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GANG CHEN: So if I give a very quick summary of what we're talking in the first lecture, we reviewed first and second law. And of course, I assume that you all have seen in various courses. And this is the maximum thermodynamic efficiency of any heat engine operating in addition to the reservoirs. One is cold, the other is hot. So that's the key point.

And then for many of you, probably first time is, I introduced if you go to the statistical side, the microscopic picture, the one quantum mechanical state, you count how many particles, either electrons or molecules or photons or lattice vibration photons. And for electrons is given by Fermi-Dirac distribution, and where this μ is the chemical potential. That's the energy you need to take a particle in or out of the system.

And e is the energy of that quantum state and t is the temperature. And then say that's for electrons where the Pauli exclusion principle will state the maximum possibility of having an electron in accordance with this one. So this number is between 0 and 1. And then I introduced also the other type of particles, those-- the photon electromagnetic wave or lattice vibration we call [? photon ?] or even molecules.

This is the Poisson [INAUDIBLE] distribution where, again, this is the energy and μ is the chemical potential. But here we're not limited by the Pauli exclusion. So in one quantum state, this number can be larger than one. And if you take out this minus 1 plus one, you go to the classical regime where we call the distribution f , doesn't have that one, the exponential distribution, that's the board's mass statistics, horizontal distribution. This is the one you see in [INAUDIBLE] nor in chemical reaction.

So that's a review of thermodynamics. And then I went on to heat transfer where we have three modes of heat transfer. The first mode is the heat conduction. The [INAUDIBLE] tells us the heat flux is proportional to the thermal conductivity and the temperature gradient. But I went one step further to show you in a very simple kinetic picture, just count the number of molecules going from left to right and right to left.

Then we have a simple expression for thermal conductivity. That's $1/3$ c is the specific heat, v is the velocity, and γ is the mean free path. That's how a lot of the molecules travel. So this is the heat conduction mode. And then we have the Newton's law of cooling for convection or the e-transfer's proportional transfer coefficient area and counter difference.

And the difference is this heat transfer coefficient is no longer property of the material. Thermal conductivity is a material property. And h depends on the flow configuration, natural convection, forced convection, what kind of geometry? And then the third mode of heat transfer was the thermal radiation. And I gave the [INAUDIBLE] model, that's the maximum emission of an object in half in the--

So this is called the Stefan-Boltzmann law. It's proportional to [INAUDIBLE] fourth power. A real object, the emission is less than that. So we normalize the real object emission by this number, we get the so-called the emissivity. So it's a number between 0 and 1. So I moved on at the end of the last lecture, I was trying to show you the thermal radiation can be derived.

There are continual, and they give you a derivation of the Planck's law again. Now we move from continuum to a microscopic picture. And once we go to microscopic, I use the same strategy, I'm going to do the counting, the number of modes, quantum mechanical state, because I already know from statistics each quantum mechanical state, how many photon. That's the Poisson [INAUDIBLE] distribution, that's the previous slide here.

So I know each mode, this is a number at this temperature. That's the number of photons in the mode. And also I commented before for photon, photon equals 0. So we'll use that. And the task is really just count the number of modes. So I will take a cubit, and I assume that the [INAUDIBLE] is perfectly reflected. And this is a [INAUDIBLE] t.

And now I want to see how much is radiation energy is contained inside this cube by counting the modes. I want to show you this because we will actually use a lot of those common techniques towards electrons, towards the photons. So it's a general way of doing the counting. And of course, we have the electromagnetic wave. So here are our basic relationships. And, you know, the wavelength times frequency equals the speed of light.

So I write it rather than wavelengths, I write it in terms of wave vector is the numerous wavelengths and the angular frequency, so [INAUDIBLE] to near relation between the amplitude and the c of the wave vector. Wave vector is vector, so it's a k_x , k_y , k_z in Cartesian coordinate. And that's what we call the dispersion relation. The relation between wave vector and frequency is dispersion.

So now I want to see how many moles in this cavity. And then my simple picture is this is the length in the x direction. and because a perfect wall, so it's reflecting and it's a wave. So the only way that this wave can exist is having 0 amplitude at the end-- on the surface of the wall. So this is-- if you have a string and you want to do the, go to think about that, you play violin and you have the nodes at the two ends and the vibrational modes at the two ends, 0.

So we have different modes. The first mode is a half and the second mode is including the [INAUDIBLE]. And I can use this as my constraint in the x direction, that's the length and a half modes or twice or in [INAUDIBLE] and half mode, half wavelengths. So with that I go to, because if we factor in the x direction is just 2π over λ , so I can write this in terms of n and L of x, the length of the box.

And if I write that k_x , I can same way write k_y and k_z , so I have a lot more numbers for this k_x , k_y , k_z . And then each set of k_x , k_y , k_z , if you think about the wave, give me a distinct wave. And that's one mode, except that the electromagnetic wave you learned before, it's a transverse wave. The sound wave is a longitudinal wave, right. When I speak here, I'm pressing the air and the direction of propagation of the wave is in the same direction as the air doing expansion and contraction. So that's a longitudinal wave.

The transverse wave, electromagnetic wave from the sun, that's direction to the Earth, the k vector. But the electric field, the magnetic field is perpendicular to the direction of propagation. And I have two, so I put a two here. And then I sum up all the modes, so n number, each set of n number, n_x , n_y , and n_z is one mole. So I count how many moles there are inside this cavity. So I just sum up all possible combinations of and each combination of one mode. Each mode has a polarization, vibration direction. And then each mode, the photon has an energy, each and every photon has an energy, that's what we got. Yes?

AUDIENCE: I have a question. How does the factor two relate to the fact that the electric field is transverse to the wave vector?

