

MIT OpenCourseWare
<http://ocw.mit.edu>

2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

Massachusetts Institute of Technology
Department of Mechanical Engineering
2.830/6.780J Control of Manufacturing Processes
Quiz 1
March 16, 2006
Solution

Problem 1

Note that the overall problem with this approach is that it assumes that all four cavities follow the same pdf. While they are clearly related by a common source of plastic for each shot, it is most probable to expect that each cavity will have a unique distribution, if only with respect to the mean value, and secondarily wrt the cavity variation. Even with this flawed approach the two measurement approaches will give different results:

- a) 1 will track time dependent changes well, since measurements will be ordered in time. As soon as a change occurs, it will be reflected in the sample measurement. 2 will track changes poorly, since the newest parts are mixed with old parts, so any change is diluted. Neither, of course, will track within cycle (cavity to cavity) changes.
- b) By averaging over all cavities (method 1) or averaging across shots and cavities (method 2) we are going to compute C_p based on a single mean and variance. However, there are actually 4 means and 4 variances, or 4 non-coincident pdf's controlling this process. By using either measurement approach, we will get a variance that is higher than any of the individual variances and a mean value that is not representative of any one cavity. As a result, we are likely to get a C_p and C_{pk} that are larger than what the best cavity can produce. More importantly, we will most likely underestimate the number of bad parts made this way, since the worst cavity will have a higher probability of producing out of spec parts than if the process were truly from single pdf.
- c) Although the average of the four cavities is a normally distributed random variable, the samples are not taken from the same random variable, which is the underlying assumption of \bar{x} charts. Also, for method 1, the samples are not of independent random variables, since the four parts from the same shot are very likely to be related (correlated). For 2, the sampling strategy erases the time history of the process. For both, die position, which is an important assignable cause, has not been accounted for.
- d) Again, an assignable cause (location) has not been accounted for. In addition, the standard deviation will not take into account any variation along the length of the sheet.

A better method would be to chart each location separately, and take samples at fixed intervals along the length that are smaller than the size of any expected disturbance. These samples would then be formed into sequential subgroups of n and charted.

The original method would be insensitive to changes at two locations that cancel each other out, and would underestimate the variation along the sheet. The proposed method would be less sensitive to small changes in the mean along the sheet.

(An even better method would be to use multivariate analysis).

Problem 2

- a) Assuming a linear relationship between W_1 , W_2 and W_3 , W_2 is given by

$$W_2 = \frac{1}{3}W_1 + \frac{2}{3}W_3$$

Assuming that the diameters at W_1 and W_3 are independent, the expected mean and standard deviation of W_2 are

$$\bar{W}_2 = \frac{1}{3}\bar{W}_1 + \frac{2}{3}\bar{W}_3 = 5.033$$

$$\hat{\sigma}_2 = \sqrt{\left(\frac{1}{3}\hat{\sigma}_1\right)^2 + \left(\frac{2}{3}\hat{\sigma}_3\right)^2} = 0.00412$$

- b) First, define a new variable $\Delta = B - W_2$. The mean and standard deviation of Δ are

$$\bar{\Delta} = \bar{B} - \bar{W}_2 = 0.007$$

$$\hat{\sigma}_\Delta = \sqrt{(\sigma_B)^2 + (-\sigma_2)^2} = .00573$$

The probability that the bearings will fit with a clearance less than 0.012 is

$$P\{0 \leq \Delta \leq 0.012\} = P\{\Delta \leq 0.012\} - P\{\Delta \leq 0\}$$

$$= \Phi\left(\frac{0 - 0.007}{0.00573}\right) - \Phi\left(\frac{0.012 - 0.007}{0.00573}\right) = \Phi(0.87) - \Phi(-1.22) = \Phi(0.87) - (1 - \Phi(1.22))$$

$$= 0.80785 - 1 + 0.88777 = 0.69662 \approx 70\%$$

Conformance rate of 70% is not acceptable. The bearings are too small.

- c) For both W_1 and W_2 , the mean is above the target value.

$$C_{pk1} = \frac{USL - \bar{W}_1}{3\hat{\sigma}_1} = 1.11$$

$$C_{pk2} = \frac{USL - \bar{W}_2}{3\hat{\sigma}_2} = 1.58$$

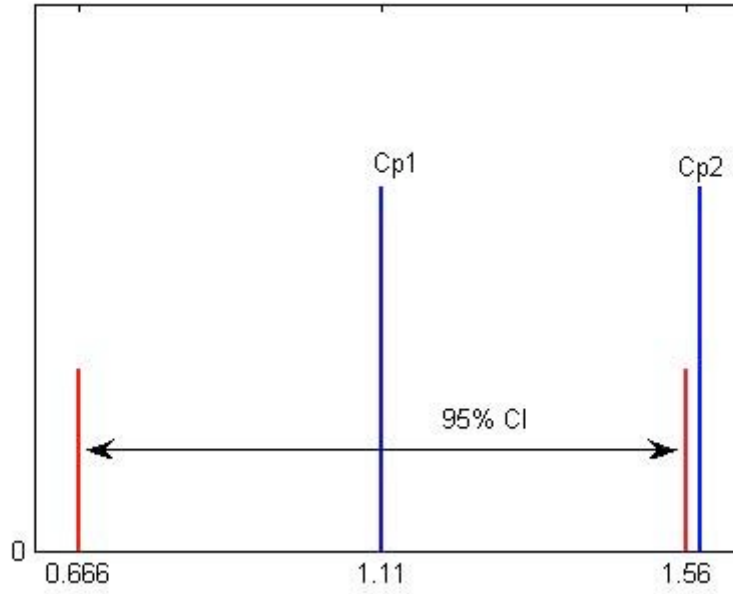
- d) The confidence interval is given by

