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DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING
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3.22 MECHANICAL PROPERTIES OF MATERIALS
PROBLEM SET 2

Due in 8 days from its assigned date

Reading

Nye, J.F., 2000, *Physical Properties of Crystals* (Clarendon Press). Chapter I, sections 2 – 3, 5 – 6, Chapter VI, and Chapter VIII, sections 1 – 4.

Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials* (John Wiley& Sons, Inc.) Chapter I, sections 1 – 2.

Ashby, M.F., *Mechanical Behaviour of Materials* (Course Notes). Sections 1 – 2.

1. (Nye 2.4) Transform the following tensors to their principle axes, using the Mohr's circle construction. Also, determine rotation and direction cosines for each transformation.

a. $\begin{bmatrix} 11.06 & 3.08 & 0 \\ 3.08 & 18.94 & 0 \\ 0 & 0 & 43 \end{bmatrix}$

b. $\begin{bmatrix} -6 & -3\sqrt{3} & 0 \\ -3\sqrt{3} & 0 & 0 \\ 0 & 0 & 10 \end{bmatrix}$

c. $\begin{bmatrix} 8 & 0 & -4 \\ 0 & 12 & 0 \\ -4 & 0 & 2 \end{bmatrix}$

2. A copper single crystal (cubic) experiences the stress state

$$\sigma_{ij} = \begin{bmatrix} 10 & -8 & 4 \\ -8 & 20 & 0 \\ 4 & 0 & 5 \end{bmatrix} (\text{MPa}).$$

a. Determine the compliance matrix. ($E_{11} = 66.7\,\text{GPa}$, $G_{12} = 75.2\,\text{GPa}$, $\nu_{12} = 0.42$) (Note $\nu_{ij} = -\dfrac{\varepsilon_j}{\varepsilon_i}$.)
b. Determine the engineering strain in the sample of copper.
c. Determine the strain energy in the sample.
d. If a hydrostatic pressure of 5 MPa is superimposed on the stressed copper sample, determine the resulting strain energy in the sample.

3. (Hertzberg 1.18) For three BCC metals, tungsten, molybdenum, and iron, compute the elastic moduli in the <100> and <111> directions. Compare the anisotropy in these three metals.

4. (Hertzberg 1.25) A cylindrical rod of steel is deformed elastically in tension to a load of 49 kN. If the original rod length and diameter are 25 cm and 15 mm, respectively, determine the rod length and diameter under load, assuming that the material possesses the following properties: E = 205 GPa, ν = 0.25.

5. In problem set 1, the principle stresses were determined for the given stress tensor.

$$\sigma_{ij} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 7 & -3\sqrt{3} \\ 0 & -3\sqrt{3} & 13 \end{bmatrix} \text{MPa}$$

   a. Find the unit normals for each principle stress.
   b. Use the unit normals to find the direction cosine matrix for the transformation from the original axes to the principle axes.
   c. Perform the tensor transformation using the direction cosines on the stress state in the problem. Explain the significance of the transformed tensor.

6. For an isotropic material, the compliance matrix constants are related by the expression $s_{66} = 2(s_{11} - s_{12})$. Derive this result using Mohr's circle and Hooke's Law.

7. A single crystal titanium carbide component is aligned such that the loading direction is parallel to the <100> direction. The component design requires that the modulus be at least 470 GPa in the loading direction. The modulus in the <100> direction is 476.2 GPa, which satisfies the requirement. However, careful inspection reveals that the single crystal is misaligned 10° toward the <010> direction about the axis parallel to the <001> direction. Because realignment of the component is expensive, determine if the misaligned component can be used and still meet the design requirement.