Collusion under Imperfect Price Information

14.126 Game Theory
Muhamet Yildiz

Model

- Infinitely repeated game with n firms
- Each firm maximizes discounted sum of its profit (δ)
- Stage Game: each simultaneously produce q_i;
- Price $p = P(\theta,Q)$, where
  - θ is i.i.d shock
  - $Q = q_1+\ldots+q_n$ total supply
- Firms observe p but not Q.
- Perfect Monitoring: $\theta = E[\theta]$
- Imperfect Monitoring: each p is possible for each Q.
- **Assumption:** Unique static NE: $(q^N, q^N,\ldots, q^N)$
- $u_i(q) = qE[P(\theta,nq)] - q^N E[P(\theta,nq^N)]$
- $u_i(q_i,q_{-i}) = q_i E[P(\theta, q_i + (n-1)q_{-i})] - q^N E[P(\theta,nq^N)]$

SPE in Trigger strategies

- There are two modes: Collusion & War
- Collusion:
  - Each produce q^*;
  - Switch to War if $p < p^*$
- War: Each produce q^N for T^* periods, followed by Collusion
- $\lambda(Q) = \Pr(p \geq p^* | Q)$

Optimal Trigger strategy under perfect monitoring

- $p^* = P(\theta, nq^*)$
- Payoff:

$$v^* = u_i(q^*)$$

- Incentive constraint: for each q_i,

$$v^* \geq (1-\delta)\, u_i(q_i, q^*) + \delta^{T^*+1} v^*$$

- Optimal SPE in trigger strategies:
  - $T^* = \infty$
  - q* = arg max $u_i(q^*)$ s.t. $u_i(q^*) \geq (1-\delta)\, u_i(q_i, q^*)\ \forall q_i$
- Main Lesson: Punish as hard as possible!

SPE conditions

- SPE payoffs:

$$v = (1\text{-}\delta)u(q^*) + \delta\lambda(nq^*)v + \delta(1\text{-}\ \lambda(nq^*))\delta^{T^*}v$$

$$v = \frac{1-\delta}{1-\delta\lambda(q^*)-\delta^{T^*+1}(1-\lambda(q^*))}u(q^*)$$

- SPE condition (IC): for all q_i,

$$v \geq (1\text{-}\delta)u(q_i,q^*) + \delta\lambda(q_i+(n\text{-}1)q^*)v + \delta(1\text{-}\ \lambda(q_i+(n\text{-}1)q^*))\delta^{T^*}v$$

$$u_i(q_i,q^*)-u_i(q^*) \leq \frac{\delta(1-\delta^{T^*})[\lambda(nq^*)-\lambda(q_i+(n-1)q^*)]}{1-\delta\lambda(nq^*)-\delta^{T^*+1}(1-\lambda(nq^*))}u(q^*)$$

- Optimal SPE: maximize v subject to (IC).
- $\lambda(nq^*) < 1$. [Price wars observed with probability 1]
- T* may be < ∞ [You may not want to punish as hard as possible.]

14.123 Microeconomic Theory III
Spring 2010