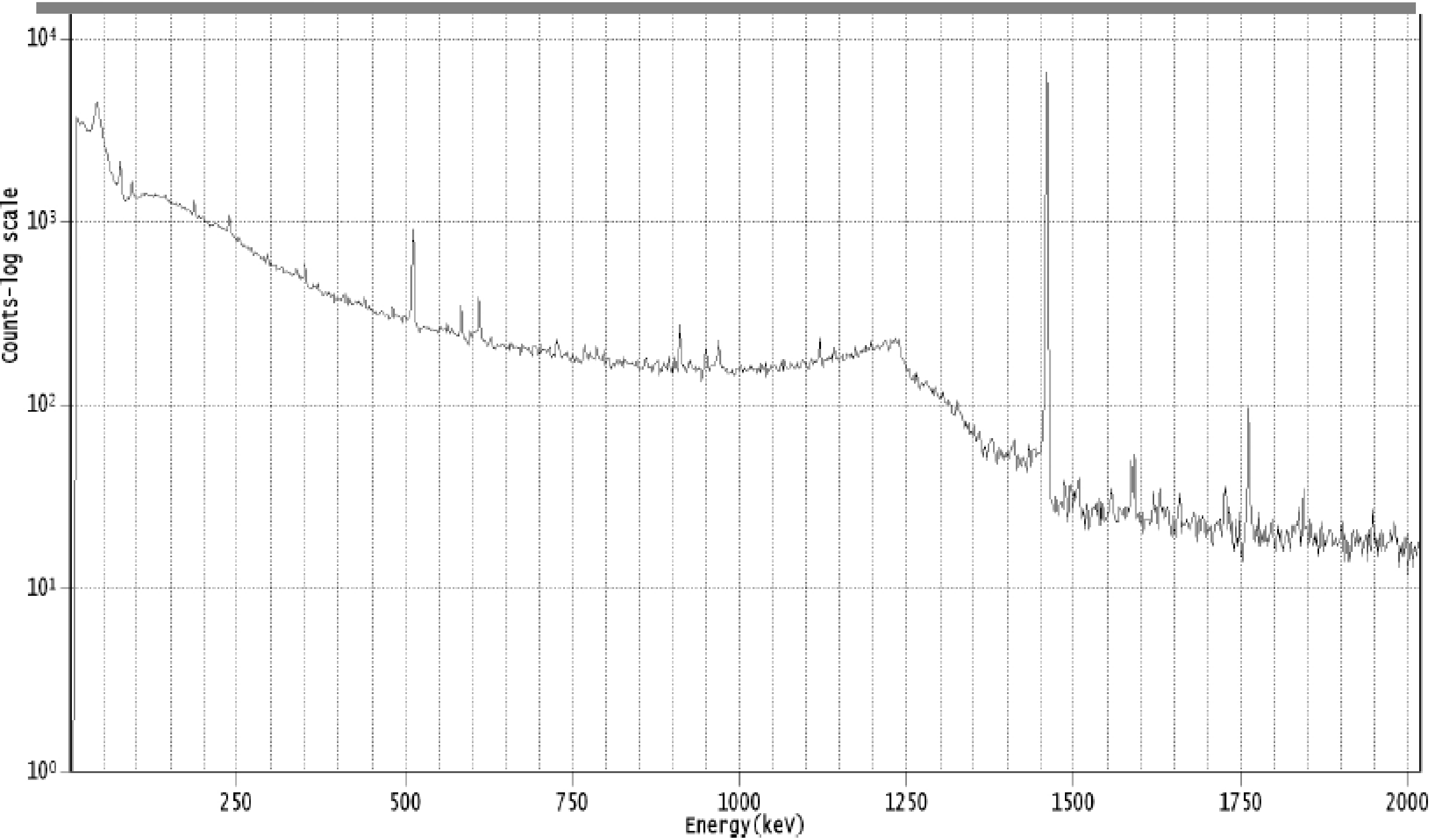
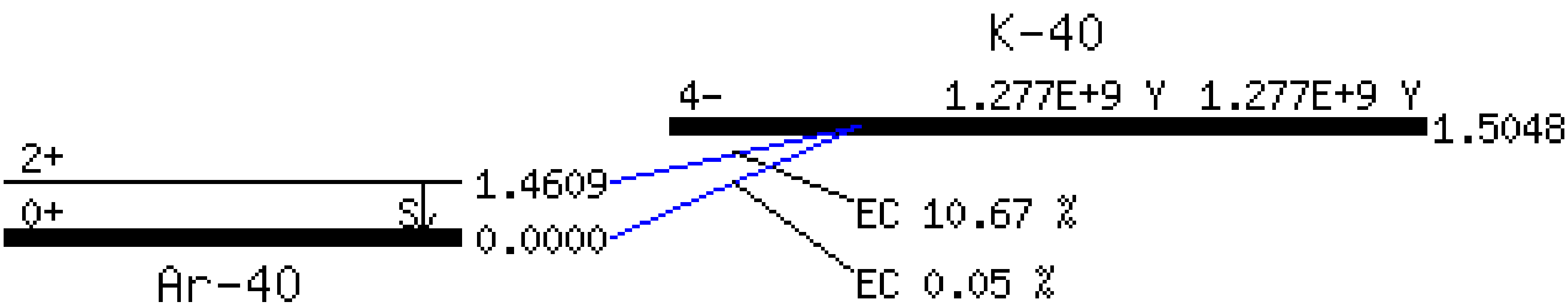
Slides for Photon Interactions with Matter

