PROFESSOR: So today, in some ways, that word for what we've been suffering through for the last few lectures because we'll actually get to see some optical instruments and how they work. So we'll basically put all of these tools and all of the terminology and the matrices and the formulas and all that, we'll put it to work for the basic optical systems, namely microscope and telescope. And we'll also look at sort of the building block of a microscope which is a magnifier lens.

But before we do that, I would like to spend some time just reviewing basically all the fundamental material before it will go into the application systems. I would like to do review the fundamental material.

So the first four or five slides-- I don't remember. I think five. They're not really new. I just brought them-- just copied them from the previous lectures as a kind of overview. And I would like, instead of just describing it again, I would like to encourage you-- I will pass on it and I would like to encourage you to ask any questions if you have on this particular topic or something related.

So the first topic is the ray transfer matrices. And again, very briefly to remind you, this comes about because at a fraction at the spherical dielectric interface is very complex to deal with, not the very complex. It is actually pretty simple. But algebraically complicated, or I should say trigonometrically, very complicated.

So because we don't want to deal with that stuff. And it also hides a lot of the intuition, we derived this paraxial approximation which is true for a pretty limited-- typically 10 to 30 degrees. Actually 30 is too optimistic, maybe 10 to 20 degrees from the optical axis. We discovered that there is a linear relationship between them, elevation and angle of a ray that is arriving from the left to the interface. And the elevation and angle of propagation of array that is departing to the right over the interface, the same rate, that is [INAUDIBLE].

And of course, the elevation is the same because of the paraxial approximation, we neglect the thickness of the instrument. So basically, no matter where the ray hits, here or here or here, it is all the same. So that helps.

And the second is that the angle, basically the angle satisfies Snell's law with respect to the normal at this sphere at this location over here. But of course, because this orientation depends on the relative position of the arrival of the ray and the center of the sphere, then we simplified it using the paraxial approximations.

And we discovered this expression over here. So the relationship between the right and the left angle, it has to do with the elevation. So if you look at this equation, it says that n-right, alpha-right equals n-left alpha-left. That's the first term.

And then the interesting part is the second one that says equals minus this quantity times the elevation of the ray. So that tells you that the more-- the higher the arrival elevation of the ray, the more it bends. And that makes sense because the higher the arrival, the larger the angle that it makes with respect to the normal to the sphere and, therefore, if you wish, the more violent the Snell's law will be. And of course, you can imagine, as you increase the angle, eventually the angle becomes so large that the paraxial approximation breaks down. But this is all true.

Before the approximation breaks down, the progression is linear. It is linear in the position. And the quantity that appears here, does anybody remember? How do you call this quantity? Optical power. Correct. Could you please push the button? Correct. It's called the optical power.

And it is actually the first instance that we saw this quantity. It is the inverse distance units. But we don't call it inverse meters. We call it-- how do we call the unit of this quantity? Diopters, right?

AUDIENCE: So professor, I have a quick question. Here we have quantity over radius r. Does this describe a hyperboloid or elliptical shaped or a spherical lens? And if so, [INAUDIBLE]

PROFESSOR: That's a good question.

AUDIENCE: For example, with the spherical or an elliptical lens that has two different radiuses. How do you account for that?

PROFESSOR: So this derivation, we actually did for a sphere. So the radius here is the radius of the spherical interface. And a very similar relationship would hold if instead of a sphere you had another ovoid, another surface of revolution which would be symmetric around the axis over here.

So for example, if I put a parabola. Imagine a parabolic bowl where this is the cross section of the parabola. But then if you spin it around this axis, it remains the same. This is called a paraboloid of a revolution.

This is actually-- I believe this was a homework or actually we'll do it a little bit later today. We'll discover that a very similar-- actually in the same equation holds, except in the case r is not that radius of the parabola if the parabola does not have a radius, but it does have a curvature near the axis. So the curvature of the parabola, actually the inverse of the curvature is-- the radius of curvature is what enters this equation.

Now we discussed. an ellipsoid. And you are right that the ellipsoid has two axes. But the ellipsoid that we saw, the two axes are oriented a different way. So in the ellipsoid that we saw, when the actual propagation is z, then we have one of the lateral axis is x. And we show that the perfect focusing element, the perfect focusing refractive element is an ellipsoid looks like this.

So of course, this has a major and minor axis. But you can see that the ellipsoid is still symmetric with respect to the optical axis. So in other words, it is still an ellipsoid of revolution around the z-axis. So I don't have to worry about 2 radii of curvature.

But it is still a valid question, what you are asking. What if I did have-- what if I do put the surface there with [INAUDIBLE] of curvature? Well, it turns out, you will seldom see this in books except basic-- I don't think I've ever seen it in a book.

But we can reconstruct what you should do. This is slightly incomplete because I'm only dealing with elevation of the ray and the angle in a cross section of the plane. And that is fine here because it is a surface of revolution.

But if it is not a surface of revolution, then I have to grow this to a 4 vector with two angles and two elevations. And then this would become a 4 by 4 matrix. And then it would have a different power in each axis, depending on the Gaussian curvature of the surface in the corresponding location where the ray hits the axis.

So you can imagine it will become a very complicated kind of situation. So typically we don't deal with this. I don't even know if optical elements are designed this way.

AUDIENCE: [INAUDIBLE]

CREW: Can you press the button, please? Oh. You cannot hear?

AUDIENCE: Sorry. I pressed it but it didn't seem to do anything. You can hear now?

CREW: Yes. Yes.

AUDIENCE: I was just saying that I haven't come across this for geometrical objects either. But I have seen it for diffraction optics, in terms of the Wigner distribution function. And what I was going to carry on to say, though, was for the geometric optics case, it might be quite interesting because it would allow you to look at the behavior of skew rays. So it might actually give some quite interesting information, I think.

PROFESSOR: Yeah. That's true, yeah. And you could also look at things like caustics, right? You know, if you make the surface non-rotationally symmetric, you get new different caustics. Forgive us. We got into a slightly esoteric conversation here. But that's a really good question. So we may end up writing a paper about this in a few months or something.

Great. Any other questions?

AUDIENCE: Isn't this problem separable in x and y?

PROFESSOR: Actually, before I answer that. Let me bring up a trivial case where you might have this situation. That's a cylindrical lens. The analysis with it is actually valid for two cases. One is when you have a surface of revolution, like a sphere. The other is if you have a cylinder, in which case, everything is invariant in the out-of-place direction.

So that's just a trivial case where you have a radius of curvature in one plane. And the curvature is-- the register is infinite in other plane. That is, we a plane, a planar surface, right? That's also called a cylindrical lens.

And of course the cylindrical lens does not focus to a point. It focuses on a line. There's no focus in here. But you can imagine if you focus on the line out. I'm sorry. So what was the question?

AUDIENCE: Yeah, the question was, isn't the problem separable in x and y. So we could just extend the matrix in y direction, and [INAUDIBLE]

PROFESSOR: Yes. I think that's true. The paraxial approximation, it is separable, yeah. But of course, if you go off the paraxial approximation, things like [INAUDIBLE] and so on, then all bets are off. They become highly coupled.

So for example, astigmatism is highly coupled. You cannot [INAUDIBLE] with-- astigmatism for those of you who are not unfortunate enough to have it in your own eyeglass prescription, astigmatism is a situation where an optical system focuses-- it's kind of difficult. I'll go into it in next week.

