22.01 Fall 2016, Problem Set 5

October 20, 2016

Complete all the assigned problems, and do make sure to show your intermediate work.

---

1 (50 points) Skill Building Problems

1.1 (16 points) Consider the three methods by which photons can interact with electrons in matter: Compton scattering, the photoelectric effect, and pair production. Use the Yip reading on photon interactions to help write your answers.

1. At which photon energies are each of these effects the most prominent? In other words, which of these effects can be neglected at which energies?

2. In figure 10.17, which electron energy shell transitions (give the numbers of the levels involved) are responsible for the discontinuities in the attenuation coefficient? Which of the three photon interaction methods is responsible for these discontinuities? Confirm that your estimates of the shell levels are correct, by looking up their energy transitions in the NIST x-ray energy transition database.

3. Why does the pair production cross section become non-zero abruptly at energies above $\frac{\hbar\omega}{2m_ec^2}$?

4. Explain why it is more likely to see a single-escape pair production peak from a NaI detector (figure 10.18), while it is more likely to see a double-escape pair production peak from a semiconductor detector (figure 10.19).

1.2 (20 points) Banana Spectral Identification

1. (8 points) Using our banana counting data, identify every single peak and major feature that you can in the spectrum. Label them with the process that created them, and calcuate their true expected energies. There may be more than one isotope found in bananas!

2. (8 points) Given your answers for 1.2.1, show the locations, energies, and physical processes for any peaks that *should* exist on our spectrum, but were for some reason not detected.

3. (4 points) Why were the peaks in 1.2.2 not detected?

1.3 (14 points) Mass Attenuation Coefficients: For these problems, refer to the NIST table of x-ray attenuation coefficients.

1. (8 points) Choose a lead apron thickness to shield dental patients from 90% of the x-rays emitted from a pure 40kV x-ray source. Do you have to consider any additional x-rays produced in the lead, and if so, account for this in your calculations.

2. (2 points) Explain the qualitative differences in the attenuation coefficients of lithium and tungsten in a quantitative manner, at the following energies: $E_\gamma < 100$keV, $E_\gamma = 1$MeV, $E_\gamma = 100$MeV. By this, we mean compare relative values of the relevant scattering cross sections, and explain any discrepancies between these and the relative values of the attenuation coefficients.

3. (2 points) What is the origin of the discontinuities in the attenuation coefficient for tungsten? Why is there more than one step change within close proximity at some places?

4. (2 points) For which energies is the *attenuation coefficient* in water higher than that in air? What about the *mass attenuation coefficient*?

2 (50 points) Noodle-Scratchers

1. (20 points) Using connservation of energy and momentum, derive formulas relating the incoming photon energy (E), its outgoing energy (E'), the electron's outgoing energy (T), and the scattering angle of the photon (θ) in Compton scattering.

   (a) ***Answer:*** $T = \hbar\omega - \hbar\omega' = \hbar\omega\frac{\alpha(1-cos(\theta))}{1+\alpha(1-cos(\theta))}; \quad \alpha = \frac{\hbar\omega}{m_e c^2}$

2. (10 points) Using photon energy and momentum relations, derive the formula for the Compton wavelength shift as a function of θ.

   (a) ***Answer:*** $\Delta\lambda = \frac{h}{m_e c}\left(1 - cos\left(\theta\right)\right)$

3. (20 points, open ended) Find the minimum and maximum values of the Klein-Nishina cross section as a function of incoming photon energy. Graph the angle of the angle of minimum scattering probability as a function of incoming photon energy, and intuitively explain the features of the graph to check your answer. The angle of maximum scattering probability is always the same... with one exception. What is it, and what physical process does that process represent?