GANG CHEN: So when I have a set of x and y and z , so in one case, the electric field is this direction. In the next case is this direction, or other is a combination of the two. So it's not a fundamental mode. So you have a factor of 2 is because in this one set of n_x , and n_y and z , you can have two possible e directions, electric field direction. So I do my summation here. And the only thing, this is the trick I will use many times. I do my summation because I don't know how to do it easily, analytically, so I'm converting that summation into integration.

So if I think about n at this [INAUDIBLE] and the wave vector is great, but if this separation between the energy is small enough, so I can say in this distance dk_x , and this is the unit of this discrete, so that's the equivalent that a continuous number of n rather than discrete number n . So this is the equivalent of summation into integration as long as this function doesn't change a lot between the [INAUDIBLE].

And I do that for y and for z , so this step is just for my future. What this step means, rather than doing this effect of two hear from 0 to infinite, which is the standing wave, OK, n is an integer, positive integer is a standing wave, I'm going to do from negative infinity to positive infinite because this is a unit function, And this way, I'm really rather than what I miss is this factor of two, I don't have a factor of two anymore.

And physically what I'm saying is the standing wave summation is the same as two propagating wave, [INAUDIBLE] propagating wave. When k_x is negative, it means going negative direction, positive, going positive direction. So this is mathematically equivalent. And later on, a lot of this minus infinity to positive infinite summation of the standing wave summation.

So it's a more mathematics. I can walk you through, but there's no physics here. It's a book doing the latter. Are you ready to hear the dk_x , dk_y , so I wrote it into Cartesian coordinate. Now it's not x , y , z , but k_x , k_y , k_z , it's the Cartesian coordinate. And three dimensional integration, the rest is just the numerator here I had before. Those numerator are l_x , l_y , l_z , so 8π cubic. So that's just a copying what I had before.

And x when we got f is [INAUDIBLE] copying. So I'm doing all the copying now to simplify again, simplify [INAUDIBLE] the integration, you can say I have a conflict, the conflict is y is integrating over k , but my function is a function of ω , so I have to transfer either y into ω into k or k into ω .

And I would like to transfer k into ω in this case, it's your choice actually, when you do it. So I transfer that k into ω , and I want to do spherical coordinate because that's easier. OK, so dk_x , dk_y , dk_z [INAUDIBLE] is $4\pi k^2 dk$ is spherical coordinate. And now you go to look at the k_x^2 , I hope I read them down in the previous slides.

OK, k_x^2 equals ω^2 divided by c^2 . So this is how I change k into ω . That's where the dispersion relation, the relation between ω and the k is very important. So k into ω , that's just ω equals ck . So dk is $d\omega/c$. And now I combine everything. I do per unit volume, how much energy we have. And I write all this other factors into the integration and this is what I have here. Everything is a mess.

And then this t , I call it density of states. What it means is how many quantum mechanical states or electromagnetic modes we have between-- around ω . And so this is the default count, energy, one photon, one state, how many photons we have. And in this state, in this region, how many states we have. So this is also the energy density.

Did I write everything together? It's just an energy density in the frequency. OK, so this is the energy density, as I wrote down. And if I want to write out the density of states, f , and the actual [INAUDIBLE] combined. That's Planck's law, Planck's blackbody radiation. The energy density per unit volume, and these were actually the start of quantum mechanics. Planck was trying to explain the blackbody radiation.

And if this is not called-- because the energy state, in fact, most time we don't talk about energy state, we talk about radiation leaving a surface, not just how many energy in per unit volume. And in that case, the definition is the intensity, energy flux per unit, solid angle. What is a solid angle? A solid angle is if you have a two dimension, you know, a polar angle, x , y .

From x , y , you want to use a circle polar coordinate, you have polar angle and in three dimension, that angle is a space, angle in space. That's a solid angle, per unit area normalized to the distance r squared. So a sphere, the area is a $4\pi r^2$. The solid angle then is 4π , because r^2 [INAUDIBLE].

In 2D, what's the ϕ ? 2π . 2π versus phase for 4π . That's the solid angle. So that's because that's the terminology we use often, so this intensity is just now, I know the energy density per unit volume, this energy is propagating at the speed of light, So is the energy density times c , and it's going radiation, thermal radiation is going in all directions.

So per unit, solid angle is a 4π . So this way, I write the speed of light times energy density divided by 4π . That's the intensity. And this is just so you go continue substitution, you get the intensity, and sometimes we do per unit wavelength interval rather than per unit frequency interval. So rather than doing $d\omega$ integration, you do $d\lambda$ integration, and because $\lambda = c/\omega$, so you convert the ω into λ .

Again, this is when you actually take a radiation course, this will be the Planck's law as written [INAUDIBLE]. And what are they? So that's a per unit solid angle. And when I do the total radiation per unit area, so this is the area, [? dense ?] area, so that's the total radiation is a π times i , that's the mathematical, we're not getting into it. With that, what I want to show you is from here, if you do your integration, you get Stefan-Boltzmann.

OK. Stefan-Boltzmann law, if you take a 2 double 5, is where we get-- we start on thermal radiation. And what I gave you is a way of going from basic concept to go to the Stefan-Boltzmann, but say, what's important is for you to look at the curve of blackbody radiation. This is very much relevant to the way we want to do solar cells or solar thermal, because the sun's radiation is about 50-- here I wrote 56 as hundred, but it's really around 5,800 Kelvin, that's the solar radiation.

It's like a blackbody, the spectrum is very similar to this curve. OK, outside the atmosphere, to go to measure the solar radiation, it's close to 5,800 kelvins curve. And at 300, I didn't plot, 300 Kelvin, this peak will be around 10. So here you can see the peak start decreasing and that relation is the wave's displacement mode. Wavelength times temperature is about, back of the envelope, three solid. If you don't remember that detail, whether it's 289 [INAUDIBLE] and in fact, every time I go back to check, I remember about 3,000.

