

1. **Grating with tilted plane wave illumination** Consider a sinusoidal phase grating of the surface relief type with complex amplitude transmission function

$$g_t(x) = \exp \left\{ i \frac{m}{2} \sin \left( 2\pi \frac{x}{\Lambda} \right) \right\}.$$

The grating is placed at the plane  $z = 0$  and illuminated by an off-axis plane wave

$$g_-(x, z = 0) = \exp \left\{ i 2\pi \frac{x}{\lambda} \sin \theta + i 2\pi \frac{z}{\lambda} \cos \theta \right\} \Big|_{z=0} \approx \exp \left\{ i 2\pi \frac{\theta x}{\lambda} \right\}$$

propagating at angle  $\theta \ll 1$  with respect to the optical axis  $z$ .

- 1.a) Describe, in as much detail as possible, the Fresnel diffraction pattern  $g_+(x, z = 0)$ .
- 1.b) Describe, in as much detail as possible, the Fraunhofer diffraction pattern.
- 1.c) Compare with the on-axis illuminated phase grating that we analyzed in class.

2. **Grating with spherical wave illumination** Consider a sinusoidal amplitude grating with complex amplitude transmission function

$$g_t(x) = \frac{1}{2} \left[ 1 + m \cos \left( 2\pi \frac{x}{\Lambda} \right) \right].$$

The grating is placed at the plane  $z = 0$  and illuminated by a spherical wave originating on-axis at location  $z = -z_0$  (where  $z_0 > 0$ .) Using the paraxial approximation for the spherical wave,

$$g_-(x, z = 0) = \frac{e^{i 2\pi \frac{(z+z_0)}{\lambda}}}{i \lambda (z + z_0)} \exp \left\{ i \pi \frac{x^2 + y^2}{\lambda (z + z_0)} \right\} \Big|_{z=0},$$

- 2.a) derive an expression for the resulting Fresnel diffraction pattern  $g_+(x, z = 0)$ ;
- 2.b) argue that the expression that you derived physically represents a coherent superposition of three spherical waves;
- 2.c) derive the  $(x, z)$  coordinates of the origins of these spherical waves;

- 2.d) explain how this diffraction pattern would change if the illumination were to move off-axis by a distance  $x_0$ , *i.e.*

$$g_-(x, z = 0) = \frac{e^{i2\pi \frac{(z+z_0)}{\lambda}}}{i\lambda(z+z_0)} \exp \left\{ i\pi \frac{(x-x_0)^2 + y^2}{\lambda(z+z_0)} \right\}.$$

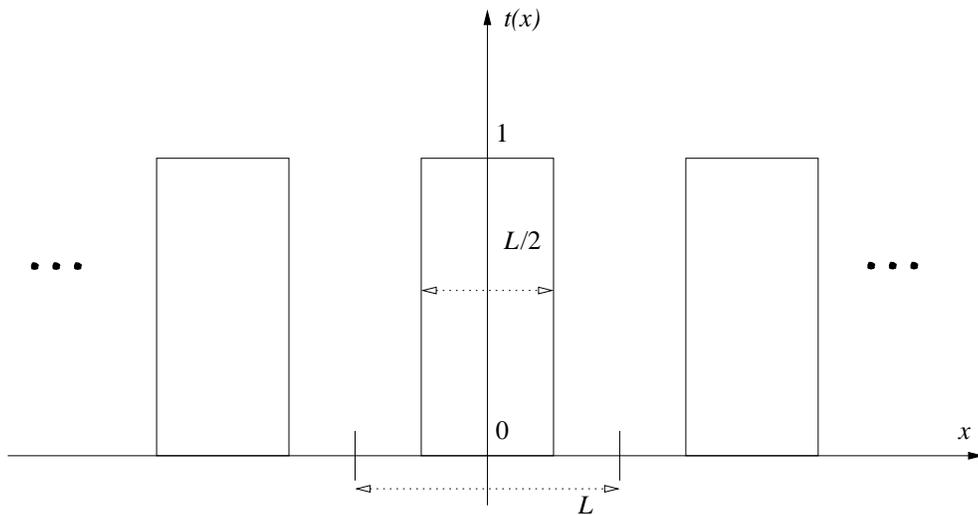
3. Consider the one-dimensional periodic function shown below. In the field of Optics, this is often referred to as a “Binary amplitude grating” of infinite extent, with duty cycle 50%.

- 3.a) Calculate the Fourier series coefficients of the infinite Binary amplitude grating in closed form. Plot the Fourier series coefficients in the spatial frequency domain; use “sticks” located at the harmonic frequencies and with height equal to the magnitude of the corresponding coefficient.

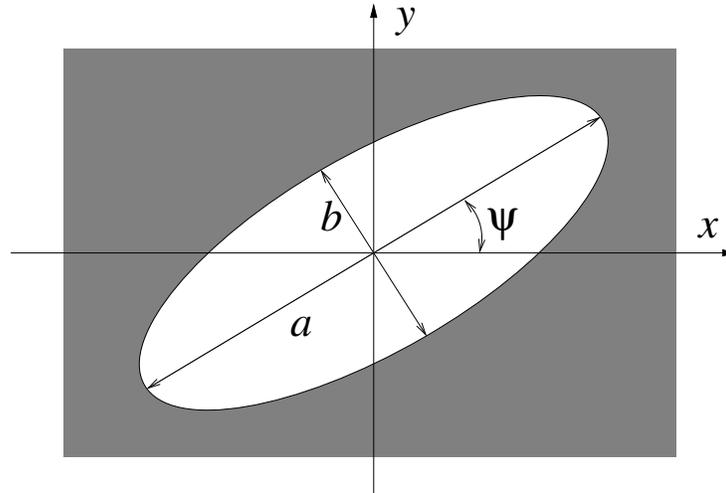
- 3.b) Write down the Fourier transform of a single boxcar, *i.e.* a single period of the Binary amplitude grating. Plot the magnitude of the Fourier transform of the boxcar as function of spatial frequency.

- 3.c) Now consider a “truncated” Binary amplitude grating that consists of a finite number  $N$  of boxcars (“grooves”), where  $N > 1$  is an integer. Use the convolution theorem to compute the Fourier transform of the truncated Binary grating.

- 3.d) Using the above results, express analytically and sketch *by hand*, without using MATLAB or other software, the Fraunhofer diffraction patterns of the infinite Binary amplitude grating, the truncated Binary amplitude grating, and the single boxcar. Comment on the similarities and differences between the three diffraction patterns.



4. **Tilted ellipse.** Calculate analytically the Fourier transform of the tilted elliptical aperture shown below, whose transmission has value one inside the tilted ellipse and zero outside. The ellipse major and minor axes are  $a$ ,  $b$ , respectively, and the major axis is tilted by  $\psi$  with respect to the  $x$  axis.



- 4.a) Sketch *by hand*, without using MATLAB or other software, the magnitude of the Fourier transform that you derived.
- 4.b) Express analytically and sketch the Fraunhofer diffraction pattern that would result from an aperture shaped like the given ellipse.

*Hint:* First express the ellipse as an anisotropically stretched and rotated circle; then use the scaling theorem of Fourier transforms.

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