It's a form of aberration where an optical system has different focusing properties for tangential rays as opposed to the sagittal rays. I don't want to confuse you right now. But it is a form of operation that has to do with skew rays that arrive in sort of off-axis and off-plane.

And it is actually very common in people. Very often those of us who wear-- does anybody know you if you have astigmatism in your prescription? Yeah. That's right. Yes. So it means that his glasses are not exactly spherical. Other questions?

The next step-- so this we did, I believe it was two weeks ago or so. Then the next thing that we did is we took this formulation basically-- oh, and before I move on let me just remind you very briefly why we did this. This looks kind of silly. It tells you that if you have a ray propagating in free space, again you can relate the elevation and angle to the left of some chunk of free space to the ray elevation and angle to the right of that chunk of free space.

That's, again, that looks silly. Because we know the ray propagates in a straight line in free space or in uniform space because of Fermat principle. And of course, what will happen is that the angle of propagation will remain the same. That's what this law says.

And also the elevation will increase by the distance times the tangent of the angle of propagation. But of course, with the paraxial approximations, instead of the tangent we put the angle itself. So this is what this equation says here. If you multiply this by this, the indices of refraction will drop out. And you're left with d, the distance times alpha. So it is really the tangent.

So this seems silly. But the reason we did it is so that when we have optical systems with a cascade of uniform space then add the refractive interface and then more uniform space and so on, then you can basically ray trace those systems simply by cascading these matrices. So this is why we did this particular pattern. But this really doesn't say anything other than rays must propagate in a straight line in uniform refractive index because that is the shortest path for them to follow.

And then what we did, after we developed this formulation, is we actually put into practice in the special case of two adjacent spherical surfaces. So typically, you have air outside. And between the two surfaces, you have glass.

And this kind of element is what we call in everyday language a lens. It does not have to be glass. For example, in the case of our eyes, it's not glass. It is actually tissue, mostly water in fact. But anyway, it is higher index of refraction than the surrounding air. So it is acting-- it is doing the same job as a lens.

And what we did then, if you recall, I will not do all of it again is but what we did is we did a cascade of the matrices for the two interfaces. And based on this, we derived the properties of this element. And what we discovered is the basic properties that if you have had a ray bundle that is arriving from infinity, therefore it is razor parallel, this type of optical element will actually bring them to a focus.

So the rays after, they will propagate a certain distance after the element. And then they will focus to a point. And we also derive the other useful quantities, we derive the elevation of this point. We discover that it is proportional to this focal distance times the annual propagation. And then we saw that there's different cases of focusing depending on the nature of the lens. Namely its optical power.

So positive lens will focus the rays at a finite distance. A negative lens will actually cause the rays to diverge. So they, strictly speaking, do not focus. But if you extend them backwards, then that create what is called a virtual image. And we will see later today what exactly this virtual image means and why it is an extremely useful concept in optical design.

For now, it appears as an oddity. What exactly is that? Why did it move-- why did it trace the rays backwards. But today, actually, you will see why this is useful. The main point to take out of here is that the lens is acting kind of like a lever. So if you have-- if you imagine that you are rotating the angle that the rays are arriving to the left of the lens, then this point is moving linearly.

It is staying on this plane, ideally at least, within the paraxial approximation. It is staying on this plane but it is moving. So as you rotate the angle, the elevation changes. And the constant of proportionality in this change is the focal length of the lens. This is a very key point to remember about lenses and how they function.

And the other thing that we said is that it can also work the other way around. If you take this picture, the rule is that the rays always go from the left to the right. It's not a rule, it is a convention. There's no physical law, obviously, that says that. But it's a convention.

But nevertheless, there is a physical law that says that if a light propagates in one direction and then you flip the time backwards, that is you reverse the propagation of the light, then the physical law actually says that the propagation should remain invariant. That is if I knew that the light propagated this way focuses, if I turn time backwards like these old movies that you see where they play the movie backwards and you see it pass, you know, moving towards the back.

If I did that, I would have the rays going like this. So this against our convention. So that's why I flipped it here. Now again, the rays are going from left to right but in this case they're going through the focus. What this picture is telling us is that if you have rays that originate at the point one focal distance behind the lens, then this will form an image at infinity.

That is, the lens will collimate this ray bundle. And it will send the rays out parallel at a 90 that obeys, actually if you look at it, is the same relationship that was obeyed before. With the next reminder sign. And why did the minus sign come about? Everything's symmetric here except for this annoying minus sign. Why has this come about?

Button. Did you do a button? Go to some desk, so you have a button.

[LAUGHTER]

AUDIENCE: The object distance, is it?

PROFESSOR: Maybe not the distance. We have distances and what else here?

AUDIENCE: The angle is negative.

PROFESSOR: The angle, that's right. The only reason this negative sign appears there is the sign convention. This angle here, we define it as positive. This angle here is the same. But that's not how we measure angles. We always measure angles this way. So therefore, this angle is negative. So to account for this negative angle, a negative sign appears over there.

AUDIENCE: I have another quick question. Let's say we have a lens where our left and our right are the same, which I believe is pretty common in optics. So from the lens makers equation you would get that 1/f is equal to 0.

PROFESSOR: Nope.

AUDIENCE: Or is that wrong? Because you get 1 minus a quantity--

PROFESSOR: [INAUDIBLE] That's not true, actually. If the red is to the left is equal to the-- well, depends how you are. Is it like this or like this?

AUDIENCE: It's like this,

PROFESSOR: Like this?

AUDIENCE: Yes, sir.

PROFESSOR: OK. Let me give you a hint. If it was really like this, we have 1 and 1. Now it is the same radius of curvature, and that would indeed have no power, no optical power. Their focal length would be infinity or the optical power would be 0. In this case, are the radii of curvature the same or not?

AUDIENCE: So one's negative and one's positive. Is that what you're saying?

PROFESSOR: Exactly. They're not the same. They're opposite.

AUDIENCE: OK. Thanks.

PROFESSOR: So they may be the same in absolute value. But they have opposite signs. Therefore in this case, you actually get a finite focal length. I'm sorry, I don't think you could see what I do. Other questions?

It was a very good question. I'm very glad you asked that. We have to be very careful when we apply these formulas to use the proper sign. So remember the curvatures by the sign convention, which I really recommend that you-- I don't have it available here. But I really recommend you go back and study it.

The convention is that if the curvature points like this, it is convex towards the left, then it is called a positive curvature. So in this case, you would write, for example, you would write r equals maybe it is 10 centimeters. In this case, because the surface now is concave, then you would say that R equals minus 10 centimeters.

So the lens make an equation in this case, we'll say-- let's say n equals 1.5 for example. Let's make it, I would say, 1/f equals 1.5 minus 1 times 1 over 10 minus 1 over minus 10. And this is whatever. I believe it is 3 or something.

Wait. This is 1/5, or there's 0.5 times 1/5. So it is 1 over 10 so it is 10. Other questions?

So this was true for what we call the thin lens, where it is thin because we assume that the distance between the surfaces within the paraxial approximation can be neglected. If we cannot do that, if we cannot neglect the distance between the two surfaces, our formulation does not abandon us, we just get a more complicated formula.

And the reason the formula becomes more complicated is because if we drop the approximation of negligible spacing then the ray refracts twice before exiting. It will diffract at the first interface and the second interface.