OK, so 3,000 micron, and that's why our eyes are sensitive to the 0.5 micron, in the solar system. And the solar radiation, you put about 6,000, 3,000 divided by 6,000, our eyes are the evolution choose us to be more sensitive to the peak of solar radiation. And during night, there is no sun. Everybody is emitting, you're emitting, I'm emitting, we're emitting 10 microns. 300 kelvin [INAUDIBLE] 10 microns.

Of course, if you take a detector, you can look at it. So you are like that, the infrared radiation measurement. But if you look at just a common-- a solar cell, silicon solar cell, the wavelengths that you can take, the shortest wavelengths is about 1.12 micron. So 1.3 is about 1.112. So that's-- this is the 1 micron. This is the 1.12. So only radiation in this range can be utilized in solar cells.

You have a lot that's wasted if you do [INAUDIBLE] or yourself, this rest of the wave length's energy cannot be used to generate the electron holes. We'll talk about that later. So that's a completely what I was talking in the review, basically heat transfer and thermodynamics. What I'm going to move on is giving you a brief introduction of thermoelectric energy conversion.

In the last lecture, you have seen a basic demonstration of thermoelectric generation. And in fact, this is the module that used to generate the power. Pass around. Take a look. And basically here we are using direct energy conversion, we're using electrons or holes in semiconductor. We have names for positive electrons and holes. And we're using this as a working fluid to carry it around from one place to another. So let's look at the phenomena, basic thermoelectric phenomena. And then we will see do a simple analysis of that device and pass it around and then look at the current and the potential application.

First system I like the effect is not new. And in fact, this were about the same time people started to investigate the electricity. And Seebeck discovered that in 1821, that when they put the two metals actually at the junction, and the one side is hot, the other side is cold, we measure the voltage. Seebeck, now we call it Seebeck effect. And Seebeck himself, actually, you go to read the history, he didn't know what's going on. He thought that was a magnetic effect, because at the time there are a lot of interest in say, magnetic Faraday or the experiment. So he saw thought that was a magnetic effect.

And about 10 years later, Peltier discovered another effect, that's the same two, say kind of junction, two metals, was pass the current through it. Actually, one side cools down, the other side heats up. He could make ice at the time. In fact, the Seebeck, if we choose this material to make a device, even at that time, a power generation device, you will get about a 2% to 3% efficiency using this material.

Somebody do the power generator, so at the same time, people were starting to develop electric motors. You can see the history was really interesting. This one actually worked out. First, there will be a lot of study, and it will be different, right. Let's say, this effect is reversible. You just pass the current first, one is one direction. It will be cooling, and the other direction you will be heating.

And so both those were experimental discoveries, comes in, gives us the theoretical prediction. And Thomson prediction is that based on the experimental discovery and understanding of the effects, he predicted that when you have a material, one side is cold and the other side of the hot. So a material under temperature difference and you pass current through it, there will be heating or cooling along the material.

So this is not char heating. Char heating is always heating. This is a reversible. It will be either heating or cooling, depending on the direction of the current. as of Thomson's, Thomson is also Law of Kelvin, and you'll probably see a lot of Kelvin work, Kelvin relation for this, Kelvin relation for that. It pops in more than 500 papers, very productive in this lifetime. So now let's get a very intuitive picture. We have to go to, for those understanding, we have to go to microscopic again and we now have to look at electrons.

Let's look at the electrons and I'm going first to just look at the isothermal, there is no temperature difference in the material. Isothermal conductor, a lot of electrons inside, and that's why you have to feel the current will flow. We all know that. So I have a current flow. Now again, I'm doing number crunching of this number. What's the current density? Current density is amp per meter squared is the charge per electron.

Hence, the per unit volume or how many electrons I have times the velocity of those electrons moving. It's a very intuitive. So you go to look at the unit, you'll be adding per meter squared. And the velocity is related. I'll give you the relation more later on through a electrical field. And this, the coefficient proportionality, coefficient is called the mobility μ , relating the velocity of the electron to the electric field. Again, this I'll give more detail derivation. And of course that will be the ohm's law, the current is proportional to the electric field and the proportionality constant is the electric conductivity.

And sometimes we don't talk about conductivity, we talk about resistivity, just 1 over the conductivity. So this is the ohm's law, and all the definitions, their units are given here. So this is when we talk about the electrical flow, we care about current. But each charge also carry heat with it. I can do the same thing for heat,

So rather than the charge, electron charge, I'm going to say the heat carried by each carrier. I don't define how much or how I write that heat yet. I'm just saying per carrier, h is the heat carried by each charge. Again number, density, velocity, they're moving. And of course I can replace the n by j the electric current. So now the heat current is proportional to electric current and the proportionality constant is the property of efficient.

So associated with the current flow, there is a heat flow. So Peltier coefficient can be defined for each individual material. You don't have to have a junction. The junction is only when you do experiment, how you observe it. OK, so this is the-- when I want to observe the Peltier effect, the best way is to put a junction there. I have two different materials, and when I pass a current through, then you can say the current has to be-- electric, current has to be continuous.

That's a [INAUDIBLE]. You have [INAUDIBLE] in and of course out. So that's the continuity, current continuity. But the heat carried by the electrons in 1 or 2 of the different from the other, so if I do my first low energy balance, I find out if I have more heat flow out, then come see it, I have the supply with me. So what does this mean, supply the heat. The lattice cools down. The atoms cool down, because I'm talking about electron,

The atom can supply the heat to the electron cool down the junction. So this cool down, and on the other side, you have to reject that heat, energy values. So this is the Peltier phenomenon, you observe experimentally. And you can say, if I reverse current, this phenomenon is reverse. So this is the Peltier effect. It's reversible.