2024

The Motivation: Explain Our Banana Spectrum!

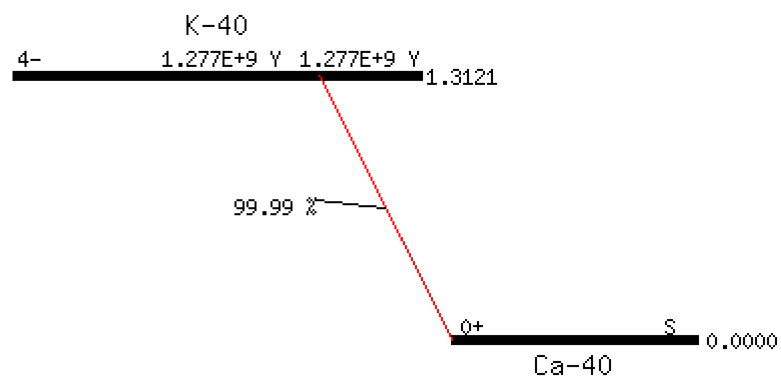


Banana Radiation Source: ^{40}K



$$^{40}K \rightarrow {}^{40}Ar^* + \beta^- + \bar{\nu} \qquad {}^{40}Ar^* \rightarrow {}^{40}Ar + \gamma$$

Don't forget the beta decays when calculating ^{40}K activity!

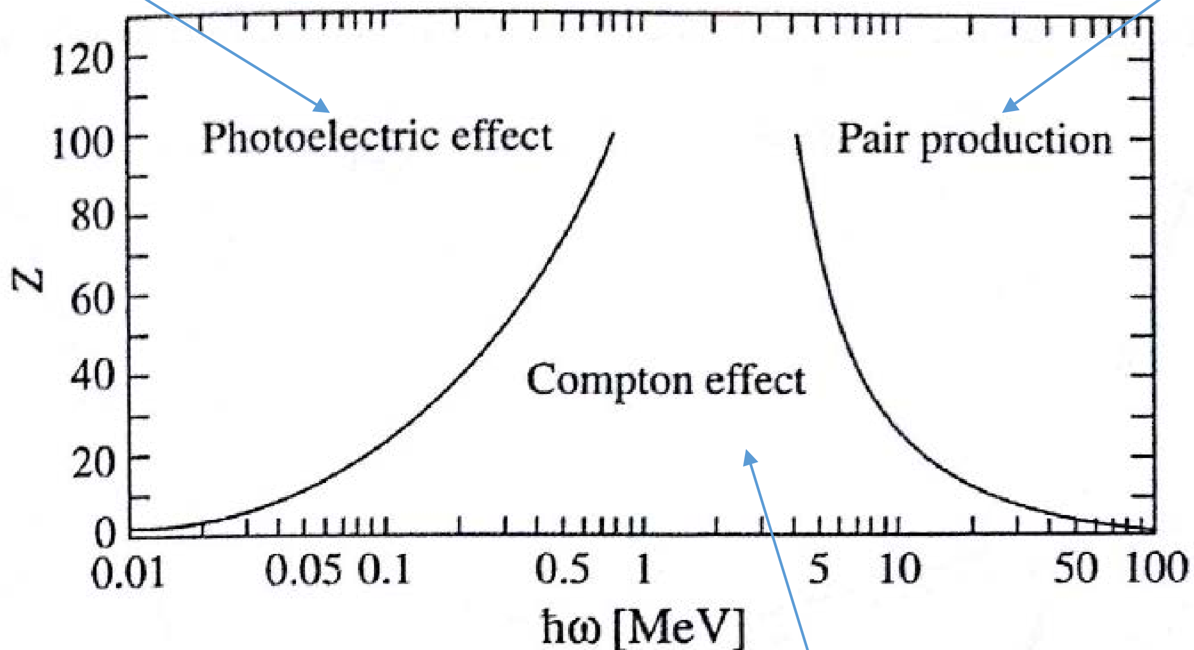


What Do These Gammas Do?