We'll get the formula for this. The thick lens, I don't want to go back into the formula. It just comes out of the algebra. What I really want to emphasize in this case is that even though the refracts doubly, we can pretend that it only refracted once. And we can do that by extending the incoming ray, extending the outgoing ray. And then what the plane, where they intersect, that is called the principal plane. Actually that is called the second principal plane.

If I do it for an incoming ray bundle that is imaged at infinity then it is the other way around. Again, it can extend the incoming ray, the outgoing ray, and again get the first principal plane. And the reason we did all this exercise were to discover principal planes and so on is because this allows us to do ray tracing as shown over there.

Can you still hear me? Sorry. This microphone has a tendency to fall. OK. You can still hear me?

AUDIENCE: Yes.

PROFESSOR: Thanks. OK. So the reason we did this exercise of effective focal length and so on is because we can play the following take. I can give you an optical system with a bunch of lenses. I don't know, whatever. Very complicated.

And I'm asking you to find the imaging condition. Here's an object. And I'm asking you where is the image. What is the magnification? Lateral, angular, and so on. This [INAUDIBLE] of principal planes, it applies as we discussed not only on a thick lens but in general in any compound optical system that contains multiple elements like this one.

And in order to utilize it, what you do is you throw out, so to speak, all of this entire system and you replace it with its principal planes. So here is, let's say that the principal planes are like this. Here's the first principal plane, the second principal plane.

And now I can do the imaging business very simply. Here's my object. If I take a ray that goes horizontal from the object, this ray, it is as if it came from infinity. So therefore this ray will go to the second principal plane, it will bend. And it will meet the axis one effective focal length to the right of the second principal plane.

Then I can take another ray which goes through the first focal point. That goes one focal length to the-- I'm sorry, did I say to the left before? I should have said to the right. Let me repeat. So this ray will go-- will hit the optical access one focal distance to the right of the second principal plane. I think I said the wrong thing before. Anyway, to the right.

Now let me take a ray that goes through the focal point one focal distance to the left of the first principal plane. I will run out of space here. But that's OK. This ray will have to hit the first principal plane and then turn horizontal. Because this is equivalent to a point source one focal distance to the left of the lens that would be [INAUDIBLE] infinity. So therefore, this ray must come out horizontal. And where these rays meet is actually where my image is located.

This construction is very useful and very basic and fundamental. And that's why the principal planes are important. Now let's try this. Any questions about this?

AUDIENCE: Since we have principal planes for the system, are we going to substitute the system by a matrix which is for single lens? Like 1, minus 1 over f, 0. But that's not going to be possible because the single lens doesn't change the angle but the system can change the position and angle both.

PROFESSOR: Yeah. The system is not equivalent to a thin lens. It's a little bit weird. This equivalent, you can think of it as having two thick lenses. One of them is operating for this ray. This is like a thin lens but for this ray. This is like a thin lens but for this ray.

That's a bit confusing to think of it like that. Right? So I so would rather not go there. But yeah, if you look at the matrix of the system, it is not a matrix of a thin lens. Of course not. Yeah. Yeah?

AUDIENCE: [INAUDIBLE]

PROFESSOR: OK. That was actually my question. So he's asking, what if the focal length is negative. How would you the ray-- this is what you're asking, right? Yeah. So that was actually my-- the next thing I was going to do also here. So let's take this out of the way. Take this out all the way too because the marker leaked. The marker leaked here. Oh, OK. We're wasting too much paper.

So let's do the same game now. But now let's say that the focal length is negative. So here is an object. And here are the two principal planes. Let's say this is the first principal plane, the second principal plane. How would the ray trace in this case? Would I have a negative focal length?

Does someone want to help me trace it? Button.

AUDIENCE: You first propagate to second principal plane and then--

PROFESSOR: So [INAUDIBLE] the horizontal ray to the second principal plane. Now what?

AUDIENCE: Come back.

PROFESSOR: Come back?

AUDIENCE: In the sense that the ray diverges from the first plane.

PROFESSOR: But we'll have to go-- well, why drawing's not very good. Let's say that this is the focal point. So [INAUDIBLE] diverge in this fashion. Right?

AUDIENCE: Yeah. Yeah.

PROFESSOR: What about the other one?

AUDIENCE: [INAUDIBLE]

PROFESSOR: First of all, where is the first focal point?

AUDIENCE: On the right panel [INAUDIBLE]

PROFESSOR: They cannot hear you.

AUDIENCE: Probably should take rays from the object to the first focal point, there's a back focal point.

PROFESSOR: So where is that?

AUDIENCE: That's on the right side of the first principal plane.

PROFESSOR: Should be somewhere around here, right?

AUDIENCE: And at the point where they intersect, you make it parallel.

PROFESSOR: So this is minus f. This is minus f. Right?

AUDIENCE: Yeah.

PROFESSOR: With the distances. OK. So then what do I do?

AUDIENCE: The point where the ray meets the first principal plane we make it parallel to the optical axis?

PROFESSOR: That's right. So what I have to do is first of all I draw a dashed line that goes to this point. This tells me which way this would be pointing. This will have to go all the way here. Yeah, that's right. And then-- So where is the image?

That's right. The image is actually where these rays are supposed to meet. Right? They never quite meet. But I have to extend one of them backwards so they would actually meet here. And notice the significant difference that here the image erect. It is upside, which is customary for virtual images.

Whereas in this case, the image if you recall the image quite clearly, it was inverted. So it is really the same approach. I haven't changed anything. But the only thing I have to do, really, is be careful with the signs. Who is to the left of whom.

And the way we keep track of that and we don't lose our head is by faithfully following the sign conventions. So the definition is that the positive focal length would be to the right of the second principal plane. OK. Fine.

If I'm given a negative focal length, that means that I should go to the left of the second principal plane. And so on and so forth. That's the principle.

What about this case? Can I have this? Of course I can. Interactive homework, you'll see a system that has that property. The second principal plane is to the left of the first principal plane. There's no rule or law against that. What we do? Nothing changed, really. I can still do my ray tracing as before.

If I were to do my rotation here, well, it would go up to the second principal plane. I would shift the ray down. Let's say this is one focal distance. And then now let me do the other one correctly. So this is the first principal plane so to go from here-- something like that. I mean, that produced a ridiculously-- Well, we're here.

I did something wrong here so the image ended up very close to the principal plane. But this is how you would ray trace the system. Any questions about the stuff I'm doing here? So when you're given an optical system it's a little bit of a judgment call. You have-- what happened. Some message flashed. Something got disconnected. Are we still there?

CREW: Yep.

PROFESSOR: So when you have an optical system there is three ways you can analyze it. One is you can just blindly write down the cascade of matrices, figure out the incoming and an outgoing angles. And then try to solve it this way. The second way is you find the principal planes. And then you apply one of these techniques that I just did on paper. And this is another way to analyze it.

The third way, which is what I think [INAUDIBLE] did in the class and was also the supplement of your nodes is where you take each length. You apply the imaging condition individually and then move on.

I will not do it again. It's in the example of your notes. But it's a little bit of a judgment call. And sometimes it's a different matter of preference. Some people are very good with matrices. So it is just better in that case to just multiply the matrices and get done with it. The principal planes that give you more intuition. So I'm personally not in favor of just blindly multiplying matrices. That's a little bit mind numbing. And also very prone to algebraic errors because we're all human.