Now look at the Seebeck effect. There's a seat here. OK. Seebeck effect, Seebeck effect is the voltage generation under a temperature difference. And you think about-- we have charge in the conductor, and then when one side is the hot, the electrons are near the hot side, have more energy they move more violently. So they will diffuse to the cosine.

So you have at the end after diffusion, you have more electrons here. When you have more electrons, the electron imbalance will happen, you have electrical field building, The internal electric field will resist the diffusion. So at the end you have a steady state for the diffusion current balance, the electric field will drive the current back. So this balance will give you the Seebeck voltage. That's the process here.

OK. The Seebeck voltage is defined, I put the [INAUDIBLE] sign. This is when you do an experiment, you want the one side is hot, the other side is cold. You put a probe, normally you say, OK, the definition, I put a hot probe on the high temperature side, positive voltage on the high temperature side, negative voltage on low temperature side. And if you do that, here is written very clearly, you should have put a negative sign in front, that's the definition.

I remember the first time I measured a sample. I just got a wrong sign, I didn't know why. Everybody measured differently, because people normally just say Seebeck voltage is the voltage divided by ΔT , but then when you do experiment, you have to see which side it falls.

Thomson effect. Thomson effect is when you have a temperature difference and you flow a current through and if you look at every point, there is a Peltier coefficient. The Peltier coefficient can depend on temperature. So that means when current flow in, the energy carried in could be different from the energy carried out. So the heat carried out could be different from current in. And then that difference must be made up, cooling in by either cool or rejected. So that's the Thomson effect and Thomson to efficient is defined the low amount of heat absorbed or rejected versus the gradient divided by temperature gradient normalized to current. [INAUDIBLE] comes into it.

OK, so we have this phenomenological explanation. And now I want to do more detail. I want to put this in more-- into more mathematics into it. I forgot to mention here, Kelvin, of course, the concept of Kelvin realized, based on his understanding, those three effects are interrelated, and he actually gives the relation between the Peltier coefficient and Seebeck temperature, and constant coefficient is a Seebeck derivative with respect to temperature.

OK. Those are [INAUDIBLE] dependent properties. Like I said, let's go the-- so let's go listen to the device, and we want to say, based on this, what are the material properties I want? So I show this device. And again, you can buy the commercially, particularly some of you may be interested in various competitions, MIT has a different competitions, and if you want to build this device, it's not expensive. This is a \$10 to \$20 commercially. you Can buy one.

And the again, the basic concept, now even cheaper device, you see a lot of legs, they are electric. There are only two electrodes. One is in, the other is out. So electrically, they are connected in series. And in fact, most of this device are made of opposite type, p and n together, a P type semiconductor, N type semiconductor.

And in the N type is electron carry the current flow and P type is called the positive charge. So when I pass a current, of course current flow against the electron, along the positive charge direction, so you can say that's in terms of device how it works. They carry heat from one side to the other. And on this side, they reject the heat. On the other side, they cool down. So you can do the refrigeration as well as power production. We show the power, here is a simple demo of the cooling. That's the price, you convert the water into ice.

OK, let's look at how the device performance depends on-- we show the summary, let's say three thermoelectric effect, but in reality, the solid state material, so there are other effects goes on, when I have 100 difference. And the first one is, of course heat conduction. On one side is hot, the other is cold, heat will flow. This heat could be heat conduction, electron can conduct heat, lattice also conduct heat. So you have both electron and lattice.

This is different than just the Peltier coefficient or Seebeck coefficient. So we have the [INAUDIBLE] law. And if it's a 1b, I integrate it, I get the amount of heat transfer is proportional to the cross section area, thermal conductivity and the temperature difference divided by the length. You see this in [? 2005. ?]

And if I group all those parameters together, AKL, I say this is the thermal conductance. We probably define thermal resistance before, but the resistance is just conductance is just the inverse of the resistance. So this is the definition of the thermal conductance in the KA or I. And just as a comment, very often we write it just look like math. But when you actually do it, it matters a lot because here, for example, the heat flow is inversely proportional to thickness,

So in the technology development, if I use a film, I is of the order micron. And if I use a bulk material, I could be probably 100 microns or millimeters, and that heat flow is a 0 order of magnitude difference for the same delta t. So whether, in reality you have different problems, sometimes it's clearly space, like the solar, solar radiation clearly is fixed. And you want to build your system to create a delta t, because it's a thermoelectric device is proportional to delta t. So that's one type of problem.

There are other cases where delta t is fixed. Give me give me an example. There are people looking to ocean geology generation. In the ocean, there is the top and bottom [INAUDIBLE] difference. You've got the different problems and you can go to back to the most basic picture to think about your system, sometimes that's always the first step. OK.

Now let's look at the device. Again, the [INAUDIBLE] device has many repeat units of this t and n. And if I have just an ideal device, how much cooling I can create. I pass a current and the cooling is the difference and the junction of the two materials. So one possible material multiplication times a negative material coefficient times current. So that's how much cooling I can create.

But in real situation, what happens is that when this site is cold, this site is hot, the heat will flow back. This is solid material. You can't get rid of that heat conduction. So it will diffuse back by conduction. That's one reality. That's a long idea factor. The other non-idea factor is when you pass the current through, I have [INAUDIBLE]. I didn't talk about [INAUDIBLE] here, right? That's unavoidable part. I don't have a superconductor. In fact, you don't want to use a superconductor here.

