MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING
CAMBRIDGE, MASSACHUSETTS 02139
3.22 MECHANICAL PROPERTIES OF MATERIALS

PROBLEM SET 10
Due in 5 days from its assigned date

1. Starting with the unsymmetric part of the Airy stress function, Eq.9.30 from the class handout, where $\chi = r^2 p(r,\theta) + q(r,\theta)$

and assuming χ is separable, i.e. $\chi = R(r) \bullet \Theta(\theta)$, derive the leading term of the asymptotic singular solution for mode II,

$$\sigma_{ij} \approx \frac{K_{II}}{\sqrt{2\pi r}} \tilde{\sigma}_{ij}^{II}(\theta),$$

following the procedure discussed in Section 9.3.2 of the handout (and in class) for Mode I. Write complete expressions for the different components of the stress field and compare your results with Eq. 9.47.

2. A piston (89 mm in diameter) is designed to increase the internal pressure in a cylinder from 0 to 55 MPa. The cylinder (closed at the other end!) is 200 mm long with internal diameter = 90 mm, outer diameter=110 mm, and made of 7075-T6511 aluminum alloy (extruded bar with yield strength = 550 MPa , and K_{IC} =30MPa-m$^{0.5}$). On one occasion, a malfunction in the system caused an unanticipated failure and the cylinder burst. Examination of the fracture surface revealed a metallurgical defect in the form of an elliptical flaw 4.5 mm long at the inner wall, 1.5 mm deep, and oriented normal to the hoop stress in the cylinder. Compute the magnitude of the pressure at which failure took place. (For the purpose of this problem assume that the stress intensity factor for the elliptical flaw is

$$K = \left(\frac{1.12}{\sqrt{Q}} \right) \sigma \sqrt{\pi a}$$

where σ is the appropriate normal stress and Q is a shape factor. The calibration for Q as a function of the aspect ratio of the flaw is given in A.7 of the Appendix section.

3. A steel used for an engineering application has a specified yield strength of 1000 MPa and a plane strain fracture toughness of 150 $\text{MPa}\sqrt{\text{m}}$.

(a) Calculate the minimum dimensions required to carry out a 'valid' plane strain fracture toughness test.
(b) Estimate the weight of the single edge cracked bend specimen and of the compact specimen of this steel which would have sufficient dimensions to provide valid plane strain fracture toughness. (See the Appendix section for K calibrations for different specimen geometries.) Assume that a/W=0.45.
(c) Estimate the load capacity of the mechanical testing machine you would need to carry out the fracture test.
(d) If the available testing machine has a load capacity of 200 kN, do you need to alter the dimensions of the specimens to obtain a valid K_{Ic}? If so, what will you do?

Exercises

16.1 Prove the statement in Section 14.3 that 1 cc of Fe oxidizes to 1.76 cc of FeO, 2.07 cc of Fe_3O_4 and 2.13 cc of Fe_2O_3. (The ratio of the volume of oxide produced to the volume of metal consumed is commonly known as the 'Pilling–Bedworth' ratio.)

16.2 Calculate the Pilling–Bedworth ratio for the formation of Al_2O_3 from Al and compare the efficacy of oxide-induced crack closure for aluminum alloys with that for steels solely on the basis of the volume of oxide produced on the fatigue fracture surface.

16.3 From the results presented in Eq. 9.49, show that the hydrostatic stress ahead of a mode III fatigue crack is nominally zero. List a few mechanistic processes for which the hydrostatic stress is likely to have a large influence on the corrosion-fatigue crack growth characteristics.

16.4 Figure 11.10 shows the crack growth characteristics of a ceramic composite under both static and cyclic loading conditions in 1400 °C air. See if linear elastic fracture mechanics provides valid characterization for the conditions of the experiments.

16.5 A hypothetical engineering alloy is subjected to high temperature fatigue crack growth at a cyclic frequency of 5 Hz. The transition time from small-scale creep to extensive creep, Eq. 16.21, under those testing conditions is 50 s. Comment on the validity of different fracture mechanics parameters to characterize the creep-fatigue fracture of the alloy.

16.6 List a set of conditions under which the similitude concept implicit in the nominal use of fracture mechanics is likely to lose validity during (*a*) stress-corrosion cracking, (*b*) corrosion-fatigue fracture and (*c*) creep-fatigue fracture.

Appendix: Stress intensity factors for some common crack geometries

This section presents a summary of stress intensity factors for some commonly used fatigue test specimens and crack configurations. These results are compiled from stress intensity factor handbooks (e.g., Rooke & Cartwright, 1976; Sih, 1973; Tada, Paris & Irwin, 1973) and monographs on fracture mechanics (e.g., Broek, 1986; Hellan, 1984; Kanninen & Popelar, 1985), where further details and derivations can be found.

