1. Eisenberg & Crothers #24

In this problem we are trying to find the chemical potential difference of sucrose at 0.01M and 0.0001M in and ideal solution.

$$\mu = \mu_o + RT \ln C$$

$$\therefore \Delta\mu = RT \ln\left(\frac{C_2}{C_1}\right)$$

$$\Delta\mu = (8.314 \text{ }^{J}/_{molK})(300K) \ln\left(\frac{0.0001M}{0.01M}\right)$$

$$\Delta\mu = -11.48 \text{ }^{kJ}/_{mol}$$

So the chemical potential of the 0.0001M solution is $11.48 \text{ }^{kJ}/_{mol}$ lower than that of the 0.01M solution.

2.
a) For the greatest degree of dimerization we want the largest K_a. Recall,

$$K_a = \frac{\sum[\text{products}]}{\sum[\text{reactants}]}$$

In our case, the product is the dimer, so the maximum amount of dimer is achieved with the highest K_a. It is worth noting that often people use K_d (the dissociation constant) rather than K_a (the association constant).

$$K_d = \frac{1}{K_a} = \frac{\sum[\text{reactants}]}{\sum[\text{products}]}$$

At 20°C, Integrilin leads to the greatest degree of dimerization. At 40°C, cRGD leads to the greatest degree of dimerization.

Now we need to calculate the fraction of integrin-ligand complexes that are dimerized at equilibrium. We are given the initial (or total) concentration of integrin-ligand complexes. The reaction for dimerization goes ad follows:

$$RL + RL \longleftrightarrow RL\text{dimer}$$

That means that $K_a = \dfrac{[\text{RLdimer}]}{[\text{RL}][\text{RL}]} = \dfrac{[\text{RLdimer}]}{[\text{RL}]^2}$.

We also know another relationship between the concentrations of RL complex dimers and RL complexes:

$$[\text{RL}]_{tot} = [\text{RL}] + 2[\text{dimerRL}]$$

Note that we need to multiply the concentration of the integrin-lingand complex dimers by two because there are two integrin-ligand complexes per dimer. We just need to combine these two equations to solve for either [RL] or [dimerRL] which we can use to calculate the fraction that are dimerized at equilibrium.

$$[\text{RL}]_{tot} = [\text{RL}] + 2[\text{dimerRL}]$$
$$\therefore [\text{dimerRL}] = \tfrac{1}{2}[\text{RL}]_{tot} - \tfrac{1}{2}[\text{RL}]$$

Now substitute this into the expression for K_a.

$$[\text{RL}]^2 K_a = \tfrac{1}{2}[\text{RL}]_{tot} - \tfrac{1}{2}[\text{RL}]$$
$$[\text{RL}]^2 K_a + \tfrac{1}{2}[\text{RL}] - \tfrac{1}{2}[\text{RL}]_{tot} = 0$$

Use the quadratic equation to solve for [RL].

$$[\text{RL}] = \frac{-\tfrac{1}{2} \pm \sqrt{\left(\tfrac{1}{2}\right)^2 - 4K_a\left(-\tfrac{1}{2}[\text{RL}]_{tot}\right)}}{2K_a}$$

$$[\text{RL}] = \frac{-\tfrac{1}{2} \pm \sqrt{\tfrac{1}{4} + 2K_a[\text{RL}]_{tot}}}{2K_a}$$

We know that the concentration must be positive, so we can throw out the negative root and keep just the positive root.

$$[\text{RL}] = \frac{1}{K_a}\left(\tfrac{1}{2}\sqrt{\tfrac{1}{4} + 2K_a[\text{RL}]_{tot}} - \tfrac{1}{4}\right)$$

We can use this to calculate the equilibrium concentrations of integrin-ligand complexes that are not dimerized. To get the fraction that are dimerized, we merely have to compute:

$$\chi = \frac{2[\text{RLdimer}]}{[\text{RL}]_{tot}} = 1 - \frac{[\text{RL}]}{[\text{RL}]_{tot}}$$