But this method of principal planes, they're a little bit less prone to algebraic errors because with these methods, you can actually test your intuition. You can see easily, whether what you are doing is correct or not. Whereas you make an algebraic mistake doing the matrix multiplications, well, it's very difficult to know where it happened.

And in fact, I confess when I wrote the supplement to the notes which is-- I don't know if you had the chance to see it, but it is the same optical system solved with the three different ways. You know, with first of all with the cascade of lenses then with matrices and then with principal planes.

So I confess that I first did it with a cascade of lenses. So I took the first lens. I imaged it. Then I used the image of the first lens as objects to the second lens and I imaged again and I found the answer.

Then I took a plan, I got a big cup of coffee, I did all the algebra of the matrices, I actually got the wrong result. It was not a green with the first result. So I went back and made it again and finally I got the algebra correct and I got the two results to agree.

But for sure, the first thing I did it with the cascade of lenses was correct. Then every time I got a different result with the matrices, I knew that I was making an algebraic mistake. Until I finally got the matrices to agree with the actual result. And I got it right.

So there reason I'm telling you this is because I've taught this class for 10 years, believe it or not. So every time when we do the quiz number one, your colleagues in the past they all try to solve the problem with matrices. Because it is easier. Matrices is a no-brainer. You just multiply and get the result.

About 70%-- over all these years, I have accurate statistics. About 70% they get stuck into some algebraic error. And then they cannot solve the problem because the measure of this is if you get the ray tracing wrong, you cannot do anything. You can not do magnification. You can not do whatever. So I would only recommend the matrices if you are really, really sharp with algebra. Otherwise, you're better off-- and actually overall, I think you're better off if you land in the physics, which is not what the matrices give you. The matrices are convenient. For example, we derived all of these based on matrices.

But after that we should let our intuition take over. And not rely just on the math, which is actually a pretty good principle in anything you do in engineering but it is especially true here. Any questions?

AUDIENCE: I have a question. So for the focal length that is negative, if you retrace the rate coming from the object say to the center of the first principle plane. So this--

PROFESSOR: No, you don't trace through center with the principal plane, you trace it to the focal point.

AUDIENCE: No, no, no. I mean, it means that, so in this case for the focal lens is a positive. So we have three [INAUDIBLE] characteristics arrays for the--

PROFESSOR: Oh, you mean this ray?

AUDIENCE: So OC, ray OC.

PROFESSOR: This ray?

AUDIENCE: Yeah. Yeah. So for the focal length that is negative, can we draw that line? So you will find that the intersection of those rays would not meet the objects you found.

PROFESSOR: Oh, it should. It should. If I had drawn it properly, it should. If you drew this ray, if you drew this ray and then you extend this ray backwards, it should still go through my image point.

AUDIENCE: No. It shouldn't.

PROFESSOR: You're right that they don't meet because this is a virtual image. What does meet is their extensions backwards.

AUDIENCE: Yeah, I know. But when you draw it, but-- so you see those angle relationship. So there--

PROFESSOR: Well, that's because my drawing is not very good. If I had done the drawing properly it shouldn't have happened.

AUDIENCE: The artwork angle is larger than the-- larger than the angle coming-- going from the say-- I mean if you take like a z, the angle relationship between this one and the one--

PROFESSOR: This one and this one.

AUDIENCE: --and the one coming from the object to the first focal point. So this angle is a larger than the array coming-- larger than the one from the goes to the first focal point, which means that if you backtrack.

PROFESSOR: This is my handwriting. This is my handwriting.

AUDIENCE: Oh, I see. I see. OK.

PROFESSOR: I forced it in order to make them eat. I did not make a good diagram. I'm sorry.

AUDIENCE: OK. I see.

PROFESSOR: Yeah. I'm not very good drafter. I'm sorry. How do you call it? Draftsman, right? I'm not a very good draftsman. OK. So I don't want to dwell on this too much. But this is basically-- we already discussed this idea that pretty much the only case where you form a real image that is the rays actually do meet at the image point, is when you have a positive lens and you place the object to the left of the front focal point of that lens.

That will form a real element. All the other combinations, they actually form virtual images, which means that what is coming out of the system is a divergent ray, a divergent ray bundle. Therefore they don't really meet. But they do meet if you extend them backwards. And this backwards intersection is what we call the virtual image.

And you can see there is different cases here. Sometimes the magnification is bigger than one. Sometimes it is less than one. These actually, can never remember. Every time I have to derive which is the case. But anyway, all the possibilities can happen except, of course, for this one where the magnification is always negative. It is always inverted.

In the virtual image, the magnification [INAUDIBLE] it is always positive. It is very much is erect. OK? Any questions about this? Real and virtual images.

And the last thing I wanted to remind you of is what we did on Monday actually. The definitions of aperture stops pupils, windows, [INAUDIBLE] and margin inlays. I really don't feel like going through all this over again. This is probably very fresh in your minds. So are there any questions for Monday's lecture that I can answer about this?

AUDIENCE: The term vignetting applies to the case when the numerical aperture changes over the field of view.

PROFESSOR: That's right. So to exaggerate it, suppose that you have an optical system where on access you get a large acceptance angle. But then somehow when you go off axis, the acceptance angle becomes very small. So now you compare this with this. Clearly, this is smaller. That is vignetting.

AUDIENCE: And that would mean-- would that be independent of placing an aperture stop at the focal plane of the lens?

PROFESSOR: What you can say about vignetting is that it generally happens when you place the aperture stop in the wrong place.

AUDIENCE: In the wrong place, OK. OK.

PROFESSOR: When a properly designed optical system, it should not happen, yeah, at least if there's a way to get around it. If you construct the optical system in a way that you cannot place the aperture stop then that's a badly designed optical system. So you generally you try to avoid it by placing your stops in a strategic location.

There's no universal rule that says you have to place the aperture stop there in order to avoid the vignette It kind of depends if you have a multi-element system.

AUDIENCE: [INAUDIBLE] If you always make sure that we place the aperture stop at the effective focal plane, then the vignetting doesn't happen? Because the rays coming at any angle will--

PROFESSOR: That is true for the case of a telescope. I'm not sure if it is true for a general optical system. Yeah, for a telescope where if you place it at the common focal point, then yes, you have-- you minimize vignetting, yeah.

OK. Before we break, let me go over what we will do today. So today we have two items in the agenda, basically. One of them is mirrors. So far with that with spherical refractive dielectric interfaces whenever I said anything about kind of reflective surfaces, like mirrors. So we'll do that very briefly. It is simply all we need is a modification of the sine conventions and we're done. So that's a fairly painless topic.

There's a little bit of terminology here which being Greek-- actually I don't know if being Greek makes sense. These are archaic words. Even in Greek, we don't use them anymore. But a system that uses mirrors is called catoptric, from the Greek-- from the ancient Greek word for mirror.

And then if it uses only refractive elements, you call it dioptic. That's also where the term diopter comes from. And finally, there is a class of optical systems that use both mirrors and refractive lenses that are very popular in two areas in astronomy and lithography. In particular, lithography.

If any of you have dealt with semiconductor lithography, semiconductor industry where they make these huge machines that write on-- write patterns on silicon to make chips and such. For example, [INAUDIBLE] for his project when he does lithography.

