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3.22 MECHANICAL PROPERTIES OF MATERIALS  
PROBLEM SET 7 SOLUTIONS

1. *At the ultimate tensile strength, the load  $P = \sigma A$  is a maximum. Show that the criterion for necking of a tensile specimen can be written  $d\sigma/d\varepsilon = \sigma$  (Considere's relationship). If the constitutive relation between stress and plastic strain can be expressed by  $\sigma = 225 \cdot \varepsilon^{0.3}$  [MPa], determine the plastic strain at the onset of necking.*

At necking,  $dP = 0$ . This condition can be written in terms of true stress and instantaneous area.

$$dP = d\sigma_T \cdot A + dA \cdot \sigma_T = 0$$

$$\Rightarrow d\sigma_T + \sigma_T \frac{dA}{A} = 0$$

Because plastic deformation does not induce a volume change, we have an expression for the change in true plastic strain.

$$dV = Ad\ell + \ell dA = 0$$

$$\Rightarrow \frac{dA}{A} = -\frac{d\ell}{\ell} = -d\varepsilon_T^p$$

Substituting the expression with true plastic strain into the true stress expression, we have

$$d\sigma_T - \sigma_T d\varepsilon_T^p = 0$$

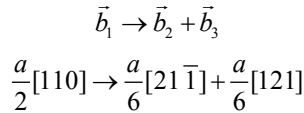
$$\Rightarrow \frac{d\sigma_T}{d\varepsilon_T^p} = \sigma_T$$

To find the plastic strain at the onset of necking, we can use the derived result for a material that deforms plastically according to the Hollomon relation,  $\sigma_T = \sigma_y (\varepsilon_T^p)^n$ , where  $n$ , is the strain hardening exponent.

The derived result is that the strain at the ultimate tensile stress occurs at a plastic strain equal to the strain hardening exponent.

$$\boxed{(\varepsilon_T^p)_{UTS} = n = 0.3}$$

2. (Hertzberg 2.2) Consider the following face-centered-cubic dislocation reaction:



- a. Prove that the reaction will occur.

A simple energy argument can determine if the dislocation reaction will occur.

$$E(\vec{b}_1) \approx G\vec{b}_1^2 = G\left(\frac{a}{2}[110]\right)^2 = G\frac{a^2}{4}[1^2 + 1^2 + 0] = G\frac{a^2}{2}$$

$$E(\vec{b}_2 + \vec{b}_3) \approx G\vec{b}_2^2 + G\vec{b}_3^2 = G\left(\frac{a}{6}[21\bar{1}]\right)^2 + G\left(\frac{a}{6}[121]\right)^2 = 2G\frac{a^2}{36}[2^2 + 1^2 + 1^2] = G\frac{a^2}{3}$$

$$\therefore E(\vec{b}_1) > E(\vec{b}_2 + \vec{b}_3)$$

The lower energy state of the two partial dislocations indicates that the dislocation reaction will occur.

b. What kind of dislocations are the  $(a/6)\langle 121 \rangle$ ?

Partial dislocations.

3. a. Why is it impossible to make a dislocation loop all of whose segments are pure screw dislocations?

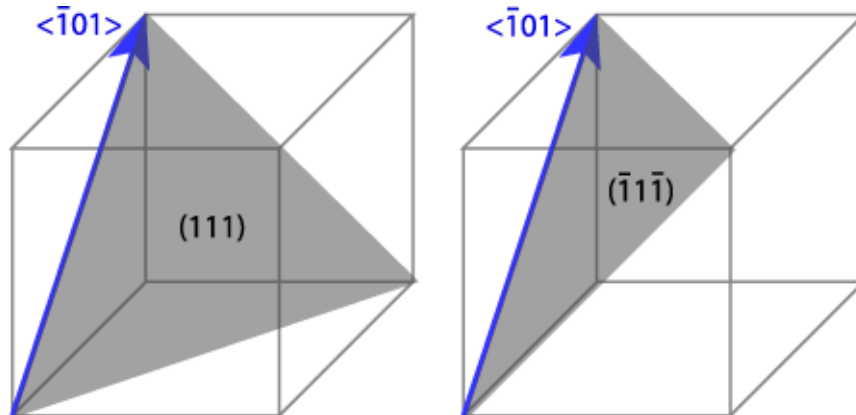
The Burgers vector of a dislocation is always the same and is independent of the position of the dislocation line. Thus, a screw dislocation only exists where the dislocation line and Burgers vector are parallel. For a loop, the dislocation line and Burgers vector will not be parallel over the entire loop, and therefore the loop will not have screw character around the loop.

