Choice Theory – A Synopsis

14.123 Microeconomic Theory III
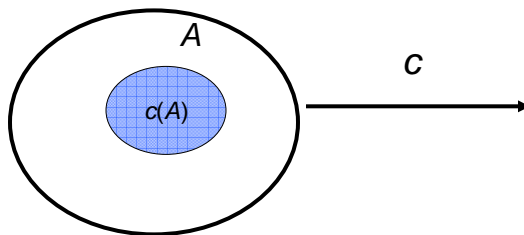
Muhamet Yildiz

Road map

1. Basic Concepts:
   1. Choice
   2. Preference
   3. Utility
2. Weak Axiom of Revealed Preferences
3. Preference as a representation of choice
4. Ordinal Utility Representation
5. Continuity

Basic Concepts

- *X* = Set of Alternatives
  - Mutually exclusive
  - Exhaustive
- *A* = non-empty set of available alternatives
- Choice Function: $c : A \mapsto c(A) \subseteq A$.
  - $c(A)$ is non-empty
- Preference: A relation $\succcurlyeq$ on X that is
  - complete : $\forall x,y \in X$, either $x \succcurlyeq y$ or $y \succcurlyeq x$;
  - transitive : $\forall x,y,z \in X$, $[x \succcurlyeq y$ and $y \succcurlyeq z] \Rightarrow x \succcurlyeq z$.
- Utility Function: $U : X \rightarrow \mathbb{R}$

Choice Function

- It describes what alternatives DM may choose under each set of constraints
- Feasibility: $c(A) \subseteq A$.
- Exhaustive: c(A) is non-empty
- Mutually exclusive: only one alternative is chosen

Preference

- **Preference Relation:** A relation $\succcurlyeq$ on X s.t.
  - complete : $\forall x,y \in X$, either $x \succcurlyeq y$ or $y \succcurlyeq x$;
  - transitive : $\forall\, x,y,z \in X$, $[x \succcurlyeq y$ and $y \succcurlyeq z] \Rightarrow x \succcurlyeq z$.
- $x \succcurlyeq y$ means: DM finds x at least as good as y
- Preferences do not depend on A!
- **Strict Preference:** $x \succ y \Leftrightarrow [x \succcurlyeq y$ and not $y \succcurlyeq x]$
- **Indifference:** $x \sim y \Leftrightarrow [x \succcurlyeq y$ and $y \succcurlyeq x]$.
- Choice induced by preference:

$$c_{\succcurlyeq}(A) = \{x \in A | x \succcurlyeq y\ \ \forall y \in A\}$$

Weak Axiom of Revealed Preference

Axiom (WARP): For all $A,B \subseteq X$ and $x,y \in A \cap B$, if $x \in c(A)$ and $y \in c(B)$, then $x \in c(B)$.

- WARP: DM has well-defined preferences
  - That govern the choice
  - don't depend on the set A of feasible alternatives

Choice v. Preference

Definition: A choice function c is represented by $\succcurlyeq$ iff $c = c_{\succcurlyeq}$.

Theorem: Assume that X is finite. A choice function c is represented by some preference relation $\succcurlyeq$ if and only if c satisfies WARP.

Ordinal Utility Representation

Ordinal Representation: $U : X \to \mathbb{R}$ is an ordinal representation of $\succcurlyeq$ iff:
$$x \succcurlyeq y \Leftrightarrow U(x) \geq U(y) \quad \forall x,y \in X.$$
Fact: If U represents $\succcurlyeq$ and $f: \mathbb{R} \to \mathbb{R}$ is strictly increasing, then $f \circ U$ represents $\succcurlyeq$.

Theorem: Assume X is finite (or countable). A relation has an ordinal representation if and only if it is complete and transitive.
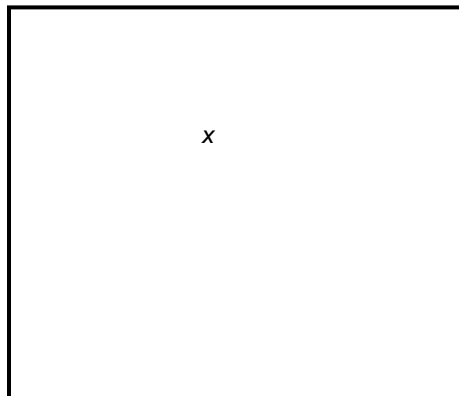
Example: Lexicographic preference relation on unit square does not have an ordinal representation.

Continuous Representation

Definition: A preference relation $\succcurlyeq$ is said to be continuous iff $\{y \mid y \succcurlyeq x\}$ and $\{y \mid x \succcurlyeq y\}$ are closed for every x in X.

Theorem: Assume X is a compact, convex subset of a separable metric space. A preference relation has an ordinal representation if and only if it is continuous.

Indifference Sets of a Continuous Preference

x

- $I(x) = \{ y \mid x \sim y \}$
- $I(x)$ is closed.
- If
  - $x' \succ x \succ x''$
  - $\varphi : [0,1] \rightarrow X$ continuous
  - $\varphi(1) = x'$; $\varphi(0) = x''$,
- Then, $\exists\, t \in [0,1]$ such that $\varphi(t) \sim x$.

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Spring 2010