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3.22 MECHANICAL PROPERTIES OF MATERIALS
PROBLEM SET 8 SOLUTIONS

1. *(Dowling 15.5) Consider metals, and polymers. Which of these classes of materials generally exhibits strong recovery of creep strain after unloading, and which do not? Briefly explain in terms of the physical mechanisms of creep why the strains are generally recovered, or why they are not recovered, for both classes of materials.*

   Strong recovery of creep strain after unloading is generally observed in polymers, but not in metals.

   Polymers generally exhibit strong recovery of creep because the stretched polymer chains will recoil to make prior creep deformation disappear with time (recovery). Creep in metals is generally not recoverable because the creep deformations are plastic and, therefore, permanent.

2. *A 20 kg screw made from a high-strength steel (E = 210 GPa, σ_o = 200 MPa) holds two rigid plates together at 300°C. The design states that the screw be tightened to a preload of 100 MPa, which may not decrease by more than 10% while in service. To determine the maximum service life of the screw, a creep test at 300°C with a stress of 50 MPa is performed and yields a strain rate of 9.5×10^{-15} s^{-1}.*
   a. *Assuming that the stresses in the screw are high enough that power law creep is the dominant mechanism of deformation (n = 4), what is the maximum service life of the screw based on the conditions given above?*
   (Hint: The rigid plates prevent any tensile strain in the bolt.)

   If power law creep is the dominant mechanism of deformation, the equation the relates strain rate to stress is given by

$$\dot{\varepsilon} = \dot{\varepsilon}_\circ \left(\frac{\sigma}{\sigma_\circ}\right)^n .$$

   From the data acquired in the creep test, the constants can be solved for.

$$\dot{\varepsilon} = \dot{\varepsilon}_\circ \left(\frac{\sigma}{\sigma_\circ}\right)^n = \frac{\dot{\varepsilon}_\circ}{\sigma_\circ^n}\sigma^n \rightarrow \frac{\dot{\varepsilon}_\circ}{\sigma_\circ^n} = \frac{\dot{\varepsilon}}{\sigma^n} = \frac{9.5\times10^{-15}\, s^{-1}}{(50\,\text{MPa})^4} = 1.52\times10^{-21}\ {}^{1}\!/_{s\cdot\text{MPa}^4}$$

   The strain rate of the bolt is combination of elastic strain and creep strain.

$$\dot{\varepsilon} = \dot{\varepsilon}_{elastic} + \dot{\varepsilon}_{creep} = \frac{\dot{\sigma}}{E} + \left(1.52\times10^{-21}\ {}^{1}\!/_{s\cdot\text{MPa}^4}\right)\sigma^4$$

   The condition that the screw holds two rigid plates requires that the total strain rate of the bolt be zero. So, the expression above becomes

$$\frac{\dot{\sigma}}{E} = -\left(1.5\times10^{-21}\ {}^{1}\!/_{s\cdot\text{MPa}^4}\right)\sigma^4 \rightarrow \frac{d\sigma}{dt}\frac{1}{\sigma^4} = -E\left(1.5\times10^{-21}\ {}^{1}\!/_{s\cdot\text{MPa}^4}\right)$$

   Integrating the above expression from the initial stress to the final acceptable stress yields the value for the service life of the bolt.

$$\int dt = \frac{1}{-E\left(1.52\times10^{-21}\ \frac{1}{s\cdot \text{MPa}^4}\right)}\int \frac{1}{\sigma^4}d\sigma$$

$$t = \frac{1}{-E\left(1.52\times10^{-21}\ \frac{1}{s\cdot \text{MPa}^4}\right)}\frac{-1}{3\cdot\sigma^3}\Bigg|_{100\,\text{MPa}}^{90\,\text{MPa}}$$

$$t = \frac{1}{-(210{,}000\text{MPa})\left(1.52\times10^{-21}\ \frac{1}{s\cdot \text{MPa}^4}\right)}\left(\frac{-1}{3\cdot 90^3}-\frac{-1}{3\cdot 100^3}\right)$$

$$t = \boxed{3.88\times10^8\ \text{sec}}$$

Based on the above considerations, the maximum life of the bolt is over 12 years.

