Lecture 8
Correlated & Sequential Equilibria

14.123 Microeconomic Theory III

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Correlated Equilibrium

Definitions

- Game G = (N, $S_1,\ldots,S_n$; $u_1,\ldots,u_n$), where
  - N = set of players
  - S_i = set of all strategies of player i,
  - $u_i : S_1 \times \ldots \times S_n \rightarrow R$ is i's vNM utility function.
- Information Structure $(\Omega, I_1,\ldots, I_n, p)$ where
  - (Ω, p) is a finite probability space
  - I_i is an information partition of Ω
- Adapted strategy profile (wrt $(\Omega, I_1,\ldots, I_n, p)$) $(\boldsymbol{s}_1,\ldots,\boldsymbol{s}_n)$ s.t.
  - $\boldsymbol{s}_i : \Omega \rightarrow S_i$
  - $\boldsymbol{s}_i(\omega) = \boldsymbol{s}_i(\omega')$ whenever $I_i(\omega) = I_i(\omega')$.

Correlated Equilibrium

- Correlated Equilibrium w.r.t. $(\Omega, I_1,\ldots, I_n, p)$ is an adapted strategy profile $(\boldsymbol{s}_1,\ldots,\boldsymbol{s}_n)$ s.t.

$$\sum_{\omega' \in I_i(\omega)} u_i\big(\boldsymbol{s}_i(\omega), \boldsymbol{s}_{-i}(\omega')\big) p(\omega' \mid I_i(\omega)) \geq \sum_{\omega' \in I_i(\omega)} u_i\big(s_i, \boldsymbol{s}_{-i}(\omega')\big) p(\omega' \mid I_i(\omega))$$

  for all ω, i, s_i.
- Equivalently, for all i and adapted $\boldsymbol{s}_i'$,

$$\sum_{\omega \in \Omega} u_i\big(\boldsymbol{s}(\omega)\big) p(\omega) \geq \sum_{\omega \in \Omega} u_i\big(\boldsymbol{s}_i'(\omega), \boldsymbol{s}_{-i}(\omega)\big) p(\omega)$$

Example

	L	R
U	5,1	0,0
D	4,4	1,5

- Ω = {A,B,C}
- I_1 = {{A},{B,C}}
- I_2 = {{A,B},{C}}
- p = (1/3,1/3,1/3)
- $\boldsymbol{s}_1$(A) = U,
- $\boldsymbol{s}_1$(B) = $\boldsymbol{s}_1$(C) = D
- $\boldsymbol{s}_2$(A) = $\boldsymbol{s}_2$(B) = L,
- $\boldsymbol{s}_2$(C) = R

Correlated Equilibrium Distribution

- Correlated Equilibrium (distribution) is a probability distribution p on S such that

$$\sum_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i}) p(s_{-i} \mid s_i) \geq \sum_{s_{-i} \in S_{-i}} u_i(s_i', s_{-i}) p(s_{-i} \mid s_i)$$

  for all i, s_i, s_i'.
- Equivalently, for all i and $d_i: S_i \to S_i$

$$\sum_{\omega \in \Omega} u_i(s) p(\omega) \geq \sum_{\omega \in \Omega} u_i(d_i(s_i), s_{-i}) p(\omega)$$

- The two definitions are equivalent!

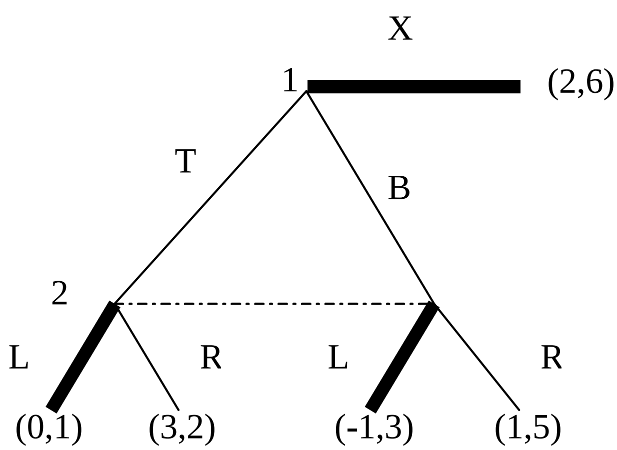
Relation to Other Solution Concepts

- If $(\sigma_1,\ldots, \sigma_n)$ is a Nash Equilibrium then $\sigma_1\times\ldots \times \sigma_n$ is a Correlated Equilibrium distribution
- If $(\boldsymbol{s}_1,\ldots,\boldsymbol{s}_n)$ is a correlated equilibrium w.r.t. $(\Omega,I_1,\ldots, I_n,p)$, $\boldsymbol{s}_i(\omega)$ is rationalizable for i.
- Correlated Equilibrium =

  Common Knowledge of Rationality +
  Common Prior Assumption

Sequential Equilibrium

What is wrong with this SPE?



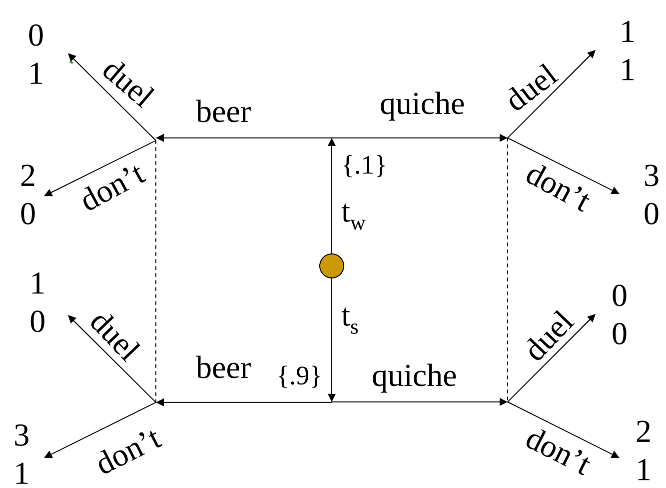
Sequential Rationality

- A player is sequentially rational (at a history) if he plays a best reply to a belief conditional on being at that history.

Sequential Equilibrium

- An assessment: (σ,μ) where σ is a strategy profile and μ is a belief system, $\mu(h)\in\Delta(h)$ for each h.
- An assessment (σ,μ) is sequentially rational if at each h_i, σ_i is a best reply to σ_{-i} given $\mu(h)$.
- (σ,μ) is consistent if there is a sequence $(\sigma^m,\mu^m) \to (\sigma,\mu)$ where σ^m is "completely mixed" and μ^m is computed from σ^m by Bayes rule:
- An assessment (σ,μ) is a sequential equilibrium if it is sequentially rational and consistent.

Beer – Quiche



Centipede game with irrationality