Yip, p. 217

Eject an outer electron

Create a β⁻/β⁺ pair

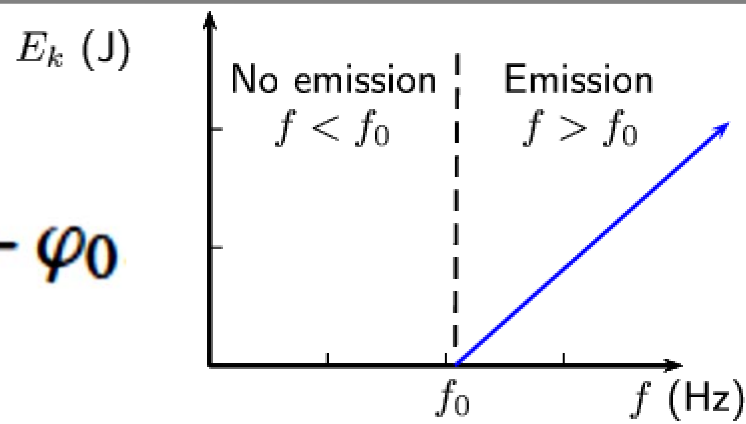


Scatter off of an electron

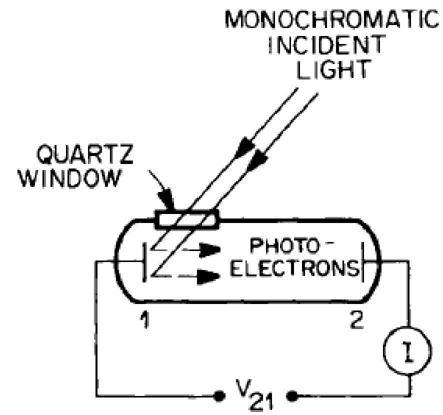
The Photoelectric Effect

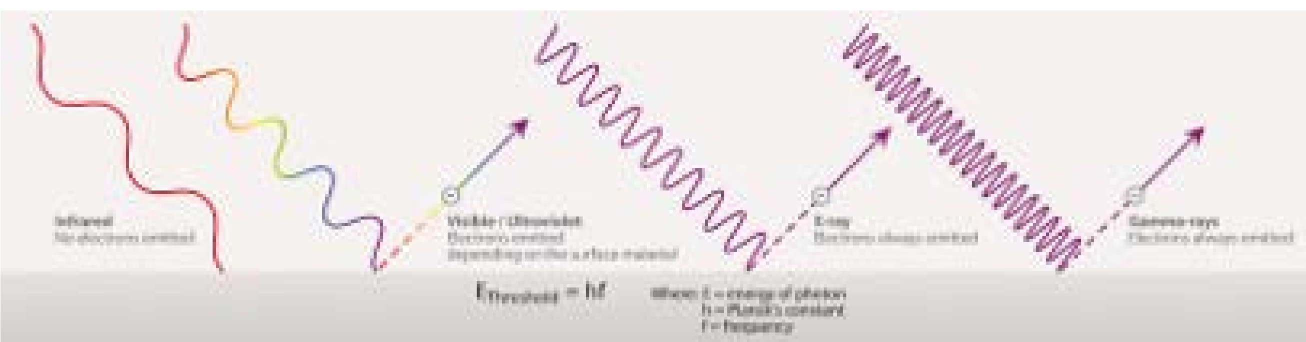
Turner, p. 174

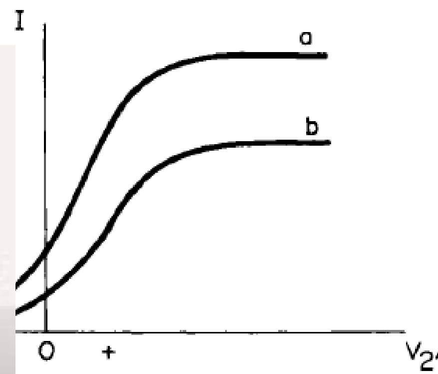
$$T_{\max} = h\nu - \varphi_0$$



http://www.everythingmaths.co.za/science/grade-12/12-optical-phenomena-and-properties-of-matter/12-optical-phenomena-and-properties-of-matter-02.cnxmlplus







