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3.22 MECHANICAL PROPERTIES OF MATERIALS
PROBLEM SET 1

Due in 8 days from its assigned date

Reading

Nye, J.F., 2000, *Physical Properties of Crystals* (Clarendon Press). Chapter I, section 1 and Chapter V.
Ashby, M.F., 1982, *Tensors: Notes and Problems* (Course Notes). Sections 1 – 3.

1. For each of the following expressions, identify the free indices and the dummy index pairs, and determine the number of expanded equations represented and how many terms each expanded equation will have.

a. $j_i = \sigma_{ij} E_j$

b. $P_i = d_{ijk} \sigma_{jk}$

c. $T'_{ij} = a_{ik} a_{jl} T_{kl}$

d. $\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}$

e. $p_i = 5$

2. Evaluate the following expressions involving the Kronecker delta δ_{ij}, the alternating tensor e_{ijk} and an arbitrary second rank tensor T_{ij}.

a. δ_{ii}

b. $\delta_{ij}\delta_{ik}\delta_{jk}$

c. $\delta_{ik}\delta_{kj}$

d. $\delta_{ik} T_{kj}$

e. $\delta_{jk} e_{ijk}$

3. Write the expanded form of the following expressions.

a. T_{ii}

b. ε_{ij}

c. $\sigma_{ij,j}$

4. Define matter tensors and field tensors and explain how they each depend on crystal structure. Give one example of each type of tensor.

5. Show that the stress tensor is symmetric ($\sigma_{ij} = \sigma_{ji}$) in the absence of body forces and accelerations. Consider a three-dimensional infinitesimally small material element and assume that stress can vary linearly along the element. (This is sometimes referred to as the *theorem of conjugate shear stresses*.)

6. Given the stress tensor below, answer the following questions.

$$\sigma_{ij} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 7 & -3\sqrt{3} \\ 0 & -3\sqrt{3} & 13 \end{bmatrix} \text{MPa}$$

a. Calculate the normal traction on the plane defined by the normal $n_i = 1/\sqrt{2}\,\hat{x}_1 + 1/\sqrt{2}\,\hat{x}_2$.

b. Calculate the three invariants of stress (I_1, I_2, I_3).

c. Calculate the principle stresses ($\sigma_1, \sigma_2, \sigma_3$). Show work.

d. Write a new stress tensor σ'_{ij} from the principle stresses (part b) in the form given below, using the convention that $\sigma_1 > \sigma_2 > \sigma_3$. Calculate the invariants of stress tensor σ'_{ij}.

$$\sigma'_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \text{MPa}$$

7. Define the terms *homogeneous* and *isotropic* in a continuous medium at the continuum level. Does one necessarily imply the other? Explain.

8. Use the given deformed element to answer the questions.

a. Write an expression for displacement ($u_1(x_1,x_2)$ and $u_2\,(x_1,x_2)$) in both directions, x_1 and x_2.

b. Determine the strain matrix for this 2D example.

$$\begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix}$$

c. Determine the rotational part of the deformation matrix.

$$\begin{bmatrix} 0 & \varpi_{12} \\ \varpi_{21} & 0 \end{bmatrix}$$