# Aspects of Design Optimization

**Based in part on 2.002 tutorial by D. Parks, 2006**

# Setting a stent to re-open plaque-clogged vessels and restore normal blood flow

Graphic removed due to copyright restrictions.

**What is the best material for a stent?**

# Identify the design parameters

- **Design visualization = Function (what does it do?) + Objective (what level of function?) + Constraints (how far can you go?)**
- **Express "best" design in terns of maximization of a performance parameter ("objective function"), p.**
- **In many cases, p is a product of functional requirements, $f_F$, geometric parameters, $f_G$, and material parameters, $f_M$.**

$$p = f_F \cdot f_G \cdot f_M$$

# Case 1. Design of compression rod (solid mechanics approach; no biology)

- Design a “strut” of solid circular cross section of length l capable of supporting a compressive force F without exceeding a specified fraction of its buckling load. Apply safety factor **SF = 2**.

$$\mathbf{F_c \leq P_{crit}/SF}$$

- Select material and cross section diameter, **d**.

# Related application

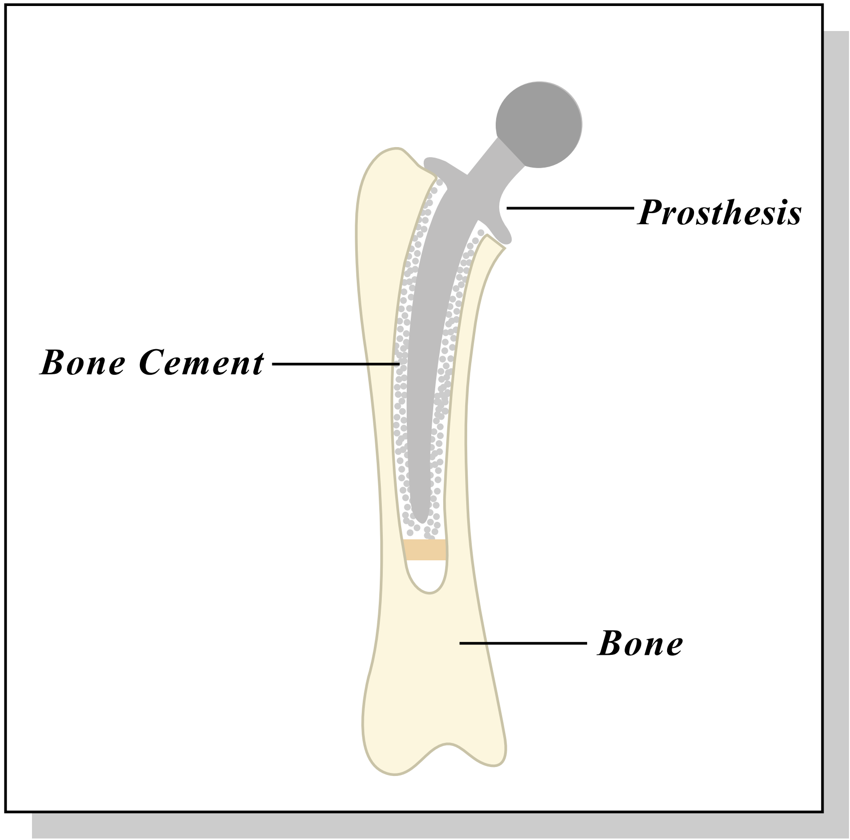


Figure by MIT OCW.

# Calculate the objective function

- Objective function (e.g., for some aerospace and biomedical applications): least mass, **m**.

$$\mathbf{m = \rho V = \rho AL = \rho(\pi d^2/4)L}$$

- Constraint: Euler column buckling load:

$$\mathbf{P_{crit} = C_1 EI/L^2 = C_1 E(\pi d^4/64)/L^2}$$

- Solve for the cross section:

$$\mathbf{d^2 \geq [SF \cdot F_c \cdot 64L^2/C_1 \cdot \pi \cdot E]^{1/2}}$$

$$\mathbf{m \geq 2[\pi \cdot SF \cdot F_c/C1]^{1/2} \cdot L_3 \cdot (E^{1/2}/r)^{-1}}$$

**m = function X geometry X material**

- For least mass, maximize Material Index, $\mathbf{M = E^{1/2}/\rho}$

# Search for maximum $\mathbf{E^{1/2}/\rho}$

| **Material** | **E, GPa** | **$\rho$, Mg/m$^3$** | **$E^{1/2}/\rho$, (GPa)$^{1/2}$m$^3$/Mg** |
|---|---|---|---|
| 1020 low-carbon steel | 210 | 7.83 | 1.85 |
| 2024-T4 Al | 72 | 2.79 | 3.04 |
| Titanium | 116 | 4.54 | 2.37 |
| Oak (parallel) | 12 | 0.6 | 5.77 |
| CFRP (lam.) | 40 | 1.4 | 4.52 |

Chart removed due to copyright restrictions.
Ashby diagram of Young's modulus vs. Density.

# Case 2. Light-weight tension member (solid mechanics approach; no biology)

- The "tie-rod" has specified length, **L**, and supports a tensile force, $\mathbf{F_t}$, while maintaining a safety factor, **SF > 1**, on tensile stress, $\boldsymbol{\sigma}$, in comparison to a material-dependent failure stress, $\boldsymbol{\sigma_f}$:

$$\boldsymbol{\sigma \leq \sigma_f / SF}$$

- Select design with least mass, m:

$$\mathbf{m = \rho V = \rho AL = \rho(F_t/\sigma)L}$$

Eventually:

$$\mathbf{m \geq (SF \cdot F_t)L(\sigma_f/\rho)}$$

where, the material index ("specific strength") that needs to be maximized is:

$$\mathbf{M = \sigma_f/\rho}$$

Identify materials having large values of M using the appropriate Ashby map.

Chart removed due to copyright restrictions.
Ashby diagram of Strength vs. Density.

# Introductory example from implant design

- Objective function: quality of regeneration of organ X (Notice neglect of alternative objective functions, e.g., benefit/risk, cost).
- $\mathbf{R_{100}}$ = describes “perfect” quality of regeneration, identical to that achieved during early fetal healing.
- $\mathbf{R/R_{100} = \varphi}$ = fractional quality of regeneration achieved. Needs to be maximized.

# Optimize scaffold design

- Regeneration achieved following use of scaffold with appropriate geometrical and biological properties that need to be optimized. Express **φ** in terms of scaffold properties:

  **φ = φ(geometry, chemical composition, pore size, pore volume fraction, duration)**

- Need to develop quantitative relations between **φ** and scaffold properties. Apply constraints. Identify scaffold.
- Continue design using other objective functions.