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3.22 MECHANICAL PROPERTIES OF MATERIALS
PROBLEM SET 3 SOLUTIONS

1. *(Hertzberg 1.7) In class we have discussed the relation between the modulus of elasticity of a group of crystalline solids and their respective melting points. Relate the modulus of elasticity to the respective coefficients of thermal expansion. (The coefficient of thermal expansion increases with temperature for most materials.)*

The fundamental concept that the elastic modulus is directly related to the bond strength between atoms in crystalline solids is essential for understanding elastic behavior. The modulus of a given crystalline solid increases with bond strength.

As crystalline solids approach the melting point, atoms vibrate more energetically and the atomic spacing increases (due to asymmetry of the potential well). The increase in atomic spacing necessarily lowers the strength of the bonds between the atoms. Thus, crystalline solids with low melting points (and relatively low atomic bond strengths) will have small elastic moduli. Correspondingly, crystalline solids with high melting points (and relatively high atomic bond strengths) will have high elastic moduli. (Elastic modulus increases with increasing melting point.)

For a crystalline solid, coefficients of thermal expansion increase with temperature. Thus, by invoking a similar argument as above, the higher the expansion coefficient, the closer the solid is relative to the melting point and the lower the modulus is. (Elastic modulus increases with decreasing thermal expansion coefficient.)

2. *For a solid-solid equilibrium phase transformation of a crystalline material, comment on the value of the modulus before and after the phase transformation.*

To reiterate, the fundamental concept is that the elastic modulus is directly related to the bonding between atoms in crystalline solids. For a solid-solid phase transformation, the type of bonding remains the same and the bond lengths change very little. Thus, for a solid-solid phase transformation, no significant change in elastic modulus is expected.

3. *As temperature increases, explain how the elastic modulus changes for crystalline materials and rubbers. Relate each type of behavior to fundamental properties of each class of material.*

Crystalline materials experience a decrease in elastic modulus as the temperature is increased. Increasing the temperature causes the atoms to move further apart, due to thermal expansion. As atomic distances increase, the modulus decreases.

Rubbers, however, experience an increase in elastic modulus as the temperature is increased. Pulling a rubber in tension causes the chains to align, lowering the order, or entropy, in the system. For rubbers, the entropy governs elastic response. As temperature is increased, entropy increases. The increase in entropy will therefore increase the resistance of the material to deformation, increasing the elastic modulus.

4. (a) Explain why aluminum and magnesium have similar melting temperatures ($T_m = 650^\circ\text{C}$) and dissimilar elastic moduli ($E_{Al} = 70\text{ GPa}$, $E_{Mg} = 45\text{ GPa}$)?

Both melting temperature and elastic modulus derive from the atomic bonding strength in a crystal. However, the modulus is also related to the atomic density (or atomic volume). The density of magnesium is substantially less than that of aluminum. Thus, the larger inter-atomic separations in magnesium can explain why the modulus is much lower.

- (b) Aluminum 7075 T6 is an aluminum alloy with 1.6% Copper, 2.5% Magnesium, 5.6% Zinc, 0.23% Chromium by weight. Explain why the modulus of aluminum 7075 T6 ($E_{Al\ 7075} = 70.7\text{ GPa}$) and pure aluminum ($E_{Al} = 70\text{ GPa}$) are nearly the same. Explain why the modulus of the alloy is slightly higher than the pure material.

The elastic modulus derives from atomic bonding strength and atomic separation distances. The alloy composition after adding about 10% of other atoms still consists of mainly aluminum. If alloying elements do not change the bonding type or atomic spacing appreciably, the modulus will not change dramatically, as is the case.

The slight increase in modulus is due to the higher average modulus of the alloying elements. Alloying with lower modulus elements will have the opposite effect.

- (c) Cobalt is a magnetostrictive and contracts upon exposure to a magnetic field. Explain the effect of a magnetic field on the elastic modulus of cobalt.

As a material contracts, its atomic spacing decreases. This increase in atomic density causes an increase in the elastic modulus. (The bonding type is unaffected by the application of a magnetic field.)

5. (Hertzberg 1.19) You are given a 135 cm long length of high-strength wire with a circular cross-sectional diameter of 0.5 cm. The wire has the following properties: $E = 216\text{ GPa}$; $\epsilon_f = 0.02$. You are to manufacture a composite rod 15 cm long and having a cross-sectional area of 6.5 cm^2 . The 135 cm long high-strength wire is to be used as the high-strength constituent of the composite. The matrix phase will consist of a polymer resin that is to be cured after the high-strength wires have been positioned. Ignoring the strength contribution of the polymer resin matrix and any residual thermal shrinkage stresses, and assuming an ideal bond between the matrix and the reinforcing phase, calculate the modulus of the strongest composite that can be made from the resin and the length of wire provided.