b. Give the specific indices of the two  $\{111\}$  planes that contain the screw dislocation with the Burgers vector  $(a/2)\langle \bar{1}01 \rangle$  in a face-centered-cubic crystal where  $a$  is the lattice parameter of the fcc crystal.

For a Burgers vector to lie in a given plane, the dot product between the Burgers vector and the plane normal must be zero.

$$\hat{n} \cdot \vec{b} = \hat{n} \cdot \langle \bar{1}01 \rangle = 0$$

The two  $\{111\}$  planes that satisfy this condition are  $(\bar{1}\bar{1}\bar{1})$  and  $(111)$ . These two planes can be equivalently expressed as  $(1\bar{1}\bar{1})$  and  $(\bar{1}\bar{1}1)$ . These planes are shown below.



4. (Hertzberg 2.4) Discuss the nature of the Peierls stress with regard to a dislocation and describe the role of the Peierls stress in determining the preferred slip plane in a crystal and the yield-strength temperature dependence of the crystal.

A dislocation moves through a crystal by individual atoms adjacent to the dislocation line shifting atomic positions by an amount defined by the Burgers vector. A consequence of the periodic structure of the crystal, the force-displacement relationship between individual atoms provides resistance to atoms shifting position and thereby causing dislocation motion. This resistance to dislocation motion due to the lattice is referred to as the Peierls stress.

The preferred slip plane is the easiest plane for dislocations to glide on. Therefore, it is expected that the plane with the lowest Peierls stress is favored. The Peierls stress is based on breaking and reforming bonds as an atom shifts position. Therefore, the planes that are spaced furthest apart (longest bonds between atoms in adjacent planes) will minimize the Peierls stress. The close-packed planes (or densest planes) have the maximum plane spacing and will be the preferred slip plane.

The trend of lower yield-strength with increasing temperature can be explained, in part, by the role of the Peierls stress. As discussed above, the Peierls stress originates from stretching and breaking bonds as

atoms shift. Increasing the temperature decreases the amount of energy needed to stretch and break bonds, thus lowering the Peierls stress and facilitating plastic deformation at lower applied stresses.

5. Consider a ductile metal with Burgers vector  $a = 3$  angstroms.
- If a typical annealed metal is plastically deformed at a strain rate of  $0.01 \text{ s}^{-1}$ , what is the average velocity of a dislocation? Calculate the average velocity of dislocations if the metal has undergone significant deformation?

Orowan's equation relates strain rate and dislocation velocity by the equation

$$\dot{\gamma}^p = b\rho\bar{v}_{dislocation}$$

For a typical annealed metal, the dislocation density is approximately,  $\rho = 10^9 \text{ 1/m}^2$ .

$$\bar{v}_{dislocation} = \frac{\dot{\gamma}^p}{b\rho} = \frac{0.01 \text{ s}^{-1}}{(3 \times 10^{-10} \text{ m})(10^9 \text{ m}^{-2})} = \boxed{3.33 \text{ cm/s}}$$

