[SQUEAKING]

[RUSTLING]

[CLICKING]

MICHAEL SCOTT ASATO CUTHBERT: Hello, computational music theorists. Our topic for the coming week is going to be on intervals and equivalents and working with what equal and the same and so on mean. I am very much trying to treat everybody in this class and everybody I meet as equals, But that doesn't mean that I think that everybody is the same. In fact, that would be a very unequal way to treat people.

So we can have this notion that equality does not mean the same thing, and that equal does not mean identical. This is something I hope we practice in our lives, but it's also very important in thinking about how we compare musical structures.

I'm asking you over the weekend to read a very interesting chapter from Dmitri Tymoczko's *A Geometry of Music* in which he talks about five kinds of equivalents, a few more, but mostly about five kinds that he calls optic, octave, permutation, transposition, inversion, and cardinality.

These ways of thinking about equality of notes or melodies or chords or other musical structures even whole pieces is going to be very important when we want to find particular musical gestures or we want to identify particular things using a computer. I'm sitting at the piano because I'm going to demonstrate a few of the things that we might think mean that something is equal or something is the same but maybe they're not identical.

But first, I want to play something that doesn't work very well on the piano. So listen to these two pieces.

[MUSIC PLAYING]

FRED ASTAIRE:

(SINGING) The way you wear your hat

The way you sip your tea

The memory of all that

No, no, they can't take that away from me

The way your smile just beams

The way you sing off key

[MUSIC PLAYING]

ELLA FITZGERALD: (SINGING) The way you wear your hat

The way you sip your tea

The memory of all that

No, no, they can't take that away from me

The way your smile just beams

The way you sing off key

MICHAEL SCOTT ASATO CUTHBERT: I asked you to listen to two pieces. Actually I just played excerpts. But did I play two pieces, or did I play one piece twice? Were those two pieces the same piece? Were they the same recording?

I think we'll think that, well, they were the same piece but different recordings, different performers, different styles, different interpretations, but there was something the same about them. So we can think of an equivalence class or a kind of equivalence between pieces. That doesn't necessarily mean that they are identical.

Now let's turn to some musical structures that we can demonstrate at the piano. So I want to talk concretely about some ways that we might think about equivalence and what do we mean by equivalence when we're talking about notes or chords or melodies or pieces. And I want to think about some of the things that we might be doing.

So, for instance, can we think of a context where notes as a group are the same regardless of the order that we talk about them? Let's start with just a C major chord C, E, G.

[PLAYING PIANO]

And if I say C3, E3, G3, that's the chord. Or if I say that it's G3, E3, C3, it's still the same chord. Or if I say that the chord is E3, C3, G3, it really doesn't matter. So this could be, when we're talking about chords, a place where there is a permutation equivalence. Doesn't matter what order they're in. On the other hand, if we're talking about a melody or something, some sort of a piece that has [PLAYING PIANO]

Then it matters if we go [PLAYING PIANO]-- sorry-- [PLAYING PIANO]

Those don't seem to be equivalent in some context that we might think of as equivalents. So there could be a context where P, permutation equivalence, is active and a context where it isn't. We can also think of cardinality, especially on the piano. It doesn't matter if I play C-E-G [PLAYING PIANO] as a chord or if I play C, C, E, G [PLAYING PIANO]. You can't even tell. Maybe I can put two fingers on the C or something like that.

But on-- in a chordal context on the piano, we would normally say, well, that cardinality doesn't really matter in terms of the sound. If you're going to be notating it, then, yeah, it matters.

We also can think of octave equivalence. [PLAYING PIANO]

That all those notes are C. When we say something like all of these things are, we're saying that they're equivalent in some way. So we're saying that in this case when I say all these notes are C, we're talking about an octave equivalence relationship.

But I can't say that all of those notes are C3-- [PLAYING PIANO]. Only that note is C3, [PLAYING PIANO] C2, [PLAYING PIANO] C4. If you were, a instrumental teacher and you ask somebody on an instrument like flute or the clarinet or the violin to play a scale in three octaves and they just played the same octave-- [PLAYING PIANO]--

You'd really say no, no, no. I asked for three octaves, so we can think of a world where there isn't an octave equivalence. And we can put these two things together that we can think C, E, G, C-- [PLAYING PIANO]

Which have a kind of world where we say, well, that's the same as C, E, G in the sense of play the tonic triad in C major. It doesn't matter if I play [PLAYING PIANO]

Those all fulfill this particular equivalence class. And there are other ones. One of the ones that Tymoczko assumes throughout is and isn't even a part of his optic, o-p-t-i-c, world is spelling, so it doesn't matter or-- C major's a little bit hard-- but it doesn't matter if I think of a single note, [PLAYING PIANO] play E-flat, [PLAYING PIANO] play D-sharp, it's the same thing. So we can think of a spelling equivalence.