A Primer on Photon Quantities

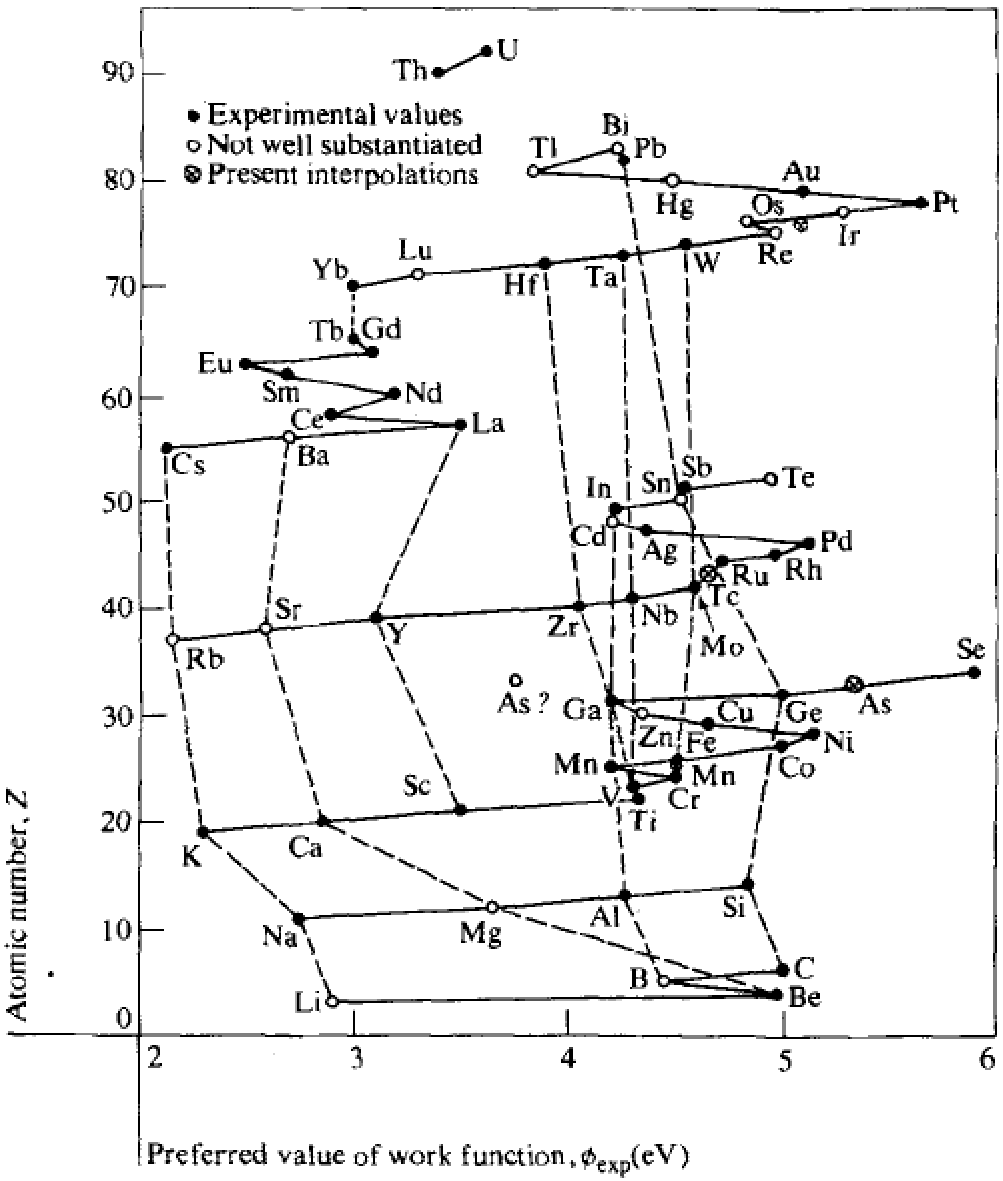
$$E = h\nu = \hbar\omega = \frac{hc}{\lambda} \qquad \nu = f \; (notation\; choice)$$

$$p = \frac{h}{\lambda} \qquad h = 6.626 \times 10^{-34} \frac{m^2 kg}{sec}$$

$$\omega = 2\pi\nu \qquad \hbar = \frac{h}{2\pi} \qquad c = 299{,}792{,}458 \; \frac{m}{s}$$