$$\hat{C}_{pk} \left[1 - Z_{\alpha/2} \sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}} \right] \leq C_{pk} \leq \hat{C}_{pk} \left[1 + Z_{\alpha/2} \sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}} \right]$$

$$1.11 \left[1 - 1.96 \sqrt{\frac{1}{9 \cdot 15 \cdot 1.11^2} + \frac{1}{2(15-1)}} \right] \leq C_{pk} \leq \left[1 + 1.96 \sqrt{\frac{1}{9 \cdot 15 \cdot 1.11^2} + \frac{1}{2(15-1)}} \right]$$

$$0.666 \leq C_{pk} \leq 1.56$$

e) At 95% confidence, C_{pk} for W_2 is outside the possible range for C_{pk} for W_1 so it is very unlikely that they came from the same population.



+

PROBLEM #3 (35%) SPC USING RUNNING AVERAGES

The classic Shewhart Chart uses sample averages taken sequentially from the run data for the process. However, this means that if we have a sample size of n , the number of points plotted on the chart is $1/n$ times the number of runs, thereby delaying or masking some of the true process changes. One solution to this problem is to use a running average of the data where a sample size of n is still used, but the running average of n points is plotted, and updated as each new run data point is measured.

Assume that a running average SPC chart is to be constructed with $n=4$. If we define the run data as a sequence x_i and that running average as the variable RA_j :

a) Write an expression for calculating RA_j recursively as each new x_j is acquired.

$$RA = \frac{1}{n} \sum_{i=k-n}^k x_i \quad \text{where } k \text{ is the current time index, thus the recursive equivalent is:}$$

$RA_j = RA_{j-1} + \frac{x_j - x_{j-n}}{4}$ that is we add the most recent data and take out the oldest, all the while keeping $n=4$ data points in the average

b) What is the expected distribution function $p(RA)$? Please explain.

Since the statistic is still the average of 4 run points at any given time, it will have the same distribution as the \bar{x} data for $n=4$, i.e. a normal distribution (owing to the central limit theorem) with $\mu_{\bar{x}} = \mu_x$ and $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$

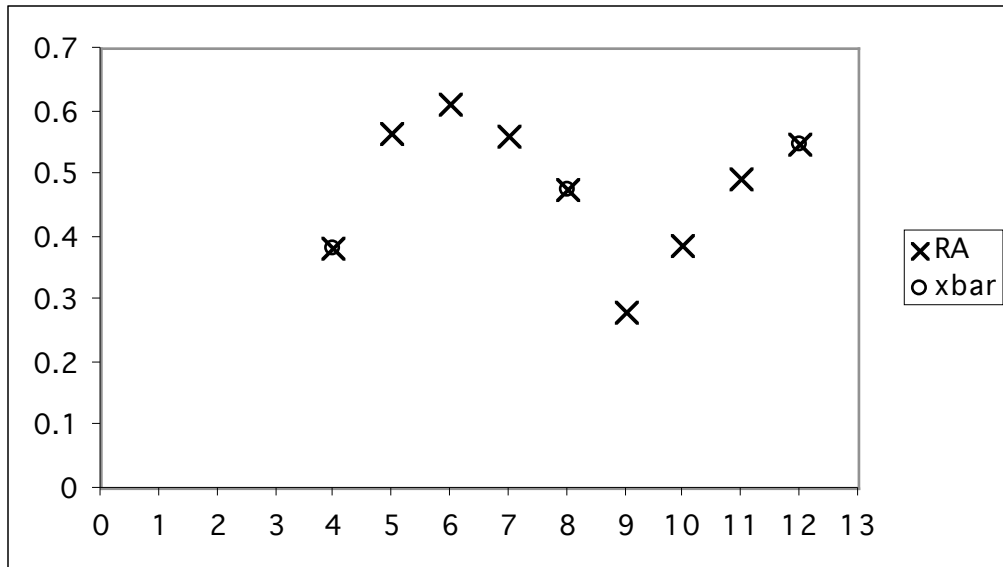
c) What are the control limits for the RA chart?

Since the distribution is the same, and we have the same variances, the control limits will be the same.

d) If you were to plot both the Shewhart \bar{x} data and RA for the same run data on the same plot, what would it look like? Please provide a sketch of at least 12 run data points to illustrate your answer.

The first observation is that the Shewhart chart will only have data at every n run points, while an RA is calculated as each new data point is recorded. However, The value of RA at each n^{th} data point will be identical to the Shewhart value (i.e. the running average = the sequential average each n data points)

On a chart this might look like:



- e) Now consider a sudden change in the mean value of the process: i.e., $\mu_x = \mu_x + \Delta$. How will the average run length (ARL) of the RA chart (and the subsequent time to detect the mean shift) compare to that of the conventional xbar chart?

The average run length is simply an expression of probability of a control limit being exceeded. This probability is determined by the distribution of the variable being plotted. Since we have already said that RA and xbar have the same distribution: $N(\mu_x, \sigma_{\bar{x}})$, then the probability of exceeding a control limit for a mean shift of Δ is the same for RA or for xbar. The difference is that RA plot a new point at each run, so while ARL may be the same for both, the time it takes for that run length is n time shorter for the RA statistic.