Solutions to PS 11

1.

(a) We have two different fatigue loads, the fluctuations (50 Hz) and the fatigue associated with the daily on-off operation of the machine. We know that for the fluctuations our $\Delta\sigma_1$ is equal to 30 MPa, and our R ratio for this case is $185/215 = 0.86$. For the on-off loading $\Delta\sigma_2$ is equal to 200 MPa with an R ratio of 0. The stress intensity factor for this geometry is given by $K_I = \sigma\sqrt{\pi a}$. We can first evaluate the critical crack size for this material and the applied loading:

$$K_{Ic} = \overset{215}{\sigma_{max}}\sqrt{\pi a_{crit}}$$

Solve to find that $a_{crit} = 6.9$ cm, which is much larger than any flaws in the plate. Now determine the applied (initial) ΔK, assuming that the largest flaws in the plate have a length $2a = 0.2$ mm (tolerance of NDE technique).

$$\Delta K_1 = \overset{230}{\Delta\sigma}\sqrt{\pi(1\times10^{-4})} = 0.53 \quad \text{MPa}\sqrt{m} \qquad R=0.86$$

$$\Delta K_2 = \overset{200}{\Delta\sigma}\sqrt{\pi(1\times10^{-4})} = 3.53 \quad \text{MPa}\sqrt{m} \qquad R=0$$

Looking at the given data for ΔK_0, for the fluctuations we are significantly below the threshold, which implies extremely slow (essentially zero) fatigue crack growth. For ΔK_2 we are above the threshold so we may assume that we are in the Paris regime of fatigue crack growth for this loading.

All of the factors mentioned play some role in the problem. The plane strain fracture toughness would be especially important of we had flaws near to the critical flaw size for the given loading. Flaws of this size do not exist in the plate. However, the *final* size of the flaw is set by the plane strain fracture toughness so it does play a role in the problem. The rate of crack growth in the Paris regime is very important since it will govern the lifetime of the plate. Also we need accurate knowledge of the near-thresold crack growth behavior, in particular the value of ΔK_0, since both the initial values of ΔK are near to ΔK_0 .

(b) From part a) Since the value of ΔK_1, is below the threshold we assume initially no crack growth due to this loading. Then the crack growth is due to the on-off operation of the machine, which can be predicted using the Paris fatigue crack growth equations assuming the values of c amd m are known. But at some point the crack could be long enough that the the value of ΔK_1 could reach ΔK_0 . We can find out the value of crack size a when this happens by setting $\Delta K_1 = \Delta K_0$, I found that $a = 0.80$ mm. → $30\sqrt{\pi a} =$

Without doing any calculations, we can say that the crack will grow very slowly out to a length $a = 0.80$ mm (due to the on-off loading) and then very rapidly from that length out to the final crack length (mainly due the fluctuations). For purposes of illustration, I set $m = 2$ and $C = 1 \times 10^{-8}$ 1/(cycle·MPa^2) and determined the total lifetime.

Compute cycles from $a = 0.1$ mm to $a = 0.80$ mm (growth only due to on-off loading).

$$N = \frac{1}{CY^2 (\Delta\sigma)^2 \pi} \ln \frac{a_f}{a_0} = 1655 \text{ cycles}$$

Since we have 1 cycle per day, this represents a time of 4.5 years!

Compute cycles from $a = 0.80$ mm to 69.0 mm (growth only due to fluctuations).

$$N_f = \frac{1}{CY^2 (\Delta\sigma)^2 \pi} \ln \frac{a_f}{a_0} = 157{,}644 \text{ cycles}$$

It seems like many cycles, but remember the fluctuations are occurring at 50 Hz. So this many cycles only requires about 3 hours. Regardless of the values of C and m you choose, the lifetime of the plate will be mainly determined by the time required for the crack to grow (under the action of the on-off loading) to a length where the fluctations can grow the crack.

2.

There are many details given in this problem and it is easy to get lost in all the information provided. To solve simply go through all the details and determine, step-by-step, what is known about the failure of the plate. We know that the load is cycled between 5 and 85 kN, and we know the area of the plate, so we can determine that $\sigma_{max} = 94.4$ MPa and $\sigma_{min} = 5.5$ MPa. The stress intensity factor for this geometry (assume a semi-infinite plate) is:

$$K_I = 1.12\sigma\sqrt{\pi a}$$

Using this, we may evaluate the critical crack length in the the plate by setting $K_I = K_{Ic}$:

$$a_f = \left(\frac{K_{Ic}}{\sigma_{max}}\right)^2 \left(\frac{1}{\pi}\right) \left(\frac{1}{1.12}\right)^2 = 0.0658 \quad \text{m}$$

So now we know the initial crack length (a = 1 cm) and the final crack length (a = 6.58 cm). The bystander reported that the crack was propagating at a velocity proportional to the crack tip opening displacement corresponding to the peak load in each cycle. This implies that that $da/dN \propto (\Delta K)^2$, which further implies that we are dealing with a Paris law crack growth regime with m = 2. We need only determine the value of the constant C to be able to finally solve the problem. This can done using the given fractographic data, which says that da/dN was equal to 2.5×10^{-7} m/cycle when a was equal to 10+2.5 = 12.5 mm. So we must have that

$$2.5 \times 10^{-7} = C(\Delta K)^2 = C(1.12)^2(\sigma_{max} - \sigma_{min})^2 \pi a$$

Use this equation to solve for C. I found that $C = 6.42 \times 10^{-10}$ $1/(\text{cycle}\cdot\text{MPa}^2)$. Now substitute into the equations for N_f for the case $m = 2$

$$N_f = \frac{1}{CY^2(\Delta\sigma)^2\pi} \ln\left(\frac{a_f}{a_0}\right)$$

Substituting the known values of $\Delta\sigma$, C, Y (1.12 for this geometry) and the initial and final crack lengths I found that $N_f = 93,886$ cycles. If the plate was loaded at 75 Hz, that converts to 4500 cycles/min and therefore the lifetime of the plate is $93,886/4500 = 20.86$ minutes. Given that the foreman left 2 minutes after the cut in the plate was made, and it took him 20 minutes to get to the plant by bike he would not make it in time to save the plate (almost but not quite . .)

This assumes that the crack is growing in the Paris regime from when it is first cut in the plate; if you assume instead that the plate was in the Paris regime only from the point when the police were called (10 minutes after the crack was made) and the bystander noted that ". . . the crack in the plate was propagating at a velocity proportional to the crack tip opening displacement corresponding to the peak load of each cycle." (Paris regime behavior) you will find that the foreman makes it to the plant in time to shut off the equipment.