The Work Function φ_0

H. B. Michaelson. "The work function of the elements and its periodicity." J. Appl. Phys. 48, 4729 (1977)

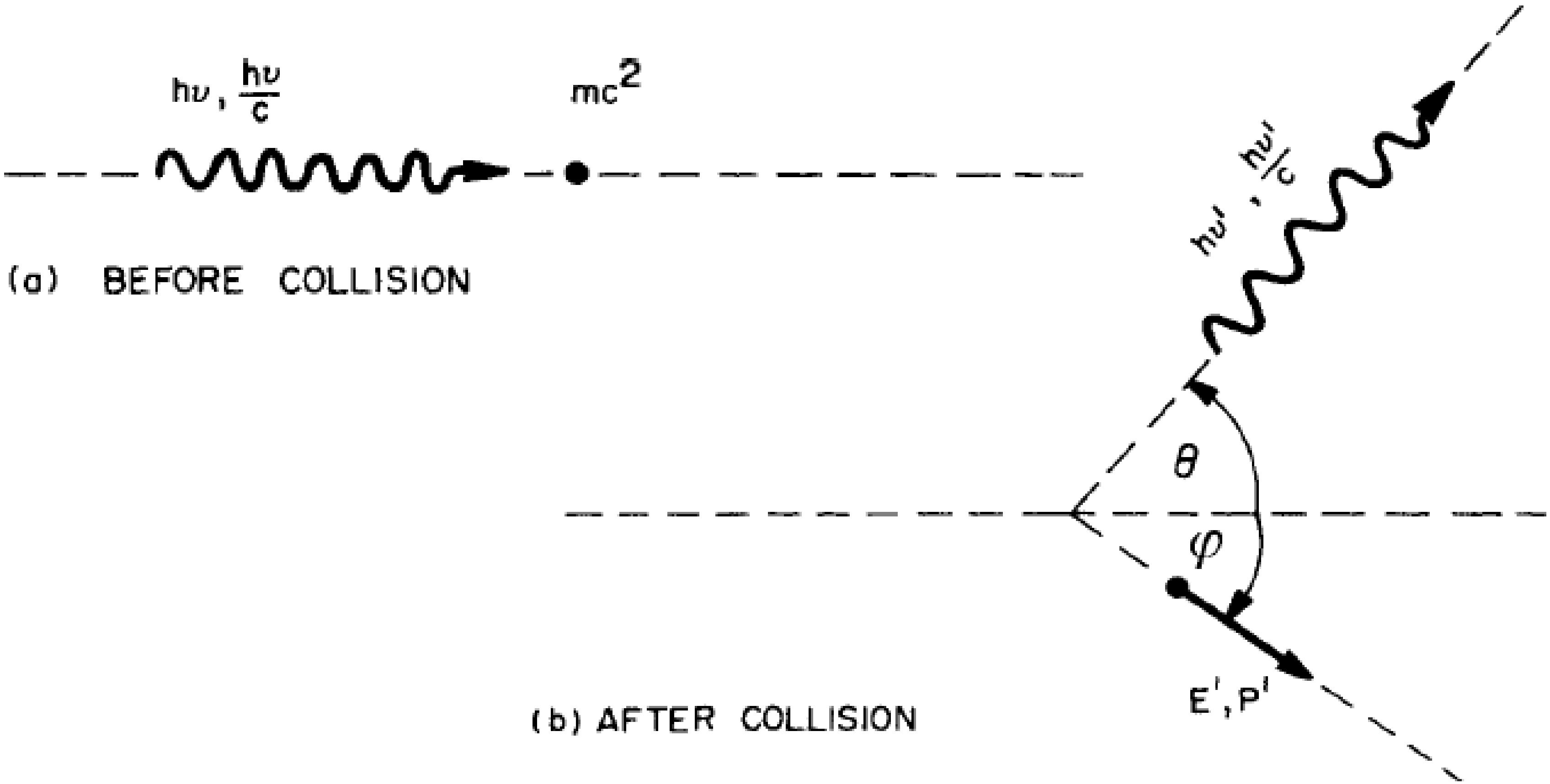


Energy needed to remove the outermost electron

Do you notice any patterns?

Compton Scattering

Turner, p. 179



Wavelength & Energy Shift

Electron recoil energy

$$\Delta\lambda = \lambda' - \lambda = c\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{mc}(1 - \cos\theta)$$

$$T = h\nu - h\nu'$$

$$T = h\nu \frac{1 - \cos\theta}{mc^2/h\nu + 1 - \cos\theta}$$

When is the electron recoil energy maximized?