Now just plug the number in. For cHArGD at 20°C:

$$[RL] = \frac{1}{1.79 \times 10^4 \, M^{-1}} \left(\tfrac{1}{2} \sqrt{\tfrac{1}{4} + 2\left(1.79 \times 10^4 \, M^{-1}\right)\left(10^{-6} \, M\right)} - \tfrac{1}{4} \right)$$

$$= 9.67 \times 10^{-7} \, M$$

$$\chi = 1 - \frac{9.67 \times 10^{-7} \, M}{10^{-6} \, M}$$

$$= 0.0334$$

The calculations are done similarly for the other ligands:

T °C	$K_{a,cHArGD}$	$K_{a,cRGD}$	$K_{a,Integrilin}$
20 - [RL]	9.67 x 10^{-7} M	9.67 x 10^{-7} M	9.16 x 10^{-7} M
20 - χ	0.0334	0.0334	0.0839
20 - [RL]	4.07 x 10^{-7} M	3.14 x 10^{-7} M	6.56 x 10^{-7} M
20 - χ	0.593	0.686	0.344

b) Here we must calculate the change in Gibbs free energy for each of the ligands at 20°C.

$$\Delta G^\circ = -RT \ln K_a$$

$$\Delta G^\circ_{cHArGD} = -\left(8.314 \, \tfrac{J}{molK}\right)\left(293K\right) \ln\left(1.79 \times 10^4 \, M\right)$$

$$\Delta G^\circ_{cHArGD} = -23.9 \, \tfrac{kJ}{mol}$$

Similarly,

$$\Delta G^\circ_{cRGD} = -23.9 \, \tfrac{kJ}{mol}$$

$$\Delta G^\circ_{Integrilin} = -26.4 \, \tfrac{kJ}{mol}$$

c) In order to determine the values of ΔH and ΔS, we need to use Van't Hoff analysis.

$$RT \ln K_a = -\Delta G^\circ = -\Delta H^\circ + T\Delta S^\circ$$

$$\ln K_a = -\frac{\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$$

So we can see that if we plot $\ln K_a$ vs $\dfrac{1}{T}$, the slope will be $-\dfrac{\Delta H^\circ}{R}$, and the intercept will be $\dfrac{\Delta S^\circ}{R}$.

T (°C)	$\dfrac{1}{T}$ (K^{-1})	$K_{a,cHArGD}$ (M^{-1})	$K_{a,cRGD}$ (M^{-1})	$K_{a,Integrilin}$ (M^{-1})
20	3.41 x 10^{-3}	1.79 x 10^4	1.79 x 10^4	5.0 x 10^4
25	3.36 x 10^{-3}	5.9 x 10^4	7.08 x 10^4	8.6 x 10^4
30	3.30 x 10^{-3}	1.95 x 10^5	2.73 x 10^5	1.47 x 10^5
35	3.25 x 10^{-3}	5.97 x 10^5	9.85 x 10^5	2.45 x 10^5
40	3.19 x 10^{-3}	1.79 x 10^6	3.47 x 10^6	4.00 x 10^5

Van't Hoff plot

y = -24154x + 92.228

y = -9550.2x + 43.414

y = -21142x + 81.944

So for cHArGD,

$$\frac{\Delta H^\circ}{R} = 21142K, \therefore \Delta H^\circ = 175.8\,{}^{kJ}\!/_{mol}$$

$$\frac{\Delta S^\circ}{R} = 81.944, \therefore \Delta S^\circ = 681.3\,{}^{J}\!/_{molK}$$

Similarly for cRGD,

$$\frac{\Delta H^{\ominus}}{R} = 24154K, \therefore \Delta H^{\circ} = 200.8\,{}^{kJ}\!/\!_{mol}$$

$$\frac{\Delta S^{\ominus}}{R} = 92.228, \therefore \Delta S^{\circ} = 766.8\,{}^{J}\!/\!_{molK}$$

And for Integrilin,

$$\frac{\Delta H^{\ominus}}{R} = 9550.2K, \therefore \Delta H^{\circ} = 79.4\,{}^{kJ}\!/\!_{mol}$$

$$\frac{\Delta S^{\ominus}}{R} = 43.414, \therefore \Delta S^{\circ} = 358.7\,{}^{J}\!/\!_{molK}$$

d) The enthalpic effects are given by ΔH° and the entropic effects are given by $-T\Delta S^{\ominus}$. In order to weigh the importance of enthalpic vs entropic effects, we will use a temperature of *300K*.