Typically, these machines, they contain-- the optics are probably taller than me. OK, I'm not that tall. But anyway, they are taller than a basketball player. And they contain typically between 20 and 30 elements. Some of those are mirrors some of those are refractive lenses.

So this type of system that contains both is called catadioptric. That's a mouthful. But that's what it's called. So there will see some examples of such systems. And we'll see them in the context of the basic imaging systems that is the magnifier lens, the eyepiece which

is basically a magnifier, then the microscope uses actually two magnifiers-- the telescope which is a slightly strange magnifier. And finally, we'll see different types of telescope.

Where is Le? Le?

AUDIENCE: Yeah.

PROFESSOR: This is the answer to your question. While you guys took a break, I drew again a careful diagram with the ray that you asked. So now you see so the red rays are the ray traces for the case where the lens is positive. But the second principal plane is to the left.

And the black lines are the rays that go to the-- that intersect the principal planes on axis. So you have to be a bit careful here. And I've got to take myself actually the first time I did it. That they-- when you depart from here, you have to meet the first principal plane at this center of the axis.

And then you start from the second principal plane. And you're drawing to the image point. And you can see there the two black lines, they do look sort of parallel, don't they?

AUDIENCE: Mm-hm. Yeah.

PROFESSOR: And of course, the image is here.

AUDIENCE: Actually it's a similar thing but I think it's wrong.

PROFESSOR: Yes, they are similar. It's a little bit difficult to see here. It is obvious that they're similar if you do the case of the single lens. Remember this diagram? That's a really bad drawing. But from here you can see that they're similar triangles. Right?

AUDIENCE: Yeah.

PROFESSOR: And then, of course, when you put the principal planes, there's just dead space between the two principal planes. But the geometrical relationships are similar triangles and all that stuff, they get presented except you are dead space. And in this example, it is even worse. The dead space is kind of reversed. Because they moved one principal plane on top of the other. You know what I mean. I think if I attempt to express it, I will confuse the issue.

Let me talk a little bit about mirrors now. So for mirrors, wherever set of the same conventions that is a little bit modified with respect to the same conventions for refraction. And the reason, of course, is that when you have a reflective surface then the rays will indeed fall and they will start going from the right to the left.

So because of that fact, then we really have to modify our sign conventions. And here I put all the cases of positive quantities. So most of these are familiar. The positive curvature is the same as before. These angles and directions here are the same as before.

The really interesting ones, the ones that happen upon the reflection. Because upon reflection, the distance has now become positive if they go to their right not to the left. And that kind of makes intuitive sense. Because now it is as if actually the best way to make sense of this is to simply unfold it.

So if you want to take this-- the part of what happened after reflection, and then found it so it could go back for one direction. Then these quantities would then would indeed remain positive. So this is the origin of the sine conventions for the mirrors.

So for example, why is this angle positive? Well, because if I flip it the other value down it would indeed be the same as a positive angle over here. Of course, this angle, if it had happened in a refractive system, that would have been-- well, it would have been forbidden to begin with because the ray is going from right to left. And that's anathema.

So having said that, I'll come back to this. But having said that, the next thing we would like to do is derive a matrix relationship for mirrors. Now long time ago, I think it was in the first or second lecture or so, we derived the ideal mirror. We said that if you have rays coming from infinity and you want to focus them onto a single point out here, like so, we said that the way you do that is with a surface, reflective surface.

And you remember, probably, that what the surface was. It was a parabola. So we wrote it in equations here. If you call this elevation s, you call this axis x. So we found the equation of the parabola was s equals x squared over 4F, where F is the focal length of the lens.

So the parabola is ideal. But in some cases, people make spherical mirrors. Actually, it is not as common as lenses. Lenses are very commonly spherical. Mirrors are very often actually paraboloidal, very close to ideal, especially mirrors used in concentrators, things like satellite antennas, solar concentrations that are used in solar energy systems and so on.

Then, because they would want to focus the light on a single point then you actually go for the parabola. It turns out, if you want to form a sort of a bigger image here than the parabola is not very good. So people actually make spherical mirrors.

So for a spherical mirror, I did a little bit of a derivation here which basically follows the paraxial approximation to connect the ideal parabolic shape and the focal length that we get from the parabola to the equivalent sphere. And again, the parabola and the sphere, they kind of look the same. If you look at the parabola, maybe it goes like this. If you look at the sphere, and you match the curvatures near the center, it might go like this.

So what we're trying to find here, what is the radius of this sphere here that matches best the parabola, the ideal parabola. So this is the ideal parabola that is given by this equation. And then here's the sphere. That's not a very good looking sphere. But anyway, let's call it a sphere.

And the question is, what is the radius of curvature of this sphere that will match the parabola ideally. So this is what I did here. If you write the equation for the sphere, well, we know-- actually, let me leave this up.

We know how to read the question for the sphere. In my chosen system of coordinates, this is x. And this is the positive axis z. So the equation for the sphere is x plus the radius, squared plus s squared equals the radius squared. So this is the question for this sphere. This is this displacement here, with respect to the center of the coordinates.

And then what do you do, this is [INAUDIBLE] slide but I will actually derive it for you here so you can see it in this sort of animated form. And then what you do is you actually solve, so you have an s equals plus/minus R minus-- what am I doing?

AUDIENCE: So you've got your axis-- your [INAUDIBLE]

PROFESSOR: Oh, I'm sorry. Yes, I did. Didn't I? Yay. I'm sorry about that. I think I better do this over. So this is s for the parabola. This is s for the sphere. So of course it would be s plus R squared plus x squared equals R squared.

So that means that s equals minus R plus minus square root R squared minus x squared. Now which signs would I keep, the plus or the minus? Well, the minus sign, you can see a little bit by inspection, the minus sign is this part of the sphere which is pretty far from the paraxial approximation, clearly, right?

So the only way to deal with a paraxial approximation is to keep the plus sign. So we'll write, then, s equals minus R plus the next step you do to derive paraxial approximations is you pull a large quantity out of the square root. So the large quantities, the radius here, and you write it like this. 1 minus x squared over R squared.

Then you apply a Taylor formula that says that square root of 1 plus small approximately equals 1 plus the small divided by 2. And note, of course, that applies also for a negative sign. So I can do like this.

And this means that s equals minus R plus R 1 minus x square over 2R squared. And if you do it carefully here you see that this kills the square. This kills this. And you end up with s equals minus x squared over 2R. And what this means now is if you compare with equation of the ideal parabola, which was x squared over 4F, it means that F equals minus R over 2.

The next question now is what does this mean. Is the focal length negative or positive? To answer this question, we have to first remember our sign conventions. Is our surface positive or negative? That is, is R a negative or positive quantity?

So if you recall, I'll go back one slide to the previous one. This is a positive curvature. if If it is pointing that way, I will not use any formal term because that may be confusing. If it's like that, it's positive.

This is not like that. It's the opposite. And so therefore here, R is negative. So you have a negative sign that came out of the math. But R is a negative quantity. So therefore F is still positive. So this is still a positive lens.

And it is clear that it is a positive lens, because it forms a real image for an object at infinity. By construction, we demanded that if I have parallel rays from infinity, these things should focus them to a real point on the axis. So therefore, clearly, this is what you would call a positive lens.

And since the lens, it actually has a matrix that is very similar to the matrix of a lens, all you have to do is substitute the focal length with the equation that we just derived for the radius of curvature. And this is the matrix that describes a mirror, a mirror concentrator.

