MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING
CAMBRIDGE, MASSACHUSETTS 02139

3.22 MECHANICAL PROPERTIES OF MATERIALS
PROBLEM SET 5

Due in 15 days from its assigned date

Reading

Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials* (John Wiley & Sons, Inc.) Chapter VI, section 3.

Ashby, M.F., *Mechanical Behaviour of Materials* (Course Notes). Section 4.

1. (Hertzberg 6.2) If it takes 300 seconds for the relaxation modulus to decay to a particular value at T_g, to what temperature must the material have been raised to effect the same decay in 10 seconds?

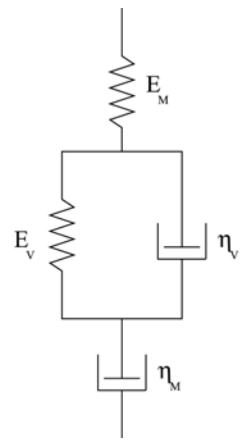
2. (Hertzberg 6.3) Calculate the relaxation time for glass and comment on its propensity for stress relaxation at room temperature. $E \approx 70\,\text{GPa}$ and $\eta \approx 1\times10^{12}\,\text{GPa-s}$.

3. Your Irish post doc asked you to determine the time for the relaxation modulus to decay to a particular value at 75°C by testing the polymer at its glass transition temperature, 0°C. She said a simple calculation using the empirical time-temperature relationship for amorphous polymers would give you the desired result. You find experimentally that the relaxation time is 217.2 seconds for the relaxation modulus to decay to the particular value. However, the refrigeration unit that was supposed to keep the temperature at 0°C (glass transition temperature) was not functioning and the test was carried out at 20°C, the ambient temperature in the laboratory. Can you use the data from this test to determine the relaxation time to decay to the specified value at 75°C? If so, what is the value?

4. (Hertzberg 6.5) The deformation response of a certain polymer can be described by the Voight model. If E = 400 MPa and $\eta = 2\times10^{12}$ MPa-s , compute the relaxation time. Compute $\varepsilon(t)$ for times to 5 τ when the steady state stress is 10 MPa. How much creep strain takes place when $t = \tau$ and when $t = \infty$?

   (Hertzberg 6.6) Compare the fractional amount of the total deformation that would occur if $t = \tau$ when $\eta = 2\times10^{12}$ MPa-s and $\eta = 8\times10^{12}$ MPa-s , respectively.

5. To improve the description of polymer behavior, Maxwell and Voigt models can be combined in series. For the four-element viscoelastic model shown below, derive an expression for the strain as a function of time for a given applied stress. Discuss the advantages of this model over the Maxwell and Voigt models.



6. Sketch a log-log plot of relaxation modulus versus temperature for an amorphous polymer with cross-links.
   a. Identify the various regions and characteristic relaxation modulus values on the plot.
   b. Explain the trends in relaxation modulus in the various regions in terms of the bonding in polymers.
   c. How would each regime change if the polymer had no cross-linking?

7. Creep compliance values for polyethylene are given.

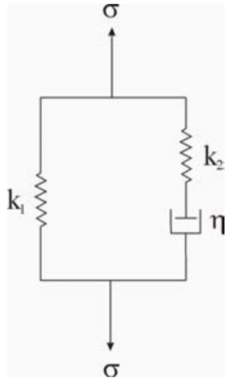
T (hours)	$J(t)$ (psi^{-1} $\times$ 10^{-4})
0	0.600
100	0.700
200	0.720
300	0.730
400	0.740
500	0.770

Consider a sample of polyethylene (cross-section 0.5 in. × 0.1 in.) for the given load history at the same test conditions as the data above.

a. Calculate the strain at 200 and 500 hours.
b. Sketch a qualitative strain-time plot.

Load (lbs.)	Duration (hours)
20	100
5	200
50	100
0	100

8. Derive the constitutive relation for the viscoelastic model shown below.

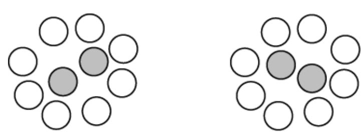


9. Evaluate the viscosity of the simple glass shown below by the following approximate procedure. Assume that half of the atoms pairs in the glass are in a position permitting an activated shear to the left (*state A*) and that the remaining half of the atom pairs are in the complementary position (*state B*). In the absence of a stress, *states A* and *B* have the same energy, *F*, and are separated by an activation energy of magnitude, ΔF^*. The stress σ_s raises the energy of atoms in *state A*, and lowers that of atoms in *state B*. The difference in energy between the two states is

$$\Delta F = 2\sigma_s \gamma \Omega ,$$

where γ is the shear strain that occurs when an atom pair stretch from *A* to *B*, (which you may take to be unity) and 2Ω is the volume of two atoms. The vibration frequency (attempt frequency) of atom pairs is ν. Calculate the rate of shear, $\dot{\gamma}$, of the unit volume of the liquid, subjected to a shear stress, σ_s, by calculating the nu,ber of atom pairs jumping, per second, from *A* to *B* and from *B* to *A*.

Assume that $\sigma_s\Omega \ll kT$ (this means that $\exp(\sigma_s\Omega/kT) \approx 1+\sigma_s\Omega/kT$) where *k*, is Boltzmann's constant and *T* is the absolute temperature and derive an equation for the viscosity of the liquid. (You may assume that a switching event, although it converts a pair of atoms in the *state A* into a pair in *state B*, also creates with the atoms surrounding it new pairs in *state A*, so that the fraction of atoms in *state A* remains constant and equal to one half.)



State A *State B*