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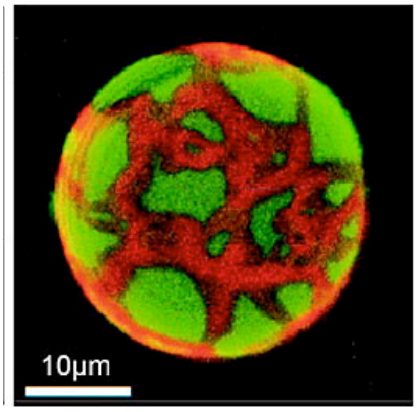
Problem Set #7
Due Wednesday, Nov 9 at 3 pm

1. **Phase separation in lipid membranes**. Just as macroscopic liquids can phase separate if mixing is thermodynamically unfavorable, mixtures of lipids within a lipid bilayer can undergo a *two-dimensional* phase separation, segregating into separate domains, as illustrated below. Such separation may have a physiological role in controlling the compartmentalization of molecules on the surfaces of cells.

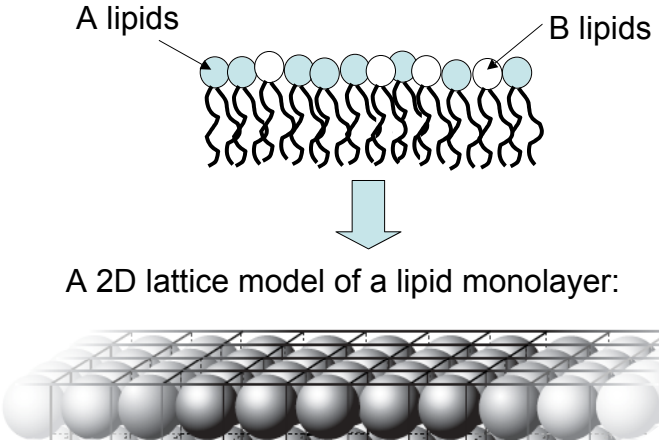


Right: Visualization of phase separation in a spherical lipid membrane shell. Segregation of two incompatible lipids labeled with different fluorochromes are observed by fluorescence microscopy. (From Korlach et al. PNAS 96 8461-8466 (1999)).

a. Consider a 2D lattice lattice as a model of one leaflet of a lipid bilayer, which contains two different lipids, modeled as beads that pack into the lattice just as we have modeled 3D liquids, as illustrated in the figure below. Would the entropy of mixing of two lipids (call them **A** and **B**, if you like) in this 2D model of a lipid monolayer differ from the entropy of mixing for 3D liquids as we derived in lecture? Explain why or why not.

b. Continuing, if we assume two lipids of interest can be described by a two-dimensional regular solution model, what parameter(s) would change in writing an equation for ΔU_{mix} for this 2D lipid layer model compared to the 3D liquid model from class? Explain why.

c. Lipid membranes formed from dilauroyl phosphatidyl choline (DLPC) and dipalmitoyl phosphatidyl choline (DPPC) exhibit regular solution behavior, with a two-dimensional interaction parameter $\chi_{2D} = 0.675$ at 30°C (Sparr et al. *Biophys. J.* **81** 1014-1028 (2001)). For a membrane composed of 1 mole total of lipids and $X_{DPPC} = 0.10$, at what temperature will the membrane phase separate? (Assume that the pair interaction energies between the lipids, ω_{AA}, ω_{BB}, and ω_{AB}, are temperature-independent).

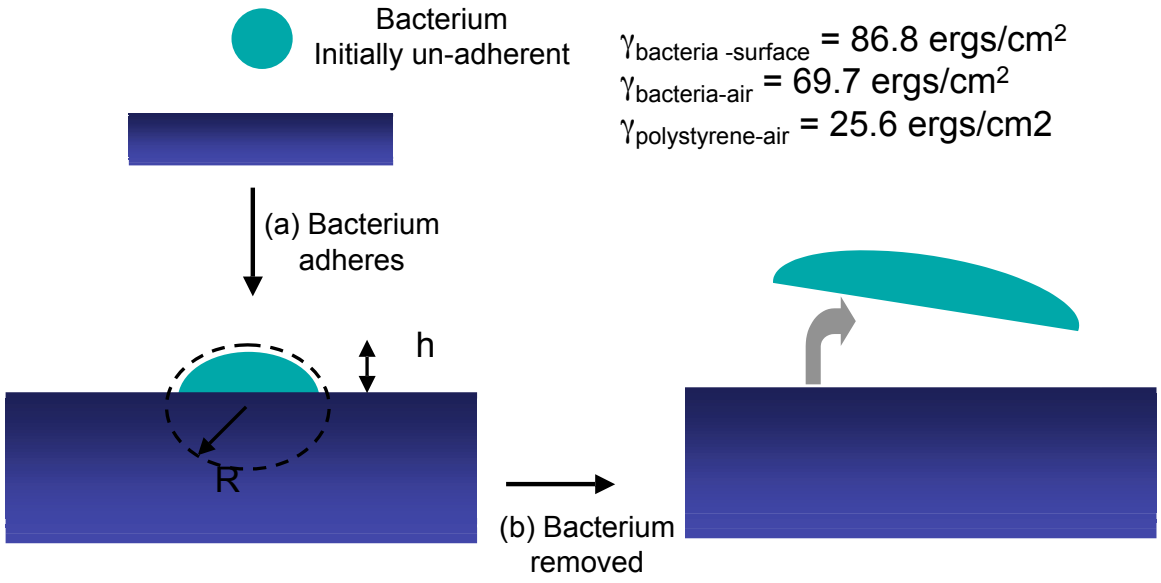


2. **Prediction of phase separation from interfacial tension**: t-Butanol is layered on the top of electrophoresis gels to prevent evaporation during the process. The interfacial tension between butanol and water has been measured as 2 dyn/cm (0.002 kg/s^2) at 20C.