Superconductor, electrical superconductor is only one quantum mechanical state. They introduce 0, doesn't carry heat. Electrons do not carry heat, in case, superconductor. OK, so there's a [INAUDIBLE] because the electric conductivity. And I have to consider both, reverse heat conduction and [INAUDIBLE]. Now, I think about [INAUDIBLE], the difference between conduction is this side is this side is called [INAUDIBLE] conduction.

The [INAUDIBLE] is uniformly generated along the leg as approximation. So in that case, [INAUDIBLE] about half go this way. The other half goes the other way. You can solve the equation and you'll find that the answer. But that's the intuitive argument also. So in that case, the real net cooling I have is, in addition to calculate, I have half of the [INAUDIBLE]. This is electrical resistance. And I have a reverse heat conduction that the thermal conductance of the two legs.

So that's the mathematics. The resistance, the electrical resistance is proportional to the resistivity of both legs area, that's and here are thermal conductance, they both p and n combine together. And now you can see the amount of heating or cooling I create depends on the current flow, right. $I^2 R$, y is a linear, the other is square.

If I drive the device too hard, I generate too much heat. I drive too little, doesn't have enough cooling. So there is an optimum. When you drive the device, there's an optimum there. It's easy to find the optimum. Firstly, I want to find out more not only for cooling but also for efficiency. So I have to calculate how much electrical energy I give in. You say, $i^2 R$. It's not that simple. There's a little trick here.

Because there is a $i^2 R$, so voltage drop is i times R , that's the [INAUDIBLE] part. But when there is a temperature difference, there is also a Seebeck voltage. So when this side, one side is cold, the other side is hot, there is an additional voltage you have to overcome, the internal review voltage, Seebeck voltage. That's the Seebeck coefficient difference times ΔT . That's the voltage you have to overcome in addition to resistance. So the electrical power is i times v . So that's the electrical I put in.

So the efficiency, according to our definition, for refrigeration, we talk about coefficient of performance, We don't talk about efficiency. So this is what I want, the cooling, this is what I pay. The pay is the power, the power is the current times the voltage. So I have $i^2 R$, that's the [INAUDIBLE]. I have to overcome the Seebeck voltage. So that's the term there. So that's the bottom part. That's what I pay. And the numerator is what do we derive, the previous slide. Including is accounted for the reverse heat conduction and the [INAUDIBLE]. So now you go to optimize it. And this-- yes?

AUDIENCE: Shouldn't Seebeck dominant be reversed? $i^2 R$.

GANG CHEN: No, you wish that's the case. You wish that's the case. But it's-- you are-- when you have a ΔT , you have to push hard. You can say, OK, we'll have a hard flow for cooling. This is a t , this is n . I know [INAUDIBLE] this way. But this side is a hot, the voltage is against here. Otherwise, you get too easy, your problem. So you go to optimize your current, that's the arithmetic, you can divide e_i for 0 and you find the optimal current, and then find the optimum, under optimal part you got to maximize efficiency or CLP.

Of course, I dropped all the math. And let me again make a comment. You go to save your [INAUDIBLE], that's easy because there's no denominator here. You got the optimal current for maximum cooling power. Here you get an optimal current for maximum efficiency, but they two do not agree with each other. And in fact, in your operation, you want to adjust your current just between you can't satisfy, you go optimal current and you don't have much cooling power.

The other point is, you look at this, you say, it's a linear term. This term is all thermal energy. That's a [INAUDIBLE] cycle for refrigeration. We wrote down [INAUDIBLE] for power generation. But this is a column so it's a column factor, it's a heat engine. And of course, you all make sure this term is less than 1. So you can't get column, and here came to this vector z . So every time people see, what's your zT ? This is where it came from, z times temperature. And this t is the average temperature between the hot and cold side.

And this is z , again, this is all coming from this mathematical derivative. And it's the Seebeck difference of the material. And the denominator is the conductance, thermal conductance times the electrical resistance. So look at it. It's not a material property at all, it'll equal the material property, Seebeck coefficient, electrical resistivity, thermal conductivity, but it also depends on the cross section area.

And they look at this function, you want to maximize your z to get a higher efficiency. So the first thing is that you go to maximize your z by geometry. You can reduce the denominator by changing your l or a [INAUDIBLE], right? That's the least you can do if you don't work in the material. And that's what you want to minimize. And if you minimize it, it turns out this is what you have to do. The k times r is the minimum when they do lens cross-section area of the two legs. So you are engineer. This is where when you do the device, you optimize it.

We then-- when we do that testing, own lab, sometimes just hand polishing, all this, trying to make it match to each other. So you want to get a better performance than others. OK, now you that your best case is an idea, that still is not a z . That's still not the individual material property. It depends on both sides, here and the end materials. So basically, if I want to make a good device, I need a good n , a good p , and the two should match each other.

One material you get, you do get the best performance, And you get about half of the performance if you have good one material. But because at the end, using the two means you have to make a lot of choices. In materials research, people just talk about one. You engineer, you know, you have to go to get both materials. Somebody has one. You say, do you have another? You're eager to us. We'll find others who has the other.

So this is when material scientists just say, I discovered material with high z , That's s for sigma or k . You want π , a material with a high electric conductivity. And so you want to minimize [INAUDIBLE]. You want a material with a high Seebeck coefficient. Seebeck coefficient fundamentally is the heat carried by per charge. OK, we'll talk more on that later. So average speaking per charge, how much heat it carrier.

And then you want a thermal insulator, a low thermal conductivity because you don't want to heat reverse back. If you know any materials, let me know. People have been searching this for centuries, good material with a good electrical thermal conductivity, poor thermal conductivity. But also you want every electron carry a lot of heat. And this has to happen in one material, [INAUDIBLE] material.

AUDIENCE: So we derived the Z_{max} for a junction.

