There's also other techniques, for example, interference lithography. There's a whole set of techniques that people use to make this grating.

I don't know. Pepe, do you know, the one that you showed, how was it made?

STUDENT: Not really. Not really.

GEORGE BARBASTATHIS: [INAUDIBLE]

STUDENT: It was the one that you used. Do you know? I don't know, to be honest. No. I think it's a binary grating and probably was done-- I don't think it was done with e-beam because it's pretty large.

GEORGE BARBASTATHIS: No. No. No.

STUDENT: It has to do with--

GEORGE BARBASTATHIS: [INAUDIBLE] suspect that way also of making these gratings by molding. That's another way. So you basically create a very expensive negative, like a mold. And then you stamp-- not quite glass. It has to be some kind of a pliable material.

That's one way. And then they can also make them by-- basically, by scratching by very accurate machines. So there are many, many different ways to make--

STUDENT: The dip-pen lithography or some kind of [INAUDIBLE]

GEORGE BARBASTATHIS: It's reactive ion etching.

STUDENT: Dip-pen lithography?

GEORGE BARBASTATHIS: Oh, Dip-pen.

STUDENT: Yeah.

GEORGE BARBASTATHIS: Yes. Except dip-pen lithography can allow you to do only-- where is it? I lost my amplitude grating. It can only make amplitude gratings, not phase gratings.

Depends. It's ink, right? So good deposit ink. And this way, you can actually absorb. You can get absorption bands.

But I don't know if you can make phase gratings with dip-pen lithography. Have to look into it. They're very popular nowadays. Again, professor [? Bagram ?] goes into this in more detail. But a very popular technique is-- it's called-- help me out here. How do you call the stamping technique?

STUDENT: Oh, we call it nanoimprint.

GEORGE BARBASTATHIS: Nanoimprint, yeah. Nanoimprint lithography, yeah. That's a very popular technique to make these kind of elements with very small periods, actually. People make [? 15 ?] nanometers or less, actually, with nanoimprint.

STUDENT: Sorry, Prof. Do we have time for one more question?

GEORGE BARBASTATHIS: I have time, but some people have classes. But yeah, go ahead.

STUDENT: OK, I'm just curious. Let's say, just now, you mentioned the sine theta cannot be greater than 1. So what happens if we shine the visible light on to a grating with a period [INAUDIBLE] 100 nanometer?

GEORGE BARBASTATHIS: Yeah. So I actually said it before. Let me repeat it again. So he's asking, what will happen if you have a grating here, like this, for example, where the period lambda now is less than the wavelength?

OK. So, if you recall, a planewave is of this form-- e to the i kxx plus kyy plus kzz-- and this, we saw when we did electromagnetics. And we also saw that because of the electromagnetic wave equation, the wave vector components here-- they have to satisfy this equation.

OK, so now, according to the equation that we had before, if this is the x direction-- and let's ignore y for now. So basically, I can neglect this term for now. And what we have is that the sine factor will equal lambda upon the period. And it's bigger than 1.

OK, mathematically, if I have a complex angle, the sine can be bigger than 1. But what does this physically mean? Physically, if you plug into this equation here, you will get that the kz squared equals 2 pi upon lambda squared minus 2 pi upon lambda squared based on this equation.

When I said that kx equals 2 pi upon lambda sine theta equals to pi upon lambda times lambda over lambda equals 2 pi upon lambda-- OK. So if I do that now, I will get that the kz squared equals 2 pi. And the quantity inside the square root is less than 0, so we'll get that the kz now will equal plus minus. It will become imaginary. [INAUDIBLE] plus minus i the square root of something positive.

So what does this mean now? I have a wave whose amplitude will be like e to the i kxx. Nothing changes here. But then it will be plus minus i times something positive. Let's call this "alpha" for gravity here. OK, this is what the wave will look like after the subwavelength grating.

So now, clearly here, if you multiply the two imaginary units, we will get a real number. So we get either exponential decay or exponential explosion. Physically, you cannot expect the wave to grow exponentially. That would mean someone is supplying power, and there's nothing like that here. So therefore, you get a wave that will look like e to the minus alpha z-- there's a z here in [INAUDIBLE]-- times e to the minus e to the plus ikxx.

So what is this now? The e to the minus alpha z's an exponential decay away from the grating. The e do the ikx is what we call a "surface wave." It propagates like this. So actually, what you get after this grating is you get a wave that propagates parallel to the grating.

I don't know if the camera can show me on this one-- probably not. But anyway, as you get the wave that propagates parallel to the grating, it is called the "surface wave," but it doesn't live very long. It decays exponentially away from the grating. That is actually called an "evanescent wave."

And it is not really part of the class. This is a very simplified description of what happens. To properly do it, I would have to take into account polarization. I would basically have to be more careful solving Maxwell's equations. And because this class is simple, basic optics, I generally stay away from those kinds of gray things. That's why I don't describe them in detail. But since you asked, I gave you a very simplified description.

We may talk about this a bit near the end, depending on how much time we have left near the end. But if we do, I will talk about this in more detail. It's also related to the question that someone has asked before about polarization. If you have a subwavelength grating, then you definitely have to take into account the incident polarization.

STUDENT: Yeah, actually--

GEORGE BARBASTATHIS: I see our audience is [INAUDIBLE] now. So maybe it's time for us to get our martinis and the rest of the others to get their coffee. OK?