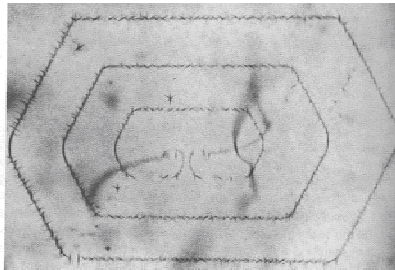
For a typical worked metal, the dislocation density is approximately,  $\rho = 10^{15} \text{ 1/m}^2$ .

$$\bar{v}_{dislocation} = \frac{\dot{\gamma}^p}{b\rho} = \frac{0.01 \text{ s}^{-1}}{(3 \times 10^{-10} \text{ m})(10^{15} \text{ m}^{-2})} = \boxed{0.0333 \text{ }\mu\text{m/s}}$$

- Explain what happens to the overall dislocation density in general as a metal undergoes increasing plastic deformation.

As a metal is deformed plastically, dislocations move and interact with each other. Trapped dislocations can act as sources (Frank-Reed sources) for dislocations (dislocation loops) that allow for further plastic deformation. Thus, as a metal is deformed, the dislocation sources increase the overall dislocation density in the metal.

6. (Hertzberg 2.5) Why do dislocation loops tend to be circular? Why, then, are they angular for silicon as shown in the figure below?



A dislocation has an energy per unit length ( $E \propto Gb^2$ ) associated with it due to the displacement of atoms around the dislocation line. In order to minimize the energy of the system, dislocation loops attempt to minimize their length. The shortest length for a loop is a circle, which is why dislocation loops tend to be circular.

In a single crystal of silicon, the dislocation loop is angular instead of circular. In the above case for circular dislocation loops, it was assumed that the energy per unit length of the dislocation is equal in all directions (isotropic). For a single crystal of silicon (anisotropic), the energy required to displace atoms about the dislocation line changes with direction. As a result, the energy per unit length varies with direction. In order to minimize the energy of a dislocation loop in an anisotropic material, the dislocation line will follow low energy directions instead of forming a circular shape to minimize the length of the loop. The preference for certain low energy directions results in angular loops in silicon single crystals.

7. *Substitutional solute atoms have a strain field about themselves, the sign of the strain depending on the size of the atom compared to a matrix atom. This strain field of a substitutional atom can interact with the stress field of an edge dislocation. Draw a schematic picture of an edge dislocation, and indicate the position around the dislocation where large and small substitutional atoms would be found. Where would interstitial atoms be found?*

The stress field below an edge dislocation is tensile. Thus the positive strain associated with relatively large substitutional atoms is accommodated when these atoms diffuse below an edge dislocation. Similarly, relatively small substitutional atoms with negative strain fields are attracted to the compressive region above the edge dislocation. See figure.

Interstitial atoms energetically favor a position at the core of the dislocation due to the free volume at the center of an edge dislocation.

8. *In rectangular coordinates, the stress field surrounding an edge dislocation is given by*

