Slides for Nuclear Mass and Stability

2024
Let's Agree on Notation

\[ ^{A}_{Z}Name^{\pm q} \]

- **A** – Atomic mass (number of nucleons)
- **Z** – Atomic number (number of protons)
- **q** – Charge (zero if not an ion)

\[ ^{10}_{5}B + ^{1}_{0}n \rightarrow ^{7}_{3}Li + ^{4}_{2}He + Q \]

is the same as...

\[ ^{10}B(n, \alpha) ^{7}Li \]
An Aside: Boron Neutron Capture Therapy (BNCT)
Explaining BNCT

• Why 30MeV
  • Look up (p,n) cross sections on JANIS

• Why beryllium?
  • Think about nuclear reactions

• How does the boron only get into cancer cells?
  • Think about the “blood/brain barrier”

• Why was boron selected for the therapy?
  • Think about range and energy loss of radiation
Reading the KAERI Table

16-S-32

- Atomic Mass: 31.9720707 ± 0.0000001 amu
- Excess Mass: 26015.981 ± 0.112 keV
- Binding Energy: 271780.656 ± 0.120 keV
- Beta Decay Energy: B- 12685.287 ± 6.782 keV


- Atomic Percent Abundance: 95.02%
- Spin: 0+
- Stable Isotope

Possible parent nuclides:
- Beta from P-32
- Electron capture from Cl-32
- EC + P from Ar-33

# Explaining Terms

1 AMU = 931.49 MeV

<table>
<thead>
<tr>
<th></th>
<th>Mass (kg)</th>
<th>Charge (u)</th>
<th>Energy (MeV/c^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 amu</td>
<td>1.660540 \times 10^{-27}</td>
<td>1.000</td>
<td>931.49</td>
</tr>
<tr>
<td>neutron</td>
<td>1.674929 \times 10^{-27}</td>
<td>1.008664</td>
<td>939.57</td>
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<tr>
<td>proton</td>
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<td>1.007276</td>
<td>938.28</td>
</tr>
<tr>
<td>electron</td>
<td>9.109390 \times 10^{-31}</td>
<td>0.00054858</td>
<td>0.511</td>
</tr>
</tbody>
</table>

- **Atomic mass**
- **Excess mass**
  \[ \Delta = M - A \]
- **Binding energy**

\[ B(A, Z) \equiv [ZM_H + NM_n - M(A, Z)]c^2 \]
Let’s Try Some Examples

Calculate the binding energy of:

$^{32}\text{S}$

$^{33}\text{S}$

$^{48}\text{S}$
Nuclear Reaction Energies

Let’s look at BNCT again…

\[ {}^{10}_5 B + {}^1_0 n \rightarrow {}^7_3 Li + {}^4_2 He + Q \]

How do we find Q?

Conserve mass and energy, of course!

**Method 1**  
Use masses

**Method 2**  
Kinetic energies

**Method 3**  
Binding energies
Binding Energy Curve


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The Liquid Drop Mass Formula

Also called the “semi-empirical mass formula”

Derive and explain on the board
Semi-Empirical Mass Formula

\[ B(A, Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta \]  

(4.10)

\[ \begin{array}{cccccc}
 a_v & a_s & a_c & a_a & a_p & \\
 16 & 18 & 0.72 & 23.5 & 11 & \text{MeV} \\
 \end{array} \]

\[ \delta = \frac{a_p}{\sqrt{A}} \]

- even-even nuclei
- even-odd, odd-even nuclei
- odd-odd nuclei
Stability Trends

The Number of Stable Isotopes for each Element or Proton Number Z

Image by MIT OpenCourseWare.
Stability Trends

The Number of Stable Isotopes as a Function of Neutron Number $N$

Image by MIT OpenCourseWare.
Stability Trends
Mass Parabolas – Plotting Stability

Odd $A$

$A = 135$

Te  I  Xe  Cs  Z=56  Ba  La  Ce

Even $A$

$A = 102$

Nb  Mo  Tc  Ru  Z=44  Rh  Pd  Ag  Cd

Odd $Z$, odd $N$

Even $Z$, even $N$

$\beta^-$  $\beta^+$  $\beta^-$  $\beta^-$  $\beta^+$  $\beta^+$  $\beta^-$  $\beta^-$

$Z=44$  Z=46

2$\delta$
An Island of Stability?


**Figure 1.** The grand nuclear landscape. Nuclei that have been experimentally identified are inside the yellow region, whereas nuclei only predicted to exist are roughly indicated by the green area. Black squares mark stable isotopes. Magic proton and neutron numbers, at which nuclei have enhanced stability, are indicated by red lines. The star labeled SHE indicates the region of superheavy elements. (Courtesy of Witold Nazarewicz.)

**Figure 2.** The dependence of alpha decay half-lives on neutron number.
An Island of Stability?


How are superheavy elements synthesized?

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An Island of Stability?


Figure 5. Decay chains for isotopes $^{293}117$ and $^{294}118$, showing the half-life and alpha-decay energy of each nucleus in the chains. Black arrows indicate alpha decay and gray arrows indicate spontaneous fission. In both cases, hot-fusion reactions between calcium-48 projectiles and actinide target materials, either berkeliium-249 or californium-249, produce compound nuclei, labeled with asterisks, that promptly evaporate off several neutrons. Toward the ends of the chains, roentgenium-281 (from $^{293}117$) and flerovium-286 (from $^{294}118$) can spontaneously fission or alpha decay into meitnerium-277 and copernnicium-282, respectively. Both end-chain nuclei undergo spontaneous fission.

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