22.01 Fall 2016, Problem Set 2

September 18, 2016

Complete all the assigned problems, and do make sure to show your intermediate work.

Part I

Skill-Building Problems (50 points)

1 Predicting Nuclear Stability

Using the KAERI Table of Nuclides and/or the IAEA Table of Nuclides, answer the following questions about sodium (Z=11):

1. Using the excess mass from the table of nuclides, calculate the ***binding energy***, and the ***binding energy per nucleon*** for each isotope of sodium listed on the KAERI table, starting with A=18. Use the table of nuclides to check your answer.

2. Graph the excess mass of each isotope of sodium as a function of mass number (A). Also graph the difference between the semi-empirical and measured nuclear binding energies. What trends do you see?

3. For each region where an increase in excess mass is seen from the most stable isotope(s) (left third, middle third, right third), briefly say why the nuclei in each region are most unstable. (Hint: What do you know about the relative number of protons and neutrons in a nucleus, and how does that help determine stability?)

2 Liquid-Drop Nuclear Models

For these questions, consider the liquid drop model of nuclear mass, which states that the mass of a nucleus can be empirically calculated as in Eq. 4.10 (p. 59) of *Nuclear Radiation Interactions* by S. Yip:

$$BE\left(A,Z\right)=a_vA-a_sA^{\frac{2}{3}}-a_c\frac{Z\left(Z-1\right)}{A^{\frac{1}{3}}}-a_a\frac{\left(N-Z\right)^2}{A}+a_p\delta;\quad \delta=\left\{\begin{matrix}\frac{1}{\sqrt{A}}\,even-even\\ 0\,even-odd\\ -\frac{1}{\sqrt{A}}\,odd-odd\end{matrix}\right\} \qquad (1)$$

1. Explain the origin of each additive term in this expression. Pay particular attention to the exponents in each one, and explain why they are what they are.

2. Why does the δ term in this expression change sign for odd/even nuclei?

3. Modify Equation 1 to empirically calculate the total rest mass of a given nucleus.

Part II

Noodle-Scratchers (50 points)

3 Recasting the Semi-Empirical Mass Formula (with answer)

Derive an expression for the most stable number of neutrons for a given nucleus with Z protons. Graph this expression as a function of Z. How does your prediction compare with the isotopes of sodium?

Answer:

$$0 = -\frac{a_s}{3} A^{\frac{-4}{3}} - \frac{4a_c}{3} Z\left(Z-1\right) A^{\frac{-7}{3}} + 4Z a_a A^{-2} - 8Z^2 a_a A^{-3} \tag{2}$$

4 Q-Values and Nuclear Power (part open-ended, part with answer)

For these questions, consider equations 4.2 - 4.6 (pp. 54-55) in *Nuclear Radiation Interactions*.

1. Show that the Q-value for a reaction can be expressed solely in terms of nuclear binding energies (derive equation 4.6).
2. Using a data extraction program like the Web Plot Digitizer to get data points from Figure 4.4 (p. 58), produce an empirical expression for the optimum number of neutrons (N) for a given number of protons (Z) in a nucleus. Comment on the quality of that fit, and explain in which regions the fit is the best, and in which the fit is the worst.

Answer: Something close to $N = 0.0055Z^2 + Z$

5 Predicting the Island of Stability (open-ended)

Does the semi-empirical formula predict the "island of stability" containing the superheavy elements (SHEs)? If so, graphically or mathematically explain how. If not, read through the article from Physics Today and suggest a missing term to the semi-empirical mass formula, which would account for the more stable SHEs. Justify your extra term by checking the improved fit with the known elements.