And of course, since it is a lens, it also forms images. So notice here, the way I drew it, these quantities are still all positive. Of course, it's not as positive because I go from the object to the right, towards the instrument.

Now notice what happens. From the instrument, I go to the left towards the image. But that is still a positive quantity because there was a reflection that occurred there. So therefore s sub I is still a positive quantity. So I can write this equation with a clear conscience, so to speak, that s0 and si are all positive qualities.

Now can you imagine when a lens might form a virtual image?

AUDIENCE: [INAUDIBLE]

PROFESSOR: I'm sorry. Yes, a mirror. So clearly the mirror here is forming a real image. Can you imagine how I could construct a mirror forming a virtual image?

AUDIENCE: If the mirror diverges and you are looking from the same side, then.

PROFESSOR: That's right. That's one way. If you make the mirror like this, now the mirror has a positive R therefore it has a negative F. Now this mirror has become the equivalent of a negative lens. Therefore it will form a virtual image.

What's another possibility for forming a virtual image with a mirror? What about a positive lens? Could a positive lens ever form a virtual image?

AUDIENCE: [INAUDIBLE]

PROFESSOR: That's right. If you put the object in between the focal point and the mirror itself, again, it will form a virtual image. That's actually a very funny to observe by yourself. You cannot quite do it with a spoon because the spoon is really too small. You have to go really close. And you cannot focus your eyes in there.

But those of you who've been at hotels-- I've never seen this in a home. But most hotels, in the bathroom they have a curved mirror. And I don't want to speculate why they put that curved mirror, there but anyway they have it. And this mirror is very lightly curved. It has a very long radius of curvature.

So it is very easy, actually, if you stand in front of the mirror, you can move back and forth. And you can switch your image from real to virtual. It is really spectacular. It is worth checking yourself in the Ritz Carlton in Boston spend $400 for a night just to see that, actually. Because it's very, very instructive from the optics point of view.

But real, it's nice, because as you go near the mirror, basically when you get between the focal length and the mirror, then you see an erect image of yourself that is virtual because you see yourself behind the mirror it is magnified. Actually, the magnification is so high that you typically only see your eye. But anyway, something like that.

If you move back, then you see yourself inverted, as you see yourself in a spoon, by the way. If you look at yourself in a spoon you almost always see yourself inverted because the spoon has a very short focal length. So you are always in this scenario when you look at a spoon.

Any questions? There's not much else to say about mirrors really. The same optics applies, with this little twist of the flipping of the signs upon inversion. So if you face a situation, all you need to do is keep cool. Don't let yourself panic or anything. It's just a matter of keeping track of the positive things that happen to the right or to the-- I'm sorry-- to the [INAUDIBLE] things that happen after reflection.

And typically with the kind of problems that we have to solve by hand, these are pretty simple. I mean at most, you might encounter one reflection. Now people who make optical design software, they have to deal with arbitrary numbers of reflections. I wouldn't want to be the programmer who has to solve that problem.

Because you know, if you have a second reflection, then the signs flip again. As you can imagine, if I stick another mirror here. So the light starts sort of zig zagging, every reflection flips the signs in the sign convention again. So it can get a bit tricky to keep track of all of these flippings. But we don't have to worry about that at this introductory level.

So let me move onto the next topic, which is a few examples of optical systems.

AUDIENCE: Hey, George.

PROFESSOR: Yes?

AUDIENCE: Sorry to interrupt. I'm going to pass around these parabolic reflecters so you can try that.

PROFESSOR: Good idea. So maybe you don't have to go to a hotel after all. [INAUDIBLE] What is the focal length of this, you know?

AUDIENCE: It's like 20 centimeters or so. You can check it out.

PROFESSOR: Great. So yeah, you try to go at 40 and 10 or whatever, and you will see the difference. It's actually quite dramatic and very interesting.

So the magnifier, as the name suggests, it is a device that we use to aid our eyes when we're looking at a very small object. And this is a-- dog it took me a lot of time to make this cartoon. So I'm very proud of it. So let me start the animation here.

So if you have a really small object, of course it will form a very small image on your retina. And very often, this image is not big enough to observe with sufficient detail. So the obvious thing that you would do if something looks small to you and you'd like to make it bigger, you don't have to know any optics. What you do is you bring it near your eyes.

But from experience, you know that this doesn't work very well. Because if you exceed the distance of approximately 25 centimeters, then you can really not focus anymore. And actually for you guys who are young it's about 25 centimeters. As we grow older, this distance becomes longer and longer. And we basically lose our ability to focus, which is a very annoying, annoying thing actually.

It has just started to happen to me. As I approach age 40, I've started noticing that I cannot focus as well as I used to. So it's a little bit annoying. But anyway it beats the alternative I guess. So I'm very happy that I reached that age. But anyway, the point is that there comes a point-- actually for me now, the only way to focus is to take off my glasses.

But even so, there comes a point where you can not focus anymore. Even you cannot focus. I bring something to 5 or 10 centimeters, you cannot focus. The question is, what could you do. So what you do is you use a magnifier.

And the magnifier is something that you put between the object and your eye. And the way it works is like this. We can do some simple ray tracing here. This is where the parameters of the magnifier, let's say it is a positive lens, the magnifier. Let's say the focal length is F and the power is the inverse of that.

So this is some ray tracing that shows how the image is formed by this lens. So all you do is you take a horizontal ray. This will go somehow through-- this guy will bend it. And let's pretend now that this ray, somehow I tune things so that this ray went through the center of the eye. So this ray went and bend, right?

But you can see very clearly now because what these lines did is it bent this ray bundle so that it meets the object over here. The result is that I got a much magnified image on the retina. That's about it. That's all there is to. By using this element, you can get bigger images.

Now if I didn't have the eye here, this is actually the most instructive thing I'm going to say today. This is really the meaning of the virtual image. If I give you this system by itself, obviously because this lens, if you look carefully, it created a divergent ray bundle. Obviously this object is between the focal point of the lens.

So therefore, this is the situation that we encountered before. And we've said that the system forms a virtual image. Well what is the virtual image? I have to take these rays that the lens produced and I have to extend them backwards.

And with a little bit of manipulation here, because this is a cartoon, it is not a real calculation. But with some manipulation, I made sure that the rays do meet. So the rays meet. And the point where the rays meet is the virtual image.

There's nothing there. Nothing. The object is here. The only physical entity is here and your eye, and the lens of course. There's nothing here. But when your eye forms an image it is as if you could take this away-- you could take away the combination of the object and the magnifier and replace it with a virtual image that this system formed.

This virtual element is acting as an object for your eye. The eye images this object. And because this object has been magnified relative to the actual object, you end up observing a magnified image.

So this is the significance of virtual images. By themselves, that don't mean anything. But because at the end of an optical system you always have the observer's-- I mean, the observer's eye length and the retina, the virtual image ends up becoming a real image, hopefully on the retina. If it becomes a real image elsewhere, you will not be able to see it. It will be out to focus.

Or even worse, if you create a virtual image and you send it to an observer so it becomes a virtual image on the retina, again you don't see anything there. Because you can never observe a virtual image unless you can image it using the optics of your eye. So this is the significance then. And we'll see in the second that everything-- microscopes, telescopes, magnifiers-- they all form of virtual images which eventually the observer's eye lens converts to a real image on the retina.