A.1 Single-edge-cracked tension specimen

The stress intensity factor at the tip of a through-thickness, single edge crack of length a in a rectangular plate of width W (and height $H \geq 2W$) subjected to a tensile stress σ (Fig. A.1*a*) is

$$K_{\mathrm{I}} = \sigma\sqrt{a}\, f\left(\frac{a}{W}\right). \tag{A.1}$$

$$f\left(\frac{a}{W}\right) = 1.99 - 0.41\frac{a}{W} + 18.7\left(\frac{a}{W}\right)^2 - 38.48\left(\frac{a}{W}\right)^3 + 53.85\left(\frac{a}{W}\right)^4. \tag{A.2}$$

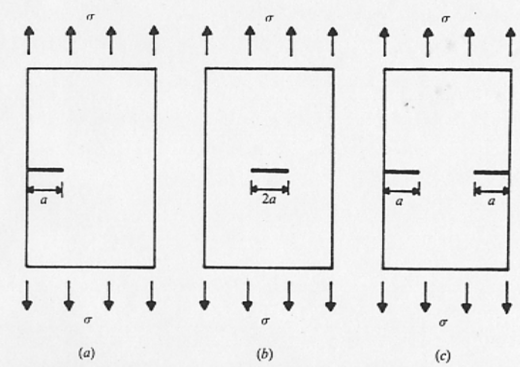

Fig. A.1. (*a*) Single-edge-cracked tension specimen. (*b*) Center-cracked tension specimen. (*c*) Double-edge-cracked tension specimen. The width of the plate is W in all cases.

A.2 Center-cracked tension specimen

The stress intensity factor for a center crack of length $2a$ in a plate (width W) which is subjected to a remote tensile stress σ (Fig. A.1*b*) is

$$K_{\mathrm{I}} = \sigma\sqrt{\pi a}\left(\sec\frac{\pi a}{W}\right)^{1/2}. \qquad (A.3)$$

A.3 Double-edge-cracked tension specimen

The stress intensity factor of a plate of width W which contains two edge cracks each of length a (Fig. A.1*c*) is

$$K_{\mathrm{I}} = \sigma\sqrt{a}\,f\left(\frac{a}{W}\right), \qquad (A.4)$$

$$f\left(\frac{a}{W}\right) = 1.99 + 0.76\left(\frac{a}{W}\right) - 8.48\left(\frac{a}{W}\right)^2 + 27.36\left(\frac{a}{W}\right)^3. \qquad (A.5)$$

A.4 Compact specimen

The mode I stress intensity factor for a compact specimen (Fig. A.2*a*) subjected to a tensile load P is

$$K_{\mathrm{I}} = \frac{P}{B\sqrt{W}}\,f\left(\frac{a}{W}\right), \qquad (A.6)$$

$$f\left(\frac{a}{W}\right) = \frac{(2+\{a/W\})}{(1-\{a/W\})^{3/2}} \times$$

$$\left[0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4\right]. \qquad (A.7)$$

A.5 Single-edge-cracked bend specimen

The stress intensity factor for an edge-cracked bend specimen of span S, width W and thickness B (Fig. A.2*b*) is

$$K_{tf=\text{"times"}I} = \frac{PS}{BW^{3/2}}\,f\left(\frac{a}{W}\right), \qquad (A.8)$$

$$f\left(\frac{a}{W}\right) = \frac{3(a/W)^{1/2}}{2(1+2\{a/W\})\,(1-\{a/W\})^{3/2}} \times$$

$$\left[1.99 - \left(\frac{a}{W}\right)\left(1-\frac{a}{W}\right)\left\{2.15 - 3.93\left(\frac{a}{W}\right) + 2.7\left(\frac{a}{W}\right)^2\right\}\right]. \qquad (A.9)$$

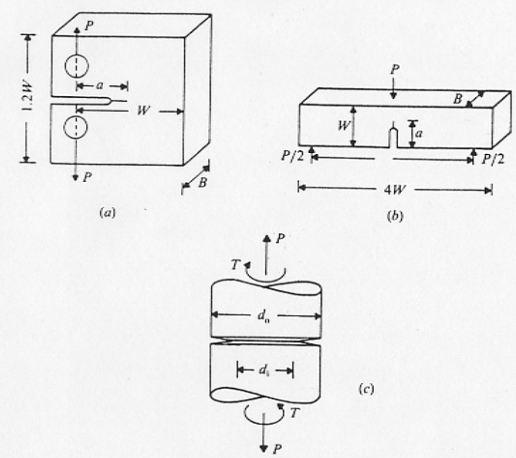

Fig. A.2. (*a*) Compact specimen. (*b*) Single-edge-cracked bend specimen. (*c*) Circumferentially cracked cylindrical specimen.