Compton Scattering Energies

$$\Delta\lambda = \lambda' - \lambda = c\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{mc}(1 - \cos\theta)$$

$\boldsymbol{At\ \theta = {}^{\pi}/_{2}}$:

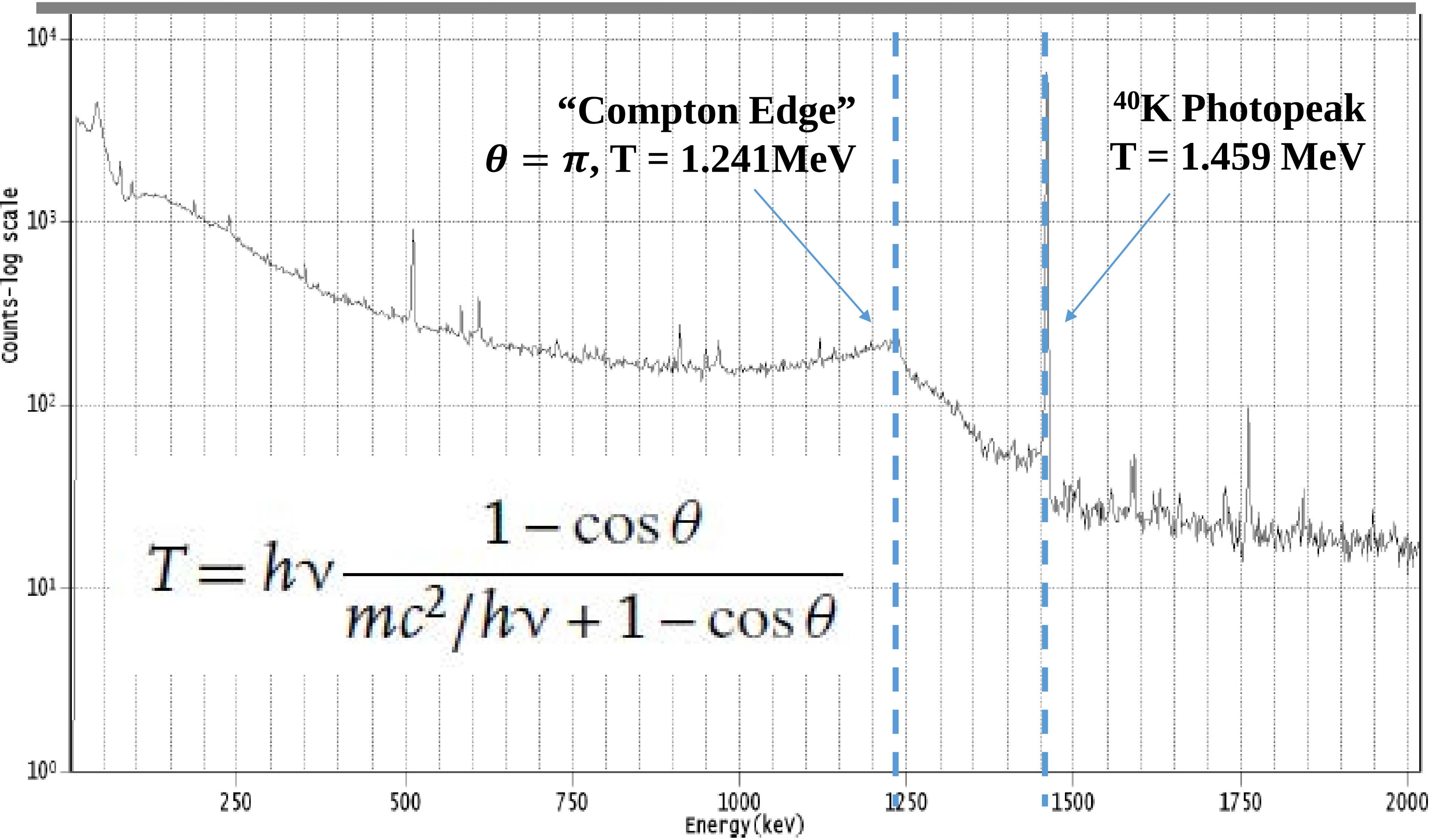
$$\Delta\lambda = \frac{h}{mc} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.00 \times 10^{8}} = 2.43 \times 10^{-12}\ \text{m}$$

$\boldsymbol{For\ h\nu = 1.332 MeV}$,

$\boldsymbol{\theta = 140°}$:

$$h\nu' = \frac{1.332\ \text{MeV}}{1 + (1.332/0.511)[1 - (-0.766)]} = 0.238\ \text{MeV}$$

