14.123 Microeconomic Theory III
Problem Set 2

The due date for this assignment is Thursday February 25 (You can return it on Tuesday March 1 without penalty.)

1. There are two urns, A and B, each consisting of 100 balls, some are black and some are red. In urn A there are 30 red balls, but the number of red balls in urn B is not known. We draw a ball from urn A with color α and a ball from urn B with color β. Consider the following acts:

$$f_{A,r} = \begin{cases} 100 & \text{if } \alpha = \text{ red} \\ 0 & \text{if } \alpha = \text{ black} \end{cases} \qquad f_{A,b} = \begin{cases} 0 & \text{if } \alpha = \text{ red} \\ 100 & \text{if } \alpha = \text{ black} \end{cases}$$

$$f_{B,r} = \begin{cases} 110 & \text{if } \beta = \text{ red} \\ 0 & \text{if } \beta = \text{ black} \end{cases} \qquad f_{B,b} = \begin{cases} 0 & \text{if } \beta = \text{ red} \\ 110 & \text{if } \beta = \text{ black} \end{cases}$$

   Let c be the choice function induced by $\succeq$. Find the sets $c\left(\{f_{A,r}, f_{A,b}, f_{B,r}, f_{B,b}\}\right)$ that are consistent with $110 \succ 100 \succ 0$ and Savage's postulates.

2. Exercise 6.C.19 in Mass-Colell, Whinston, and Green (Assume that the asset returns are independent.)

3. Exercise 6.D.3 in Mass-Colell, Whinston, and Green

4. Consider a monopolist who faces a stochastic demand. If he produces q units, he incurs a zero marginal cost and sells the good at price $P(\theta, q)$ where $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ is an unknown demand shock where P and C twice differentiable. Assume that the profit function is strictly concave in q for each given θ, and $P(\theta, q) + qP_q(\theta, q)$ is increasing in θ, where P_q is the derivative of P with respect to q. The monopolist is expected profit maximizer.

   (a) Show that there exists a unique optimal production level q^*.

   (b) Show that if the distribution of θ changes from G to F where F first-order stochastically dominates G, then the optimal production level q^* weakly increases.

   (c) Take $P(\theta, q) = \phi(\theta) - \gamma(q)$. Suppose that there are two identical monopolists as above in two independent but identical markets. Find conditions under which the monopolists have a strict incentive to merge and share the profit from each market equally.

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