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3.22 MECHANICAL PROPERTIES OF MATERIALS
PROBLEM SET 4

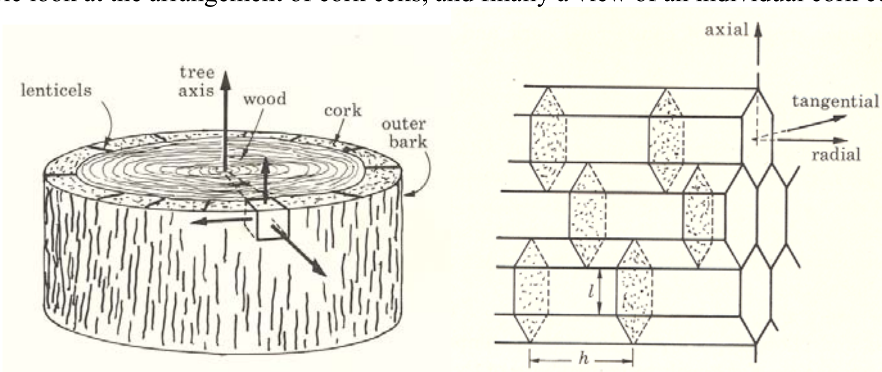
Due in 8 days from its assigned date

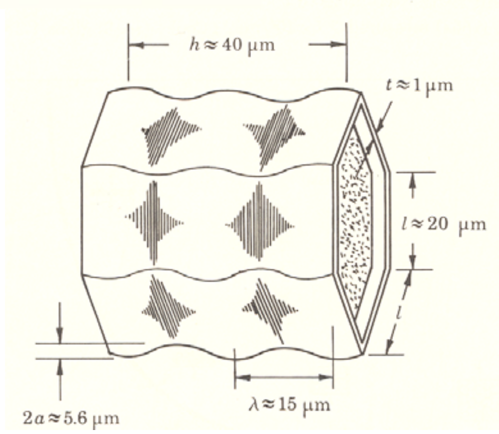
Reading
Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials* (John Wiley & Sons, Inc.)
Chapter I, sections 1 – 2.

1. Consider a 500 nm thick aluminum film on a 500 μm thick silicon wafer, 200 mm in diameter. After deposition at 60°C, the wafer is cooled to room temperature. What is the radius of curvature of the film? Is the film in tension or compression? (E_{Si} = 150 GPa, ν_{Si} = 0.17, $\alpha_{Si} = 3\times10^{-6}\,^{\circ}C^{-1}$, E_{Al} = 69 GPa, ν_{Al} = 0.33, $\alpha_{Al} = 23\times10^{-6}\,^{\circ}C^{-1}$)

2. A bilyer is composed of a 1 mm thick coating ($E = 350$ GPa, $\alpha = 9\times10^{-6}\ ^{\circ}C^{-1}$) on a 5 mm substrate ($E = 70$ GPa, $\alpha = 23\times10^{-6}\ ^{\circ}C^{-1}$) and has a width of 5 mm. The bilayer has zero curvature at the initial temperature of 20°C. The bilayer is then heated to 100°C. Calculate the resulting curvature and the maximum tensile and compressive stresses. Where do the maximum internal stresses occur?

3. The structure and mechanics of cork: The structure and properties of cork are approximately axisymmetric. The schematics below illustrate a macroscopic view of cork as it exists on a tree, a microscopic look at the arrangement of cork cells, and finally a view of an individual cork cell.





Ignoring the natural variations in cell structure, we can model this cellular material as a linear-elastic solid by

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} .$$

Axisymmetry reduces the number of independent compliances to five (note that the axis of symmetry is the x_1 axis).

$$\left.\begin{aligned}
\varepsilon_{11} &= S_{1111}\sigma_{11} + S_{1122}\sigma_{22} + S_{1122}\sigma_{33} \\
\varepsilon_{22} &= S_{1122}\sigma_{11} + S_{2222}\sigma_{22} + S_{2233}\sigma_{33} \\
\varepsilon_{33} &= S_{1122}\sigma_{11} + S_{2233}\sigma_{22} + S_{2222}\sigma_{33} \\
\varepsilon_{23} &= (S_{2222} - S_{2233})\sigma_{23} \\
\varepsilon_{31} &= 2S_{1212}\sigma_{31} \\
\varepsilon_{12} &= 2S_{1212}\sigma_{12}
\end{aligned}\right\}$$

Measurement of four of the five compliances is straightforward. Simple tensile or compressive tests are done on the cork along the axis of symmetry (x_1 axis) and orthogonal to this direction (x_2 and x_3 axes) to measure the Young's modulus and Poisson's ratio in these directions.

$$\left.\begin{aligned}
&1/E_1 = S_{1111} \\
&1/E_2 = 1/E_3 = S_{2222} \\
&\nu_{12}/E_1 = \nu_{13}/E_1 = \nu_{21}/E_2 = \nu_{31}/E_2 = -S_{1122} \\
&\nu_{23}/E_2 = \nu_{32}/E_2 = -S_{2233}
\end{aligned}\right\}$$

The modulus G_{23} in the x_2, x_3-plane is obtained from these measurements by

$$1/G_{23} = 2(1+\nu_{23})/E_2 = 2(S_{2222} - S_{2233}) .$$

To determine the fifth compliance value, S_{1212}, which is related to the shear modulus, G_{12}, we rotate the cork through 45° about the x_3-axis and cut a cube with one face normal to the x_3-axis, and the other two at 45° to the x_2-axis and x_1-axis. A simple compression test in the new x_1 direction then gives a new Young's modulus, E'.

a. What is the relationship between the shear modulus, G_{12}, and S_{1212}?
b. By rotating the S_{ijkl} through 45° about the x_3-axis, obtain an expression for S'_{1111} in terms of the compliances in the original orientation.
c. Use your answers from parts (a) and (b) to find an expression for the shear modulus G_{12}, in terms of experimental parameters E_1, E_2, ν_{12}, and E'.
d. For the isotropic case, what does the equation derived in part (c) become?

4. An isotropic sample of material subjected to a compressive stress σ_z is confined so that it cannot deform in either the x- or y-directions.
   a. Do the stresses occur in the material in the x- and y-directions? If so, how are they related to σ_z?
   b. Determine the stiffness $E' = \sigma_z/\varepsilon_z$ in the direction of the applied stress in terms of the isotropic elastic constants E and ν for the material. Is E' equal to the elastic modulus from uniaxial loading? Why or why not?
   c. What happens if the Poisson's ratio for the material approaches 0.5?

5. Consider a flat plate of isotropic material that lies in the x-y plane and which is subjected to applied loading in this plane only. Such a plate is under plane stress, so that $\sigma_z = \tau_{yz} = \tau_{xz} = 0$ MPa .
   a. Does the thickness of the plate usually change when the plate is loaded?
   b. Under what conditions does the thickness not change? That is, when is this state of plane stress also a state of plane strain?