Attitudes Towards Risk

14.123 Microeconomic Theory III
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Model

- $X = \mathbb{R}$ = wealth level
- Lottery = cdf F (pdf f)
- Utility function $u : \mathbb{R} \to \mathbb{R}$
- $U(F) \equiv E_F(u) \equiv \int u(x)dF(x)$
- $E_F(x) \equiv \int x dF(x)$

Attitudes Towards Risk

DM is

- risk averse if $E_F(u) \le u(E_F(x))$ $(\forall F)$
- strictly risk averse if $E_F(u) < u(E_F(x))$ $(\forall$ "risky" $F)$
- risk neutral if $E_F(u) = u(E_F(x))$ $(\forall F)$
- risk seeking if $E_F(u) \ge u(E_F(x))$ $(\forall F)$

DM is

- risk averse if u is concave
- strictly risk averse if u is strictly concave
- risk neutral if u is linear
- risk seeking if u is convex

Certainty Equivalence

- $CE(F) = u^{-1}(U(F))=u^{-1}(E_F(u))$
- DM is
  - risk averse if $CE(F) \le E_F(x)$ for all F;
  - risk neutral if $CE(F) = E_F(x)$ for all F;
  - risk seeking if $CE(F) \ge E_F(x)$ for all F.
- Take DM1 and DM2 with u_1 and $u_{2.}$
- DM1 is more risk averse than DM2
  - $\Leftrightarrow$ u_1 is more concave than u_2,
  - $\Leftrightarrow$ $u_1 = \varphi \circ u_2$ for some concave function φ,
  - $\Leftrightarrow$ $CE_1(F) \equiv u_1^{-1}(E_F(u_1)) \le u_2^{-1}(E_F(u_2)) \equiv CE_2(F)$

Measures of Risk Aversion

- absolute risk aversion:

$$r_A(x) = -u''(x)/u'(x)$$

- constant absolute risk aversion (CARA)

$$u(x) = -e^{-\alpha x}$$

- If $x \sim N(\mu,\sigma^2)$, $CE(F) = \mu - \alpha\sigma^2/2$
- relative risk aversion:

$$r_R(x) = -xu''(x)/u'(x)$$

- constant relative risk aversion (CRRA)

$$u(x) = -x^{1-\rho}/(1-\rho),$$

- When $\rho=1$, $u(x) = \log(x)$.

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