GANG CHEN: This is the only-- Z_{max} . You need a computer device in these two materials, Well, theoretically, you don't need two material, you can do one material. In fact, in that case, what people choose, the other material, they choose a superconductor. Just don't do anything. So that's how they characterize the one mature if you want to do a good job. All right, let's see, this one is immature research, you don't-- you can't wait to get too mature and positive,

You get one but mature taking many years, so you do a simple material to say this is a measure of how good is this material. But as an engineer, you want to build a device, you have to say, where is the other material? I mean, I need both materials. So those are the comments that is made to optimize [INAUDIBLE] optimize index.

And there is two types. And then you optimize the device performance, load matching by current. This is a [INAUDIBLE]. So here is some typical [INAUDIBLE] for the business, that's the commercial device mostly you buy. You can only buy this type of material based device. Seebeck [INAUDIBLE] is about 220 microwatts per kelvin. Electric [INAUDIBLE] fence is around 1.51 [INAUDIBLE]. This is all Si units. It's good to keep in mind, example of a micro [INAUDIBLE] including the [INAUDIBLE] zte is about 1.

And we'll talk how good is that. Now, what's the problem? If you have all this library, you say, OK, this is-- I think, a [INAUDIBLE] 2 millimeter long electrical interface, cross-section, one millimeter square. So it's a small, if you look at the device that pass around, that's dimensioned roughly, each leg is only 20 million ohms, 20 million ohms, very small. You go to buy resistors, actually very hard to buy this small resistor.

What it means? It means if you want to build a single device or two legs, because current is a voltage divided by resistance, It puts your [? heart, ?] huge [? heart, ?] you need a big power supply. So you go to look at you in your lab, have a big power supply, a small driver, small device. And at the end, that means cost, so everything you have to consider. And that's why some people do micro devices. You can actually shrink your area,

You do a smaller cross-section area where you put a lot of those together, in that case, you get a higher resistance by putting more of the [INAUDIBLE] together. So in the device, I showed the cost around, this is about 100. So if you multiply, 100 is only two ohms, the total resistance if you own this sort of device. So it's a small resistance, current device.

So that's the reason to put them in series is putting getting higher resistance so that they are supplied easier. Let's say it's still pretty large current. And then you put them ceremony in parallel, because the parity from one side to the other, electrically [INAUDIBLE].

AUDIENCE: When you optimize your system, then you really get that simple number, 1, 1, 2.

GANG CHEN: Simple number of 1, 1?

AUDIENCE: [? 100 ?] millimeter by [? 200 ?] millimeter.

GANG CHEN: Oh, you mean-- no, no, no, no, no. This you can charge. You can make a long leg to test it. You will still get the same delta t.

AUDIENCE: But when you optimize the system?

GANG CHEN: Optimize means also material cost, The? Real world is not just you have to match to your application. You don't have to fix to the geometry I pass around. And that geometry is in commercial production because in fact, each application may be different, but the heart of the market is too small, so people are just selling more standard modules. Well, anything you go along a standard, you have to talk to the manufacturer and see whether they can manufacture.

OK, so this is the example, this we built before. This is actually measuring two legs. There's a lot more difficult than measuring device, because any time you put a thermocouple there, you have a feed loss. Feed loss is your performance may diminish. If you want to say, what's your maximum delta t or that, so it's a lot of time brewing and doing the experiment, you have your leg is mediums. So anytime you have missing [INAUDIBLE] the industry, they do much more, much better.

I look at a production. I was really amazed at what I'm seeing the worker can do, versus what we can do in the lab. That's a different environment. But this is the $1p1n$, and this is a measurement curve. You can see it's a large term. In this case, we go one fatigue along the line through that device. By [? line ?] absent, the use of aggregate 1 million or 2. So these were-- What's amazing with that small thing, on this side is 100 degree, the other side is 0.

So you can derive a lot of the other things you have. The efficiency is not very good. OK. The material is dependent-- mentioned this before. An the analysis I gave you is for constant temperature. So if you do real device design, you have to go solve differential equations and match the property, consider all the temperature dependent properties, and that I'm not going to cover in depth. Anybody wants to do that, you can just do the energy balance in differential form.

And this is the heat conduction term. This is the constant term. This is [INAUDIBLE]. So that's a differential equation locally. And very often people use differential analysis to incorporate companies. And then the boundary has a plural term and the positive term. So you have a boundary condition and a differential equation. And so, again, this is a [INAUDIBLE].

Y'all know what. I want to talk in details. And now let's look at where those technologies are. I have the expression for CLT and you do the same analysis about power. And you get the efficiency for power generation. And we [INAUDIBLE] power generation and getting ZR inside, and this factor is slightly different from CLT, but you go to check, it's probably your first one. And this is a part of that,

Again, it's a thermal engine. The [? power ?] pans to whatever your material. You can try to [? power, ?] you went big on this, you have 5gt, you can go to [INAUDIBLE]. And so this is not-- so I would say, not so accurate. I mean, it's like around 95 to 96. And this is the accurate ratio of hot to cold in Kelvin. And this is the efficiency. And what I put down here is the thermal activities at the bottom.

OK, there's another [INAUDIBLE] picture. First, have to do a lot of research, And the power plant is about 40%. The normal coal fired power plant is 40%, Internal combustion engine is about 20%, 25%. OK. What I need now for the [INAUDIBLE] is [INAUDIBLE]. Depends on amount of silicon, about 6%, single crystal silicon about 18%

Once you think about that, then your picture becomes a better. You can, in fact see the [INAUDIBLE] use a thermoelectric power generator and that's about 6% to 7% in the deep space missions. And now in the lab, there are people developing similar generator using a zt bottle, one material that is around 40%, copper, 40%, uranium 2 level of that's actually very attractive [INAUDIBLE]. So it really is an application because you don't-- at this stage, we're not thinking you can't it's not at that level to say I'm going to replace major form over in the sense of like, we're not dealing here with an internal combustion engine, but there are many different sources where you can potentially [INAUDIBLE].