A.6 Circumferentially cracked cylindrical specimen subjected to tension and/or torsion

The mode I and mode III stress intensity factors for a circumferentially cracked cylindrical specimen (uncracked ligament diameter $d_{tf=\text{"times"}i}$, outer diameter d_o, and $D = \{d_i/d_o\}$) subjected to tension and torsion (Fig. A.2*c*) is

$$K_{\mathrm{I}} = \frac{2P}{\pi d_i^2}\left\{\frac{\pi d_i}{2}(1-D)\right\}^{1/2} \times$$

$$\{1 + 0.5D + 0.375D^2 - 0.363D^3 + 0.731D^4\}, \qquad (A.10)$$

$$K_{\mathrm{III}} = \frac{6T}{\pi d_i^3}\left\{\frac{\pi d_i}{2}(1-D)\right\}^{1/2} \times$$

$$\{1 + 0.5D + 0.375D^2 + 0.3125D^3 + 0.273D^4 + 0.208D^5\}. \qquad (A.11)$$

A.7 Large plates with elliptical and semi-elliptical cracks

Consider a plate specimen containing an elliptical crack. Let the major and minor axes of the ellipse be $2c$ and $2a$, respectively, as shown in Figure A.3. The stress intensity factor for the embedded elliptical flaw varies along the crackfront as a function of the angle ϕ (see Fig. A.3b). When the dimensions of the cracked body are much larger than a and c,

$$K_{\mathrm{I}} = \frac{\sigma\sqrt{\pi a}}{\Psi}\left(\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right)^{1/4}, \tag{A.12a}$$

where Ψ is the elliptical integral of the second kind, which is given by

$$\Psi = \int_0^{\pi/2}\left\{1 - \left(1 - \frac{a^2}{c^2}\right)\sin^2\phi\right\}^{1/2} \mathrm{d}\phi. \tag{A.12b}$$

K_{I} is maximum when $\phi = 90°$. Using a series expansion for Ψ, it can be shown that

$$\Psi \approx \frac{3\pi}{8} + \frac{\pi}{8}\left(\frac{a^2}{c^2}\right). \tag{A.12c}$$

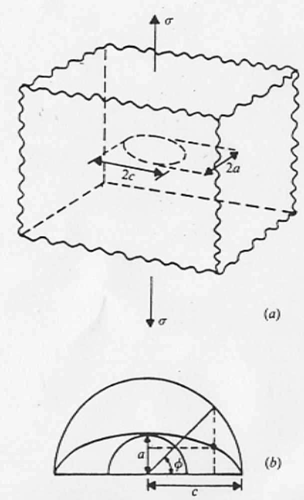

Fig. A.3. (a) A large plate containing an embedded elliptical crack. (b) Details of the crackfront.

When $a = c$, we obtain the solution for a circular (penny-shaped) crack. In this case, Eqs. A.12 reduce to

$$K_{\mathrm{I}} = \frac{2}{\pi}\sigma\sqrt{\pi a}. \tag{A.13}$$

The solutions given by Eqs. A.12 can also be applied to the case of semi-elliptical surface cracks. For the semi-elliptical surface flaw (thumb-nail crack) in a finite size plate, the stress intensity factor at the mid-point (i.e. end of the minor axis, $\phi = \pi/2$) is

$$K_{\mathrm{I}} = \frac{1.12\sigma\sqrt{\pi a}}{\sqrt{Q}}, \tag{A.14}$$

where the pre-multiplier 1.12 is the free-surface correction factor, Q is the flaw shape parameter extracted from Ψ in Eq. A.12b, $2c$ is the surface length of the crack, and a is the maximum depth (at $\phi = \pi/2$) of the crack into the material. $Q = \Psi^2$ in the elastic limit, $\sigma/\sigma_y \to 0$, where σ is the applied stress and σ_y is the yield strength of the material. Additional corrections may have to be made to Eq. A.14 to account for the proximity of the free surface to the crackfront (depending on the relative magnitudes of a and the specimen thickness B) and for crack-tip plasticity. The modified value of Q incorporating the plasticity correction is usually taken to be $Q \approx \Psi^2 - 0.212(\sigma^2/\sigma_y^2)$.