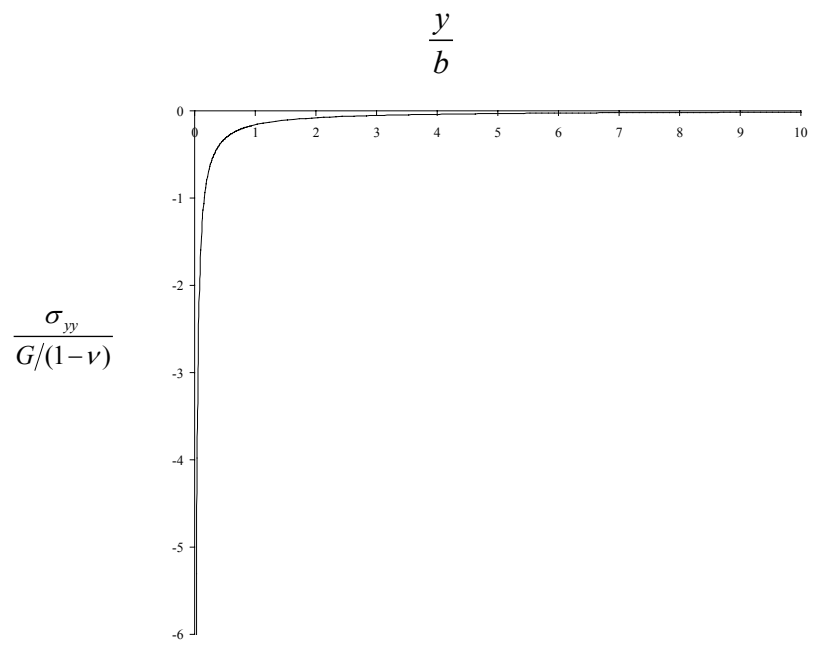
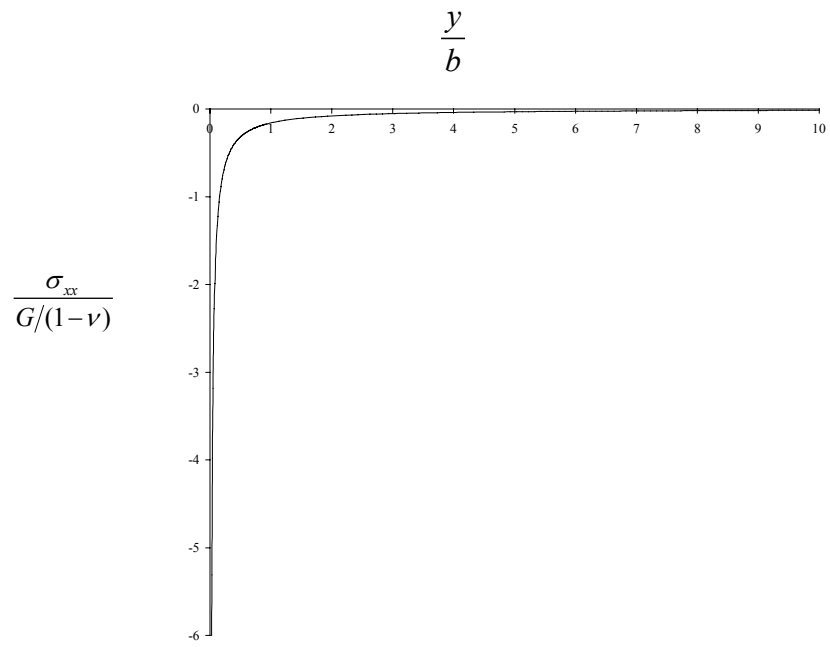
$$\begin{aligned}\sigma_{xx} &= -\frac{Gb}{2\pi(1-\nu)} \frac{y(3x^2 + y^2)}{(x^2 + y^2)^2} \\ \sigma_{yy} &= \frac{Gb}{2\pi(1-\nu)} \frac{y(x^2 - y^2)}{(x^2 + y^2)^2} \\ \sigma_{xy} &= \frac{Gb}{2\pi(1-\nu)} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2} \\ \sigma_{zz} &= \nu(\sigma_{xx} + \sigma_{yy})\end{aligned}$$