The strongest composite will have the reinforcing fibers oriented along the axis of the composite rod. From the 135 cm long high strength fiber supply, it is possible to prepare 9 lengths for the composite rod.

$$\frac{135\text{ cm}}{15\text{ cm}} = 9\text{ lengths}$$

The cross-sectional area of these 9 lengths is

$$A = 9 \times \frac{\pi}{4} d^2 = 9 \times \frac{\pi}{4} (0.5\text{ cm})^2 = 1.767\text{ cm}^2.$$

The volume fraction of the reinforcing fiber phase is calculated to be

$$v_f = \frac{1.767\text{ cm}^2}{6.5\text{ cm}^2} = 0.272.$$

Neglecting the stiffness of the matrix phase, the modulus of the composite is

$$E_c = v_f E_f = (0.272)(216\text{ GPa}) = \boxed{58.75\text{ GPa}}.$$

6. (Hertzberg 1.23) From Table 1.9, we see that the stiffness of Nylon 66 + 25v/o (volume fraction) carbon fibers is 14 GPa, whereas the stiffness of an epoxy resin + 60v/o carbon fibers is 220 GPa. If the elastic modulus of carbon fibers is 390 GPa, speculate on the nature of the two composites in question in terms of fiber length and fiber orientation. Also, calculate the lower bound modulus for the Nylon 66 + 25v/o carbon fiber composite.

For simplicity we can ignore the stiffness contribution of nylon and epoxy in the two composites. For the case of the ideal epoxy resin-carbon fiber composite modulus,

$$E_c = v_f E_f = (0.6)(390 \text{ GPa}) = 234 \text{ GPa} .$$

This result compares well with the reported value of 220 GPa and indicates that the carbon fibers are long, well bonded to the matrix, and oriented parallel to the loading axis.

On the other hand, for the case of the nylon 66 matrix-carbon fiber composite, the ideal composite stiffness is found to be

$$E_c = v_f E_f = (0.25)(390 \text{ GPa}) = 97.5 \text{ GPa} ,$$

which is considerably larger than the reported value. Therefore, one must conclude that either the fibers are not oriented parallel to the loading axis or the fibers are very short.

The lower bound modulus of the nylon 66 matrix-carbon fiber composite can be calculated from isostress analysis. The composite modulus in the direction normal to the reinforcing phase is given by

$$E_c = \frac{E_f E_m}{v_f E_m + (1 - v_f) E_f} = \frac{(2.9 \text{ GPa})(390 \text{ GPa})}{(0.25)(2.9 \text{ GPa}) + (1 - 0.25)(390 \text{ GPa})} = \boxed{3.857 \text{ GPa}} .$$

7. *Thermostatic bimetals: A thermostatic bimetal consists of two metals layers with different coefficients of thermal expansion bonded together. When subjected to a temperature change, the bimetal undergoes a change of shape that is mechanically exploited for control of temperature or some other function. To design a bimetal, it is advantageous to bond a low expansivity alloy with a high expansivity alloy. Guillaume (1898) discovered that an iron alloy with 35.4 wt% of nickel yielded a material with a very low thermal expansion coefficient over a useful range from 0°C to 100°C of $1.3 \times 10^{-6} (\text{°C})^{-1}$. This alloy was named invar to reflect its invariant dimensions over most atmospheric temperatures.*

(a) *Explain why lead, aside from environmental reasons, cannot be used as one of the two metals in the bimetallic strip in thermostats.*

The design of a bimetal requires that all deformation is reversible, hence elastic. Because lead has a very low yield stress, the bimetal would respond elastically only over small temperature variations. Thus, it is impractical to build a thermostatic bimetal with a limited function temperature range.

(b) *Derive an expression for the variation of curvature of a brass (90% Cu–10% Zn)–Invar thermostatic bimetal as a function of temperature change, ΔT . Assume that the brass and Invar layers have the same thickness.*

$$(\alpha_{\text{Cu-Zn}} = 20 \times 10^{-6} (\text{°C})^{-1}, E_{\text{Invar}} = 141 \text{ GPa}, E_{\text{Cu-Zn}} = 117 \text{ GPa})$$

From the derivation in class for a bilayer with equal layer thicknesses of $h/2$,

$$\kappa = \frac{24\Delta\alpha\Delta T}{h \left(14 + \frac{E_1}{E_2} + \frac{E_2}{E_1} \right)} .$$

Substituting our known constants, we obtain

$$\kappa = \frac{24(20 \times 10^{-6} (\text{°C})^{-1} - 1.3 \times 10^{-6} (\text{°C})^{-1}) \Delta T}{h \left(14 + \frac{141}{117} + \frac{117}{141} \right)}$$

$\kappa = 2.739 \times 10^{-5} \Delta T / h$

(c) *If the temperature is increased, identify the direction in which the bimetal curves.*

If the temperature is increased, the brass will expand more than the Invar. To match displacements at the interface, the brass will develop a compressive stress, while the Invar develops a tensile stress. Thus, the bimetal will curve toward the Invar side.

(d) *Discuss why rapid fluctuations in temperature would affect the ability of the thermostatic bimetal to provide accurate measures of instantaneous temperature.*

The derivation for curvature of a bilayer is based on a uniform temperature in bilayer. For rapid fluctuations, the time to reach equilibrium is finite due to thermal diffusivity. The time needed to reach equilibrium limits the ability of a thermostatic bimetal to measure instantaneous temperature.