For cHArGD, $\Delta H^{\ominus} = 175.8\,{}^{kJ}\!/\!_{mol}$ and $-T\Delta S^{\ominus} = -(300K)(681.3\,{}^{J}\!/\!_{molK}) = -204.4\,{}^{kJ}\!/\!_{mol}$. Since $T\Delta S^{\ominus}$ is larger than ΔH°, ΔG° is negative and the dimerization is dominated by entropic effects.

For cRGD, $\Delta H^{\ominus} = 200.8\,{}^{kJ}\!/\!_{mol}$ and $-T\Delta S^{\ominus} = -(300K)(766.8\,{}^{J}\!/\!_{molK}) = -230.0\,{}^{kJ}\!/\!_{mol}$. For Intrgrilin, $\Delta H^{\ominus} = 79.4\,{}^{kJ}\!/\!_{mol}$ and $-T\Delta S^{\ominus} = -(300K)(358.7\,{}^{J}\!/\!_{molK}) = -107.6\,{}^{kJ}\!/\!_{mol}$. Again, in both of these cases $T\Delta S^{\ominus}$ is larger than ΔH°, so the dimerization is dominated by entropic effects.

3. Dill & Bromberg 14.4

a) For a first order approximation of w_{AA}, assume that the surface tension (γ) does not vary with temperature. So we will pick a surface tension of

$$\gamma = 70\,{}^{dyn}\!/\!_{cm}\left(10^{-5}\,{}^{N}\!/\!_{dyn}\right)\left(10^{2}\,{}^{cm}\!/\!_{m}\right) = 0.07\,{}^{N}\!/\!_{m}$$

We know that for the lattice model of surface tension

$$\gamma = -\frac{w_{AA}}{2a}$$

Now we just need an estimate for the area of a water molecule. Since bond lengths are one the order of $1\mathring{A}$, let us assume an area of about $a \approx 5A^2 \left(10^{-10} \, \frac{m}{\mathring{A}}\right)^2 = 5\times10^{-20} \, m^2$

$$
\begin{aligned}
w_{AA} &= -2\gamma a \\
&= -2\left(0.07 \, \frac{N}{m}\right)\left(5\times10^{-20} \, m^2\right) \\
&= -7\times10^{-21} \, J \\
&= 4.2 \, \frac{kJ}{mol}
\end{aligned}
$$

b) This attraction is mainly due to hydrogen bonding between water molecules. A small portion of this attraction is likely due to van der Waals forces.

c) A simple explanation is that as water is heated, there is more energy available for the molecules to overcome their attractive forces. Thus the effect of the attractive force is decrease, and the surface tension also decreases.

4. Dill & Bromberg 15.5

a) In general, the mixing entropy is given by

$$
\Delta S_{mix} = -RN\left(x \ln x + (1-x)\ln(1-x)\right).
$$

The maximum mixing entropy is attained when $x=(1-x)=0.5$. Let us use this value to obtain an upper limit on the mixing entropy.

$$
\begin{aligned}
\frac{\Delta S_{mix}}{N} &= -2Rx\ln x \\
&= -2\left(1.987 \, \frac{cal}{molK}\right)(0.5)\ln(0.5) \\
&= -1.378 \, \frac{cal}{molK}
\end{aligned}
$$

Since this is much less than the entropy for dissolving benzene in water, we know that the entropy for mixing makes up only a small portion of the total entropy.

b) The remainder of the entropy is probably due to the way in which water interacts with benzene. Since benzene is unable to form hydrogen bonds with the water, the water molecules are likely self arranged into so-called "solvent cages" around the solute (benzene) molecules in order to maximize the number of hydrogen bonds formed. The mixture pays a large entropy cost for this type of organized structure.