The next thing that we'll do now is we'll put some math around this topic. And this is a little bit unfortunate because the book is generally very good, the textbook that we have, Hecht. I do think that in this particular topic, the textbook is a little bit confusing.

So I'll try to walk you through the derivation in the textbook and hopefully I will make it less confusing. But if, when you go back home and you read the textbook, you confused again, please bring your questions on Monday so we can talk about this again.

So I tried to make it less confusing but still I'm using the notation in the book. So you don't-- in addition to the confusion in the book, you don't have to overcome notational confusion. So the definition of the magnifying power which cannot be argued against because it is a definition is it is the ratio of the angle subtended by the tip of the object before and after magnification.

Actually the subscript here, U and A, U stands for Unaided, that is the eye without the magnifier. And A stands for Aided. That is the eye with the aid of the magnifier lens. And now we need to place symbols to all of these distances here. And I think this is where the book is a bit confusing. Because it tries to do everything at once.

But anyway, so I need this symbol for the distance between the object of the lens. Then I need the symbol from the lens to the eye. And in the symbol from the lens to the virtual image, which is of course si, and in this case you would expect the side to turn out to be a negative number, because it is a virtual image. And finally you use the symbol L for the distance between the virtual image and the eye. So L is, of course, that simply the sum of these two distances with a proper sign convention.

The first thing to note is that you don't really need all these notations because the magnifying power is very simple to calculate. As we did in the past, all you have to do is use similar triangles. Because you can see that the magnifying power-- the angle a sub a, the [INAUDIBLE] angle, equals the ratio of the size of the virtual image over what? Look at the red line. From this ray angle, over the big distance L.

And of course there's a tangent. I should have a tangent there. But I dropped the tangent because of the paraxial approximation. Similarly for the unaided eye, it equals the ratio of the natural height of the object to the distance between the object and the lens.

Now for the unaided case, the convention is to use the near point of the eye, that is the 25 centimeter distance beyond which the eye cannot focus. Because presumably, if you wanted to get the maximum possible unaided magnification, you really have to bring the object within this 25 centimeters. So that's why we use d0. So that's a standard actually. It is 25 centimeters.

And when you play around with this, you discover that it equals the ratio of these two distances, which is easy to do. And this is what we used to call-- what did we call this quantity when we did the imaging condition? Button.

AUDIENCE: Lateral magnification.

PROFESSOR: That's right. This is what we used to call the lateral magnification. It is the ratio of the size of the image over the size of the object. So this is what they say here. And of course this is the lateral magnification. We have the questions for it in terms of the image and distances. And we can substitute these equations and we can actually derive a number of equivalent expressions for the magnifying power.

So now, none of this is particularly intuitive. And then the book goes ahead and gives two possible uses of the magnifying lens that are not particularly intuitive either. So I will not even bother to deal with them. One of them is when you put the magnifier one focal distance away. I don't know if it's necessary to do that.

The other is when you put the magnifier-- you stick it to your eye, which is not a very good idea because there's all kinds of germs around so you don't want to stick magnifiers onto your body. The way we use magnifiers in general is the number three description of the book, which is why we place the object at one focal point behind the magnifier.

So if the magnifying focal length is 5 centimeters-- actually, that's a pretty long. But anyway, you place it 5 centimeters from the object. What do you accomplish this way? What happens if you place an object at one focal distance in front of the lens. That's right. You form an image at infinity. Actually, in that case, you form a real image with infinity.

Now is that good or bad? In the context here, you form the image with infinity and you still have an observer with his or her eye lens and the retina. Let me draw this. Now we draw the specific case, according to the book lingo, this is case number three, si equals F.

So what does it look like? Here's my magnifier. Here is my eye lens. And of course the eye kind of looks like this. And this is the retina. And here is the optical axis. And here is the object.

So let's look at the tip of the object. Since it is one focal distance away from the lens I will get a parallel ray bundle. Is that good or bad?

AUDIENCE: It's s0 equals F, right?

PROFESSOR: Yes. Thank you. So is that good or bad?

AUDIENCE: If we talk of aberrations, it's good.

PROFESSOR: Yeah. Let's not go into aberrations. In our simple geometrical optics, is that situation happy? Are we happy that we did this? We're happy for a number of reasons.

AUDIENCE: Yes. Because it will perfectly focus at the point.

PROFESSOR: That's right. First of all, this is assumed to be a corrected eye. So if it was me, that would be me wearing my eyeglasses. If it is me without my eyeglasses, I have a problem. You'll see in a second what problem.

But if it is a healthy eyesight where the eye can focus perfectly, then this indeed will form an image on the retina. What is the other good news about this situation? The fact that the image is at infinity, is this good news or bad news?

AUDIENCE: Your eyes remain-- your eye remains unaccommodated.

PROFESSOR: That's right. We learned that the eye is most relaxed when it observes remote objects at infinity. The reason it cannot focus closer is because I have to strain the lens of my eye. And as I get older, my ability to strain it diminishes.

But this is true for everybody, young or old. When you try to look at something close by, you get tired. If you read a book like this, your parents probably told you-- they told me. Don't read the book like this, you get the headache. And indeed you do. If you read for a long time you get a headache from very close.

So the reason this is a happy situation is because the eye can be totally relaxed as if it was observing something very far away and still see the object with magnification. And how much is the magnification? Well, we'll skip the math. The math is fairly simple here. But it turns out to be simply the product of the near point, the 25 centimeters, times their power of the lens.

And the only penalty you pay, the reason, actually, the book covers this case over here is because this is not the maximum magnification you can get with a magnifier. You can get slightly higher if you stick the magnifier next to your eye. But you never stick the magnifier to your eye, actually. Unless you're adventurous. If I did this, I would damage my glasses.

So the message, perhaps, important. But what I really hope that you get out of this is the significance of the virtual image and why the magnifier functions by forming a virtual image that is erect and magnified. And this also tells you why you will use a positive lens as a magnifier.

If you recall, it is actually one of the previous slides. A negative lens also forms a virtual image. But unfortunately, it is not magnified. The magnification, it is de-magnified. The lateral magnification is less than 1. So we would be out of luck in this case.

The next element is actually the same. It is no different than the magnifier. But it comes under the name eyepiece because in optical instruments such as binoculars, telescopes, microscopes, and the like this is the last lens that you look into the instrument with your naked eye. Now I realize that this is old fashioned talk. Modern instruments, you seldom look at with a naked eye. You actually have a camera. So you put a digital camera and you register the image digitally.

However, even a digital camera functions like a retina. So in other words, even a digital camera would have an eyepiece and then a condenser. And then it would go onto the chip. So really the situation here applies to digital imaging as well as human, as well as instruments meant to be used by humans.

So the eyepiece is basically the last element in the instrument, just before you get to the actual observation stage. And because the eyepieces were traditionally designed for the humans, they are typically designed like a magnifier with the same principle that I described before that forms an image of infinity.

So therefore the observer, when he or she looks into the eyepiece, they can use an unaccommodated eye and observe the magnified image without having to strain their eyes. So this is the sort of standard use of an eyepiece.

And there's a second condition for the eyepiece which will become a little bit more apparent in the next slide when I talk about the microscope. It is this one over here. This looks a little bit cryptic. It says the center of the exit pupil eyepoint where the observer's eye is placed at 10 millimeters from the instrument. This doesn't mean much. But I will describe what this means in a second, when I talk about the microscope. Not withstanding this last point over here, any questions about the eyepiece or the magnifier or anything of that sort?