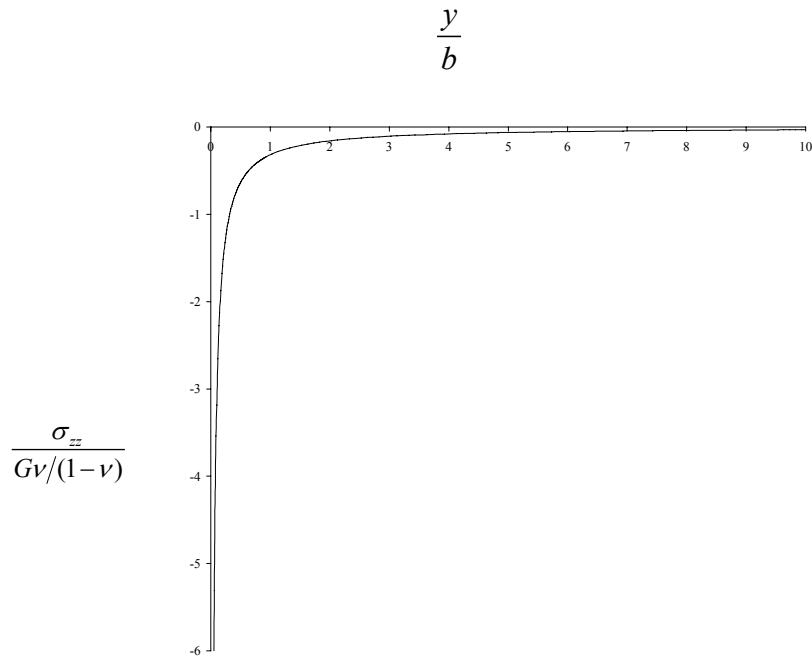
*Plot the stress field along the positive y-axis. Explain why the theory breaks down at the extrema ( $y \rightarrow 0, \infty$ ).*

Along the y-axis implies that  $x = z = 0$ . So, the stresses become a function of  $y$  only.

$$\begin{aligned}\sigma_{xx} &= -\frac{Gb}{2\pi(1-\nu)} \frac{1}{y} \\ \sigma_{yy} &= -\frac{Gb}{2\pi(1-\nu)} \frac{1}{y} \\ \sigma_{xy} &= 0 \\ \sigma_{zz} &= -\frac{Gb\nu}{\pi(1-\nu)} \frac{1}{y}\end{aligned}$$

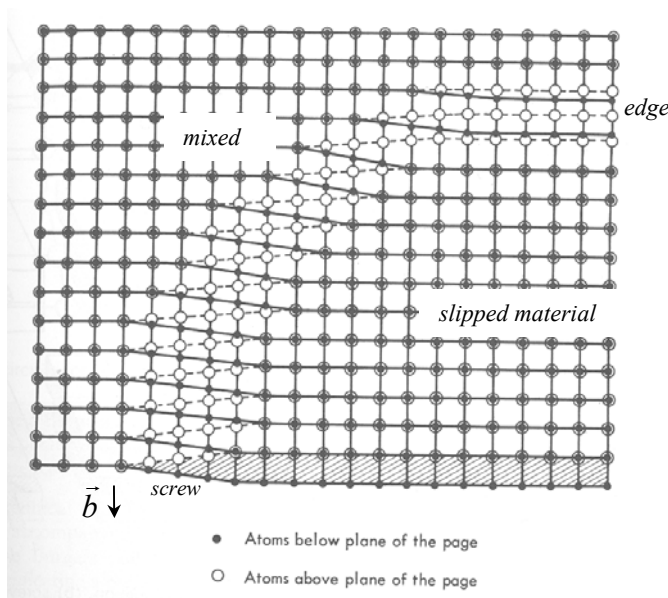
The dimensionless plots on the following pages illustrate the stress profile above an edge dislocation.





From the plot we see that the stress approaches infinity as  $y \rightarrow 0$ , and that there exists a positive stress due to the dislocation as  $y \rightarrow \infty$ . If either of these conditions were true, the strain energy due to the dislocation would be infinite. Because the strain energy of a dislocation must be bounded, the theory cannot hold for  $y \rightarrow 0$  or  $y \rightarrow \infty$ .

9. Using the figure below, answer the following questions.
- What is the Burgers vector for the dislocation shown?
  - Assuming that all atoms shown were originally above (or below) each other, indicate the region of slipped material.
  - Define the regions of the dislocation, whether screw, edge or mixed.



10. Explain why a second hardness measurement made adjacent to a previous measurement indicates a higher hardness.

The previous hardness measurement leads to plastic deformation near the indented area. This plastically deformed region is thus work hardened. The work hardened area around the previous indentation results in a higher hardness when indented a second time.