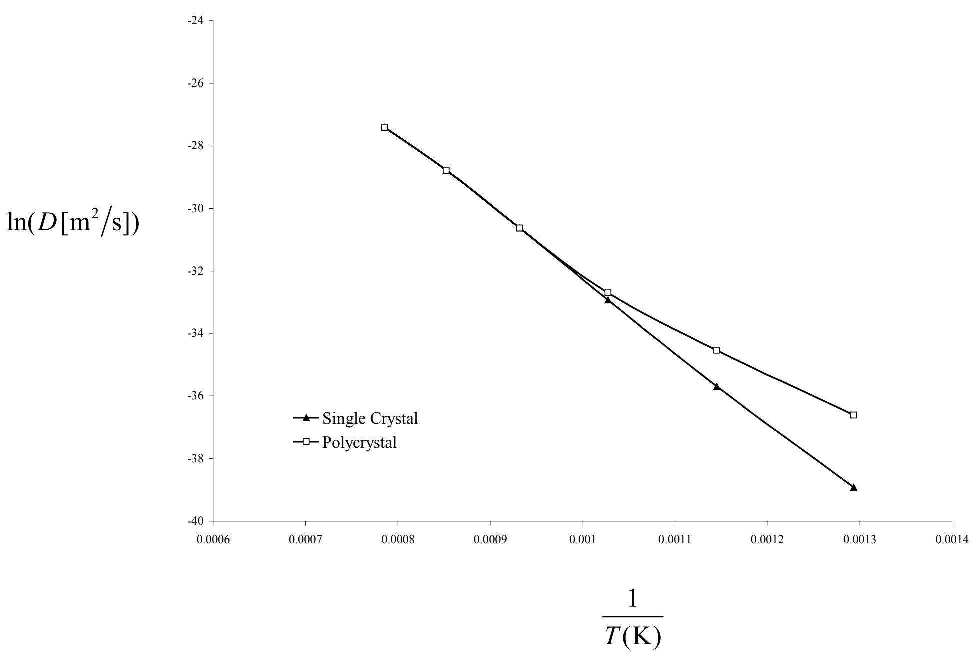
b. *If the temperature increases dramatically (1000°C), can we use the data above to estimate how long the screw will maintain the designed preload? Explain.*

The data above is not sufficient to determine the service life of the bolt. The data is only applicable for systems that operate at the test temperature of 300°C. Another creep test at 1000°C would be necessary to determine the relation for power law creep at this temperature.

3. Below is a table of self-diffusion values for silver single crystals and polycrystals over a range of temperatures.

Temperature (K)	$D_{\text{Single Crystal}}$ (m^2s^{-1})	$D_{\text{Polycrystal}}$ (m^2s^{-1})
773	1.26×10^{-17}	1.26×10^{-16}
873	3.16×10^{-16}	1.00×10^{-15}
973	5.00×10^{-15}	6.31×10^{-15}
1073	5.00×10^{-14}	5.00×10^{-14}
1173	3.16×10^{-13}	3.16×10^{-13}
1273	1.26×10^{-12}	1.26×10^{-12}

a. *Make an Arrhenius plot of the diffusion rate as a function of temperature.*



b. Calculate the activation energy for diffusion in a single crystal of silver.

The temperature dependence of diffusion is given by the equation

$$D = Ae^{-\frac{E_A}{kT}},$$

where A is a constant, E_A is the activation energy and k is the Boltzmann constant. Taking the natural logarithm of each side yields

$$\ln D = -\frac{E_A}{k}\frac{1}{T} + \ln A .$$

From this equation, we see that the activation energy can be determined from the slope of the line, m = –22920 K, in the Arrhenius plot.

$$E_A = -k \cdot m = -(1.38 \times 10^{-23} \frac{\text{J}}{\text{atom K}}) \cdot (-22920K) = \boxed{190 \frac{\text{kJ}}{\text{mol}}}$$

c. Comment on the differences in diffusion rate between single crystalline and polycrystalline silver.

Single crystal diffusion involves no grain boundaries, so diffusion must occur in the lattice only. Polycrystalline diffusion involves both lattice diffusion and diffusion along grain boundaries. At low temperatures, where the activation energy for lattice diffusion is much greater than the available thermal energy, grain boundary diffusion can still occur, which is why the polycrystalline sample has a higher diffusion rate at low temperatures. At high temperatures, thermal energy allows lattice diffusion to occur. Because lattice sites are much more numerous than the grain boundary sites, it is logical that diffusion is equal in polycrystalline and single crystal materials.

4. Consider a pure nickel component with a mean grain diameter of 0.1 mm. If the component is subjected to a shear stress of τ = 100 MPa and creep deformation must not exceed 10^{-8} s^{-1}, what is the maximum operating temperature? If the safety factor is increased such that creep deformation cannot exceed 10^{-10} s^{-1}, what is the maximum operating temperature?

The operating temperatures can be determined from the deformation-mechanism map for pure nickel with mean grain diameter of 0.1 mm. For a creep rate not to exceed 10^{-8} s^{-1}, the operating temperature must not be greater than ~300°C. For the increased safety factor, the maximum operating temperature is ~225°C.

5. Turbine engines in jets can reach temperatures of 1300 K. At high temperatures, creep due to grain boundary sliding must be prevented. How is grain boundary sliding prevented in turbine blades rotating at 10,000 rpm?

Turbine blades are single crystals. This means that the entire blade is one grain, thereby eliminating the possibility of grain boundary sliding. If turbine blades were made from a polycrystalline material, the blades would creep significantly until catastrophic failure when the blades deformed enough come in contact with the outer rim of the engine.