And there is refrigeration on this before. and CLT, again, depends on temperature. And a CLT of thermal electric compared to fossil refrigerator is about a factor of 5. So that's actually probably the oddest really want to say the application, say really replacing the screwdriver, I think probably took a long time.

But again, there may be other applications where you can think about [INAUDIBLE]. And in fact the current market, and I'll show later is mostly [INAUDIBLE]. So this is where really the engineers, you can really say, I know the technology is where it makes sense. There are different areas that it could make sense, and commercially, mostly standard size, standard size people offer different sides, but the geometry is very similar.

The height could be in the case of our generator for about 5, 6 millimeters, and the coolers, the typical millimeter in height in the neck, and the top is a ceramic and copper bottomed ceramic. And [? the head, ?] the next the next are interconnected by copper. So the technology wise, they have to put a diffusion barrier between the copper and material, because copper easily get into the material that composes in the [INAUDIBLE].

In terms of applications, this has one standard small cooler, this is because when you reduce the compressor, the compressor actually, the smaller you make, the hotter-- the hotter you make it. When you make a very small compressor, of course, the northern part is [INAUDIBLE] and this becomes a more attractive, so this becomes more attractive.

A lot of the dormitory [INAUDIBLE] refrigerators made for somewhere like this. And still the efficiency, I think, is still not as competitive, but the cost could be very attractive. And then it can be as small as just a little vial. And the people, in fact, that put a lot of multi-stage to create a lot of-- each stage is a maybe-- this to about six feet together, you can get a commercially at least 50, [INAUDIBLE] pounds.

So the infrared detector were also needed to be cool with [INAUDIBLE]. But the main laser diode has a somewhat like a cooler. This is because the wavelengths in communication, telecommunication, the wavelengths of this laser are [INAUDIBLE]. So they actually use some energy to stabilize the temperature, not much for the [? human, ?] but for the [INAUDIBLE] macro DNA, we have to compare the initial [INAUDIBLE] naturalization and recycling.

And probably a really good commercial example is this company for the American. And they started putting a device, this is where you have to look at where it makes sense, They started to manufacture around 2,000, somewhere like six, and in some high automobiles. Now they have a market, about 3 million models, include the car seats. I still have this in my car. It's a luxury item and but pretty well successful.

And now the DOE is trying to even get rid of the main air conditioner. And the idea is that the main air conditioning automobile will require about 6, 7 kilowatt. And the purpose is to turn the engine on, the air conditioning on. You want to cool the whole compartment in about 5 minutes, but normally you do need a high compression, large compressor.

So the question is whether they can use this distributed cooler into the automobile. I'm not sure whether it will be successful, but there [INAUDIBLE]. And in fact, this is a-- when you think about where they make sense. They find the seats, not your offices. Your offices, you move around. Once you get the car, it's there. They plug in the power because this means the power there. Right? So any other places it could be using that--

AUDIENCE: Movie theater

GANG CHEN: Since that's probably too big. I was thinking about the bed. I say there are actual products there. You go, I was thinking before, I looked at the wrong products already. I guess the key there is whether it's a physiologically comfortable and also how you reject that. If you reject it to the same rule, does it make sense at the end? So there are-- those are all the places where if you're an engineer, you got to think about where it makes sense and where you have a product people accept.

And with the more product into the market, of course there's a material is also improving that you have potential, further advancing the technology. Power generation, the main driver in the past is space missions. So in once you go beyond the Mars, the solar radiation diminishes. So in Mars, most of the power still come from the solar panels, although there are some electric cooler or thermoelectric generator.

And sometimes those generator proposal was actually to keep the electronics warm, amazingly, in you go to say beyond in the-- it's very cold in the night. There's no power. You need the electrical power to heat up the electronics. So the space mission, most of the deep space mission are thermoelectric power. And that's a good news because Galileo, for example, send back signal for more than 20, close to 30 years.

Galileo, eventually the signal was not lost a lot because the power generator, just because of the mission was abandoned. Voyager, all those were based on thermoelectric generator. And in there, they use a nuclear force. So actually, I went to one deep space conference in our pocket. That's interesting, in the conference outside, there are people, space missions. So there are outside people protesting, because it's the most of the space missions are nuclear power.

OK. A terrestrial application. This is a standalone generator where, for example, this company, global thermoelectric they become good business because they sell those power generators along to big oil companies. The oil company needs to monitor their pipelines. The pipelines, let's say you go through Alaska. There's no electric power there. So they actually use the oil to generate the heat and the power generation.

Body power watch. OK. I don't think it's very commercially successful. The company first company trying to market it was a Japan company, Seiko. I went to visit them at the time, and they said \$3,000 is the modern power watch. I for one. I say it's amazing that the watch only 1 microwatt, 1 microwatt, that's what I learned from the [INAUDIBLE]. The temperature difference is 1 degree Kelvin, a degree Celsius to develop for this watch.

And there was a lot of companies who see, [? Carlheim ?] I think I didn't go to check her watch, but I heard now they are still on the market. They are selling \$300. This one, went to-- I visited and they say, well, what's in the Smithsonian? So today it's free. But I think [? Carlheim ?] it's is still about \$300. I don't have much. I don't need a watch.

But some people are developing body power for, you think about the low power electronics. Some of these devices don't really require a lot of people. At the other day, I met an MIT professor in the department, I know [INAUDIBLE] student there. He has-- he said that he can generate 300 microwatts, which is pretty good. And I saw Philips trying to also develop it, the body power.