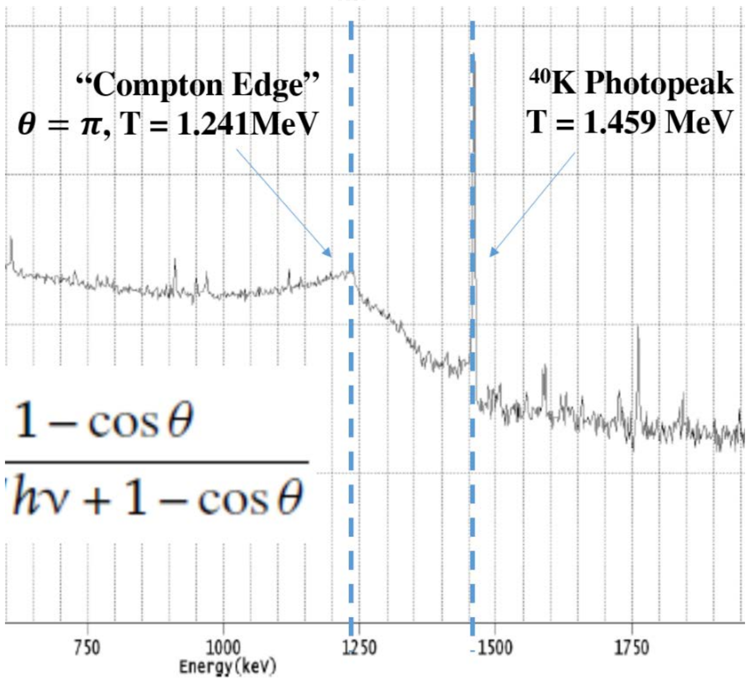
Let's Look at the Spectrum Again



The Real Quirk of Physics Here… Constant Compton Edge at High E

$$T = h\nu \frac{1-\cos\theta}{mc^2/h\nu + 1 - \cos\theta}$$

As $h\nu \to \infty$ at $\theta = \pi$,
$T \to h\nu - 0.255 MeV$

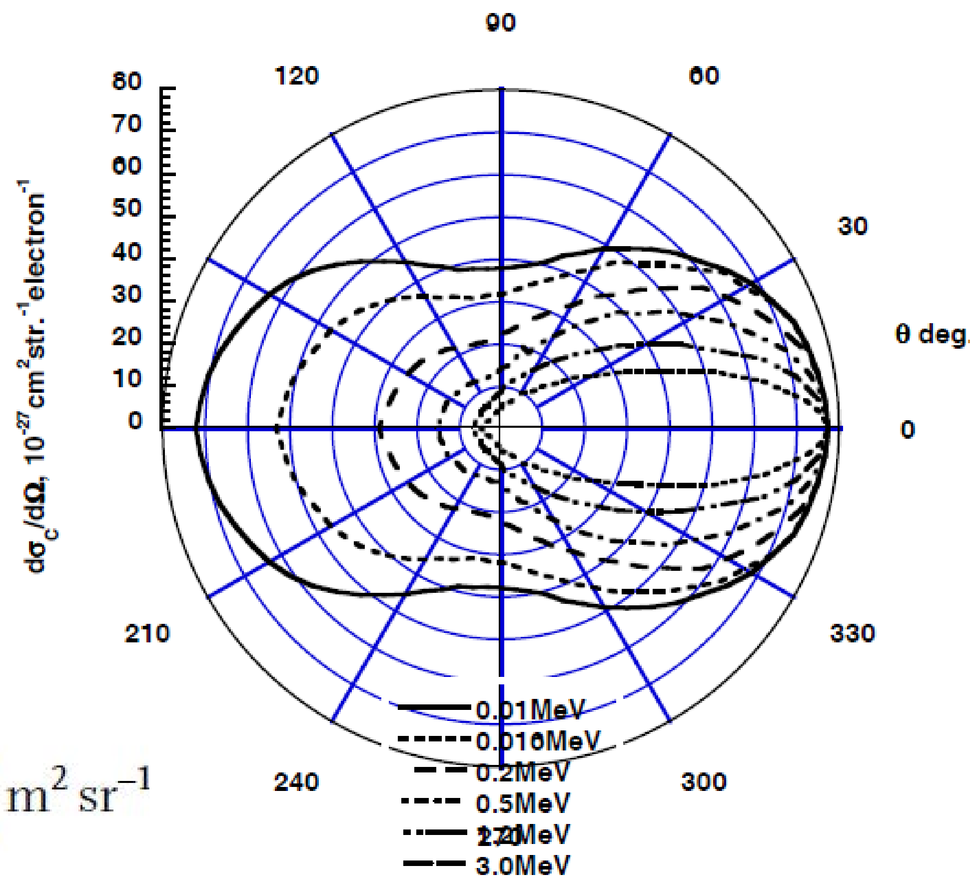


Maximum difference between Compton Edge and Photopeak locations

Angular Differential Cross Section: The Klein-Nishina Formula

R. D. Evans. "Compton Effect," in Handbuch der Physik XXXIV, Tlugge, Ed., Springer-Verlag, pp. 218-298 (1958)

Describes the probability of scattering into a given *angle*



$$\frac{d_e\sigma}{d\Omega} = \frac{k_0^2 e^4}{2m^2c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'}+\frac{\nu'}{\nu}-\sin^2\theta\right)\ \mathrm{m}^2\,\mathrm{sr}^{-1}$$