Now if you were to be in theory one class and you're asked to write a C minor triad, and you write C [PLAYING PIANO], D-sharp, G, you're probably going to get marked wrong because that's a context where there is not spelling equivalence.

We can also think of a world where there's a transposition equivalence. So, for instance, [PLAYING PIANO]

And [PLAYING PIANO]

We can say, oh well, those are both the same piece. Those are both "Mary Had a Little Lamb." So that there's a transposition equivalence that makes that piece still "Mary Had a Little Lamb."

And we definitely use that when we're talking about chord types-- so we talk about [PLAYING PIANO], that's a major triad, [PLAYING PIANO] that's a major triad, [PLAYING PIANO] that's a major triad-- so that we can have-- talk about a concept like major triad that takes into account that there's a transposition equivalence.

Probably the one that people like the least and think that it's not really a context that we want to be working with is the inversion context, and when we play something in a tonal context like a C major triad [PLAYING PIANO] and then we invert it so let's see from [PLAYING PIANO] E to G is a minor third, so we'll go from G, B-flat is a minor third and E to C is a major third. So B-flat to D is a is a major third.

So we can say [PLAYING PIANO] in an inversion and transposition contexts that those-- well, I guess those are not even a transposition context-- that those two are equivalent and that might go, whoa, whoa, whoa, what are we talking about.

But there are some places in post-tonal theory where that's important, but we can even think about it even in the tonal or in this case a pre-tonal context. Here I'm going to play four pieces. One goes-- well, I'll do the one that we just played. [PLAYING PIANO]

Here's another piece. [PLAYING PIANO]

And here's another piece. [PLAYING PIANO]

Play that one more time so you get your ears. [PLAYING PIANO]

And then we can have a piece like this. [PLAYING PIANO]

I played in some sense four different melodies or something, but maybe I only played three different pieces because we can recognize, oh, those last two are the same piece but one is upside down from the other. This is actually a piece that is really famous in the 15th century called "L'homme Armé" or "The Armed Man," and composers at that time liked to play around with it. They transposed it. They did diatonic transpositions so that you play in-- well, different modes. So instead of [PLAYING PIANO], here you play some-- maybe you play something like [PLAYING PIANO]

And it's a slightly different piece but still the same piece, so you can think of a diatonic transposition equivalence. But they also definitely played with what I was showing the inversion equivalence context so that [PLAYING PIANO] at [PLAYING PIANO] are in some ways the beginning of the same piece.

We're going to be asking you in the problem set to see if you can come up with other contexts where we talk about, well, that's the same thing like is this a C major triad. [PLAYING PIANO] Good. Well, this is a fancy piano, so I'm going to change the temperament to meantone. And now we go. [PLAYING PIANO]

Maybe I'll resume. You can't hear the difference as much, but we can go to a slightly different one even [PLAYING PIANO] or [PLAYING PIANO]

And those are all the notes C. E, G. They're not exactly the same, but we think of them as the same. So we can think of a temperament context or a tuning context where we allow things to be equivalent.

So I think this is the background that you need in order to go through the reading of Dmitri Tymoczko on optics and equivalents. As usual, you're going to be implementing your code in Python using musical objects either that you create or that you have unlocked in music21. And when you're doing this, you think about what two objects are. What are two notes that Python defines as equal or that I have defined in music21 defines as equal? What do we mean by that?

And you'll see that each object in the computational language generally has only one built-in equivalence set that defines what the equal equal operator or whatever it is means. And, well, there's actually a second one that is operator that defines something else. What do we mean by these two objects are the same-- is the same item is what I wanted to say. But-- and what choice was made, and how could we have made a better choice? And who's going to hate me for the choice I made, and who would have hated me if I made another choice on defining equality in a note or in a court?

Thanks for listening to this video. And now let's get started with reading what Dmitri Tymoczko has to say on this subject.