AUDIENCE: Why is the eye [INAUDIBLE] 10 millimeter when we cannot see?

PROFESSOR: OK. What this really means is that 10 millimeters is the design distance that you're supposed to place your eyes with respect to the instrument. When you look at the microscope, for example, most people don't do that. But you're supposed to be at about 10 centimeters from the last surface, right? So that is called the eye relief.

And what this is really saying is that when you look at this, you're not supposed to see any blockage of the image. And you're not supposed to see vignetting. There's two things that can happen. If you place your eye at the wrong point-- as you know, if any of you have looked at a microscope, if you are not positioned properly you cannot see the entire field of view. That's one problem.

You see a black hole. And you kind of have to move around and you can never see anything until you go to the proper place and then finally you see everything. So this is the eye relief. This is by design where you should place your eye.

The other thing that can happen if you place your eye at the wrong place or if the microscope is not properly designed is vignetting. So this requirement clearly assures that there's no vignetting when your eye is placed at eye relief distance, that is at 10 millimeters from the instrument. And of course, you don't have to worry about focusing because the microscope is supposed to have infinite conjugate, right? So you just use with an accommodated eye.

And I suspect, I don't know if it's true, but I suspect-- there are many reasons why they put over there. But also, it's kind of convenient that when you look at the microscope you're so close to the microscope tube itself that you cannot focus and you cannot see that. So you can basically see what you're supposed to observe but you don't see the actual instrument because it is out of focus. So that's kind of convenient.

So [INAUDIBLE] this about the microscope. Here is a microscope. Here's what it looks like. It is deceptively simple. Because there's, of course, a lot of engineering design that goes into making all of these elements. But the principle is surprisingly simple. It consists of basically two magnifiers. One of them is called objective. The other is called eyepiece. But they both function as magnifiers.

So the job of the objective, this is the first lens. It is called objective because it is placed near the object. And its job is basically to form an intermediate image that is magnified to begin with. What is it magnified by? Well, here we simply have-- actually this is a real image. So we simply have a positive lens forming a real image. We can compute the magnification. It is equal to minus si over s0, just like the regular formula.

And then we have the eyepiece for which the image that the objective produced actually now acts as the object. So this intermediate image is re-imaged by the eyepiece. But the eyepiece is placed one focal distance. F sub e here is the focal length of the eyepiece. It is one focal distance from that intermediate image. And therefore the final image is formed at infinity.

Now the way this is drawn for this one, what is prominent in this picture is, of course, the on-axis ray. And there's no magnification to talk about because this is just an axis. Nothing interesting is happening. What is really interesting is if you look at an off-axis point, which is at the bottom here. It's a little bit difficult to see. It looks better in the book actually.

So if you trace this off-axis point, then you find that its final image is this ray bundle that is coming off-axis into your eye, this parallel ray. That is the final image of the microscope produced of this off-axis objective point. You can see it has entered the eye at a very large angle. Therefore it will form a big image. So that's how the microscope magnifies small objects so that you can observe them.

So then the magnification of the microscope, very similar to the case of the objective, is defined as the ratio of this angle, the angle that the microscope produced for the off-axis object, for the same angle that the eye would observe unaided, in a very similar case as the magnifier.

So in order to compute this now, it's not very difficult to do. We need two elements. One is the lateral magnification of the objective. Because recall, the objective actually forms a real image. So we have one. Here is the object. And here is its image.

So we need, first of all, to find out where the intermediate image appears with respect to the optical axis. That's the lateral magnification of the objective. That is whatever. Well, I will say in the second why this 160 appears.

But the second element that we need is the angular magnification of the eyepiece. Because this will allow us to say how this distance over here translates into angle that goes into the eye finally.

So this one is actually a familiar formula. This is the same formula you had for the magnifier. So go back one. We said that-- actually, it is over here already. We said that the magnifying power of an eyepiece or a magnifier equals the near distance for the eye, which is about 25 centimeters, times the power of the lens, of the positive lens that they use. So this is the same formula.

In microscopes, they don't use exactly 25 centimeters. They use 25.4 centimeters. So this is the standard. And how much is that in inches? Is it 10 inches? No. 11, 12 inches. 10 inches. I thought it would be 22-- oh, that's pounds. I'm sorry. That's pounds. There's 10 inches then.

So this is, then, the same formula that we had for the magnifier. Where did this come from? Well, if you remember one of the ways to add the lateral magnification is like this-- 1 minus si over F. So you can write this as 1 minus si over F equals F minus si over F. And now I have to-- Ah, that worked.

So minus si-- I am sorry. It is F minus si over F. So F minus si is actually the distance between the objective lens and the-- it's basically this distance L. That's what I'm trying to get to. So this is known as the tube length of the microscope. It is the distance between the objective lens and the location where you put the eyepiece.

Of course, the eyepiece is this whole thing. So that's where you stick the eyepiece. So this is standard. In microscopes, it is 160 millimeters and is known as the tube length. And this is what appears in this formula over here. So basically, you can see that if you pick the focal length to be sufficiently small, then you can get very high magnification in the order of hundreds, maybe even 1,000.

Let me ask two questions now. First of all, what I just said. What do you think might stop me from getting a focal length that is, let's say, 1 micron? I pick a focal length to be 1 micron, and then I have-- these are all in millimeters. So 160 millimeters over 1 micron, that would be what? That would be about 160,000. And then another micron here, I would get a magnification in the order of 1 billion. What stops me from doing that?

First of all, just to get back to, what is it from your experience? What is a typical magnification that you get in microscopes? How about 1,000 orders of magnitude? But if you apply this formula here, you could easily get a magnification of 1 billion by picking the focal length to be 1 micron. What is wrong with my statement? Clearly, there's wrong. Otherwise, people would be doing it.

AUDIENCE: Is it because you can't see past the wave length of light? That 1 micron is the most you could focus because that's the--

PROFESSOR: That is in the correct direction, yes. The other possible reason is the failure of the paraxial approximation. But the paraxial approximation may fail all at once. But I can always do something more sophisticated to do non-paraxial calculations.

Maybe you're saying that if the paraxial approximation fails, then this formula fails as well. So then you don't get as good magnification. I think you guys both captured-- the two of you captured the answer in its entirety You have another one?

AUDIENCE: I don't know if it's another. To make such a high-- such a short focal length lens we'll have to make a very big lens.

PROFESSOR: That's right. So you have to be very highly curved. Remember the focal length of the lens is proportional to the radius of curvature. So you would have to make a lens with a radius of curvature equal to 1 micron. And that's when you get into problems like the-- what's your name again? [INAUDIBLE] answered the question before.

AUDIENCE: Dean.

PROFESSOR: Yeah. This is what Dean answered. If you try to make a curvature that is as sharp as 1 micron, then basically you get into the range of very strong diffraction of light. So basically all these geometrical optics that we do here, they fail. In fact, you discover if you did make such a very small element that the light does not really focus. We'll see what it does. But it does not propagate.

So reasonable focal lengths are in the range of 1 centimeter, a few millimeters. These are typical microscope objective focal length. And the eyepieces are similar. And that's why you get magnification on the order of 1,000.

The next topic is the telescope. But I think I've exhausted your endurance.