11. At 20°C, an alloy yields at 150 MPa when strained at 0.03 s<sup>-1</sup>. The same alloy yields at 145 MPa when the strain rate is decreased by a factor of ten and the temperature is kept the same. What test temperature would have to be used with a strain rate of 0.03 s<sup>-1</sup> to reach a yield strength of 145 MPa? (Assume that the energy per bond is 500 kJ/mol.)

The relevant equation to use for the temperature and strain rate dependence of the yield strength is

$$\tau(T, \dot{\gamma}) = \tau_0 \left[ 1 - \frac{kT}{Q_b} \ln \left( \frac{\dot{\gamma}_0}{\dot{\gamma}} \right) \right],$$

where  $k$  is Boltzmann's constant ( $1.38 \times 10^{23}$  J/K) and

$$Q_b = 500 \frac{\text{kJ}}{\text{mol}} \cdot \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ atoms}} = 8.039 \times 10^{-19} \frac{\text{kJ}}{\text{atom}}.$$

Using the two yield strengths at different temperatures, we can find the constants  $\tau_0$  and  $\dot{\gamma}_0$ .

$$\tau(T = 293 \text{ K}, \dot{\gamma} = 0.03 \text{ s}^{-1}) = \tau_0 \left[ 1 - \frac{(1.38 \times 10^{23} \text{ J/K})(293 \text{ K})}{8.039 \times 10^{23} \text{ J/atom}} \ln \left( \frac{\dot{\gamma}_0}{0.03 \text{ s}^{-1}} \right) \right] = 150 \text{ MPa}$$

$$\tau(T = 273 \text{ K}, \dot{\gamma} = 0.003 \text{ s}^{-1}) = \tau_0 \left[ 1 - \frac{(1.38 \times 10^{23} \text{ J/K})(273 \text{ K})}{8.039 \times 10^{23} \text{ J/atom}} \ln \left( \frac{\dot{\gamma}_0}{0.003 \text{ s}^{-1}} \right) \right] = 145 \text{ MPa}$$

$$\tau_0 = 431.727 \text{ MPa}, \quad \dot{\gamma}_0 = 66.47 \times 10^{54} \text{ s}^{-1}$$

Use the constants determined above to find the temperature at which the yield strength is 145 MPa for a strain rate of 0.03 s<sup>-1</sup>.

$$\tau(T, \dot{\gamma} = 0.03 \text{ s}^{-1}) = \tau_0 \left[ 1 - \frac{(1.38 \times 10^{23} \text{ J/K}) \cdot T}{8.039 \times 10^{23} \text{ J/atom}} \ln \left( \frac{\dot{\gamma}_0}{0.03 \text{ s}^{-1}} \right) \right] = 145 \text{ MPa}$$

$$T = 433.29 \text{ K} = \boxed{160.14^\circ \text{C}}$$

12. (Hertzberg 4.7) Maraging steels are relatively soft upon quenching from the austenitizing temperature range but strengthen greatly following exposure to a reheating treatment at intermediate temperature. Given that the carbon level of such steels is typically less than 0.03%, whereas the alloy contains additions of Ni, Mo, and Ti, speculate as to the probable strengthening mechanism that controls the strength of this class of alloys.

In maraging steels, the carbon concentration is so low that the solid solution of carbon in the steel does not influence the strength of the steel significantly. (Ferritic steels derive their strength from solid solution strengthening of carbon.) If solid solution strengthening were present, then the steel would not be relatively soft after quenching from the austenitizing temperature. Strength does increase following exposure to an intermediate temperature (aging) after quenching. We can speculate that the strengthening comes from the precipitation of a second phase (NiMo and NiTi) due to the presence of the alloying elements. Therefore, the probable strengthening mechanism in maraging steels is precipitation hardening. (The term *maraging* steel originated from the class of steel, *martensitic* alloys, and the strengthening process, *aging*.)