And in fact, it might be worth the calculation to say, I think Philips is trying to develop a few, where you put the body the delta t right. And one of the prototype I saw is more like around the head. Maybe it's good if you generate enough power for the poor kids to read like LED, now, how much power LED is. It's interesting to check.

This we were actually talked a lot recently, not because this will create a lot of [INAUDIBLE] in the thermoelectric generator, but this will be a technology that will really make the full area of people [? burn. ?] Because look, a lot of places, people are still cooking using wood. I know where I grew up, say not my place now, but when I grew up, I used the wood.

And in this wood, if you ever use a wood cooking, you will know that the smoke doesn't burn efficient And what the [? Phillip ?] develop is a fan generated power from the temperature difference of the burning and then drive the burning process clean. So it's a clean, clean wood-burning, wood-burner. And now they-- I heard that they're trying to market, but they still, this is about 10 to 20 bucks and people still couldn't afford it and \$20.

And so their market is now first person to get to world health organization by giving the people free. So there are a lot of-- this one, if we can gather, all the people use it, it's a tremendous, their analysis actually of CO2 problem right, because about 2 billion people are out of Greek in the world. A lot of people burn this.

And if you make those who still efficient, reduce a lot significantly CO2 consumption. So the point is, at this stage, you have to think about where it can make sense technologically. Well, at the same time, of course, none of you are interested in developing more efficient, better materials.

So I go to a lot of material conference, several industry conference. And one sense in the past, the last conference was very different. But in the past, you see more than 90% of the talk are materials, and not many talk on systems. So this is where I think the system, the engineering actually can make a good contribution there, because a lot of times people just say, I have the [INAUDIBLE] make a difference. But ZR is only half of the story.

Because if you look at particular system, right, where the cost will come from, most likely it's not in the generator. It's an important part, but it will not be the dominant one. So this is an object to be cooled. The difference of similar electrics from solar is a solar cell, you put it in the sun, that's your system. You manufacture out. You just need a frame and put their electrical and the rest is-- it's about half the cost or balance of system, the inverters or those other stuff.

But there's a much simpler system. Here for thermal system, you have to establish, you have to be able to maintain the temperature difference, so that means you have to take the heat out or put the heat in. And I once did a calculation for system for. And what I found is the electric, thermal electric material is only about a few cents per watt, electrical. But the cost come from all the aluminum that, I didn't do a very good thermal design, but that cost us a few dollars per watt because how you maintain the DC and do the cooling could be easily dominant in your cost. Not always, you have to do a good design.

And the other is if you think about the temperature, so you need, for example, household temperature is about 22. Think about your refrigerator. And inside the refrigeration, refrigerator, let's suppose you want a 0 degree. So you say, I need a 22 degree solar heat. But that means you [INAUDIBLE] 22 is your refrigeration compartment inside, your air outside. Now you have to transfer the heat to the cold side. So this cold side must be lower temperature than 24,

So let's suppose the 15 degrees, I give you degree. And then the far side, I rejected him. You don't want [INAUDIBLE] reject has to go through the error. So I have to have another theology from here to here. I give another 5 degrees. OK, that's 12 plus 22. So I need 39 degrees rather than 22 degrees.

Now you go to 39, COP versus 22 COP will be very different. So your system is really a very critical part of the consideration. That's the significant difference of those thermal systems, it could be more expensive than the simulated side. Sensor, if you think about [INAUDIBLE], right? [INAUDIBLE], let's suppose you want-- this is a hot gas, and you put a similar [INAUDIBLE] exhaust and then you cool this side.

And you see what temperature this system will offer. Then you suppose the device operate at a uniform temperature, not a good design. OK let's say, I suppose, the device offering uniform temperature, then the [INAUDIBLE] gas must have a higher temperature at exist. This temperature of the gas must be higher than this, otherwise heat cannot go from gas to the hot side.

OK. Heat only go from high temperature to lower temperature. So that means if the gas is 600 degree, the amount of heat I can take out is from 600 degree to whatever this happened. So if you do an optimization, it's about half, because you need to take the heat out of the gas, put into the device. So that means your [? half-side ?] operator is 300.

So you can't do your calculations, 600 degrees, t what's my [INAUDIBLE]? What's my efficiency? And of course, you can be more innovative. You say, OK, I don't operate at the same time, but actually I operate at a higher temperature and my efficiency system [INAUDIBLE]. So there you got a lot of system design that depends on material availability and cost, weight, particularly when you do automobile.

And with that cost, and so a lot of really-- so what I say is there's space for both sides. There are a lot of problems. Engineers can make an important contribution and a lot of problems with the material, [INAUDIBLE] improvement of chemical conditions. You look at-- say this is not a serious work on the other side, because you look at where the potential to use it, but maybe car generation most people not working [INAUDIBLE] because it is too small.

And the reality is automobile company says 3% fuel saving, you'll be happy to think about it. 3% fuel is less than 1% system. So if we are exhausted, one third of the wasted energy there, one third, let's take one half of the heat, if I can intercept the half of the heat. So that's one sixth. So my system efficiency, if it's 6%, which you usually can be done with a [INAUDIBLE] not system device, 6%.

But then the question is, when you put all this together, your [? half-side ?] your co-side is really the temperature, you know, if you internalize naturally. So you have to think all this, put them together and make you have the impact. Yeah. So the [? BMW ?] is a very aggressive things, but if you put in a market when [INAUDIBLE] is before and after [INAUDIBLE]. It's really up to you to find that where it's not competitive against-- I say not competitive against main [INAUDIBLE]. Let's say there are a lot of places where you can be [INAUDIBLE]. Any questions? [INAUDIBLE]