    a. At this temperature, what is the maximum mole fraction of water that can be dissolved in butanol? You may assume that the interfacial area of a molecule is a = 9 x 10^{-20} m^2 and that the effective coordination number z = 8. At this temperature kT = 4 x 10^{-21} J (note that 1 J = 1 kg-m^2/s^2).

    b. Your answer in part a tells you that butanol and water are not miscible at a 50/50 mole fraction ratio. If you add 10 mol of butanol and 10 mol of water to a flask, mix well, and wait until the phases separate, what will the mole fraction of water be in the butanol phase? What will the mole fraction of butanol be in the water phase?

3. **Bacterial adhesion as a surface thermodynamic phenomenon**.
    a. Using the simplified geometry and surface tension data given below to model the adhesion of *E. coli* to a substrate, calculate the free energy change when this bacterium binds to a polystyrene surface. (note: 1 erg/cm^2 = 1 mJ/m^2). Take R = 5 μm and h = 0.5 μm.
    b. Calculate the work required to remove the bacterium from the substrate.



4. **Cryopreservation**. DMSO or glycerol are often used as cryoprotective agents in freezing cells and is typically added at ~10 vol%.

(a) What is the freezing point of water containing glycerol at a volume fraction of 10%? The molar mass of glycerol (CH_2OH-$CHOH$-CH_2) is 92 gm/mol and the density of glycerol at 20C (the temperature at which the 10% solution is made) is 1.25 gm/cm^3. The density of water at 20C is 1.0 gm/cm^3.
(b) If the glycerol is instead added to a salt solution containing 0.1M salt, does the freezing point depression due to the glycerol differ from that you calculated in part a if the glycerol-salt solution interaction is "ideal"? If it is not ideal?

5. **Freezing point as a molecular weight yard stick**. Do you think that freezing point depression is a good way to measure the molecular weight of a protein that is expected to be ~100,000 gm/mol? What would be the freezing point depression of a solution containing 1 g of such a protein in 10 gm of water?

6. **Plant thermodynamics**. Consider the problem of how plants might lift water from ground level to their leaves. Assume that there is a semipermable membrane at the roots, with pure water on the outside, and an ideal solution inside a small cylindrical capillary inside the plant. The solute mole fraction inside the capillary is x = 0.001. The radious of the capillary is 0.01 cm. The density of the solution is 1 gm/cm^3. What is the height of the solution at room temperature? Can osmotic pressure account for this rising water? (Dill and Bromberg, *Molecular Driving Forces*)

7. Dialysis is common laboratory procedure used to remove low molecular weight solutes from protein solutions. Example situations include: removal of removal of free, unreacted label after fluorescently labeling a protein; and removal of urea from a denatured protein solution to allow refolding of the protein. The protein solution is sealed inside a dialysis bag or tubing and placed in container with a large excess of pure water, as shown in the figure. The system is then allowed to come to equilibrium. For this problem, consider an idealized system where the only solute in the dialysis bag is the low molecular weight solute (i.e., no protein is actually present.)

(a) Explain why the combined system of the protein solution (system "A") and the water (system "B") may be considered to be an <u>isolated</u> system.
(b) Use a lattice model, with each individual water molecule occupying a single site in the lattice, to show that the entropy of System B is zero at the start of the experiment. Designate the number of water molecules N_{WB} and the number of total lattice sites M_B.
(c) Use a lattice model to express the entropy of system A at the start of the experiment in terms of the number of water molecules in system A, N_{wA}, the number of solute particles, N_s, and the total number of lattice sites in system A, M_A. For your final expression, use an appropriate approximation to eliminate factorial expressions. You may assume that the water molecules and solute molecules are comparable in size.
(d) Now write an expression for the entropy of the combined system under the conditions where water and solute can freely pass through the dialysis membrane. For every solute molecule that crosses over to system B, a water molecule must cross over to system A. (You may want to keep track of the number of solute molecules by noting that $N_S = N_{SA} + N_{SB}$ and the number of water molecules by noting that $N_{WA} + N_{WB} = N_W$).
(e) Show that you can write the entropy expression in terms of one system variable (i.e., one parameter that changes value during the progress of the experiment), and briefly describe

how to determine the equilibrium condition in terms of that variable and constants in the system. [You do not need to work out the equilibrium condition (lots of algebra involved) but feel free to predict what it is if you like.]

(f) Does the internal energy U of the system change? Explain why or why not.