Angular Differential Cross Section: The Klein-Nishina Formula

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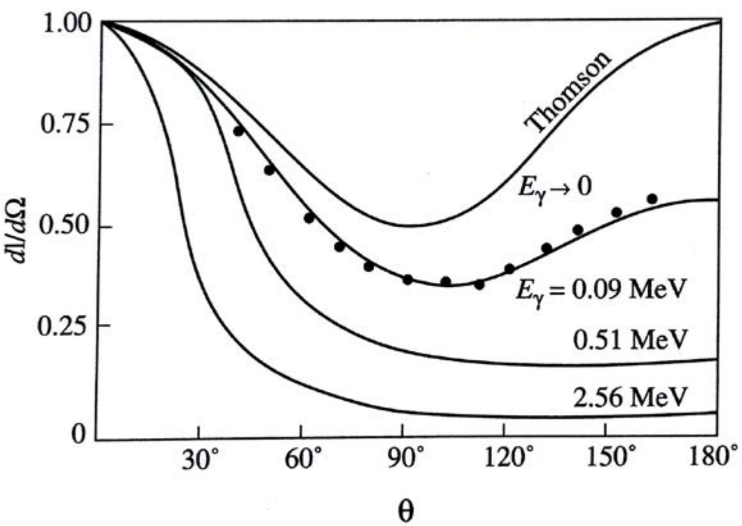
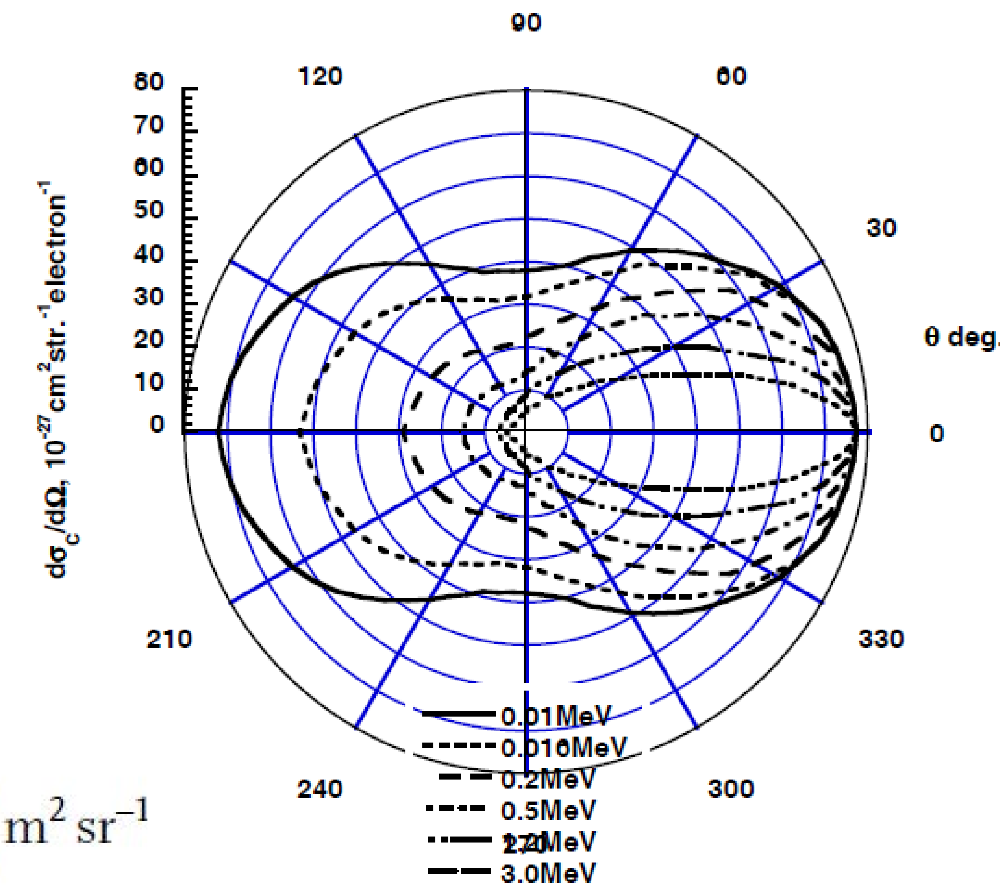
A simplified version… and the Thomson scattering (low energy) limit



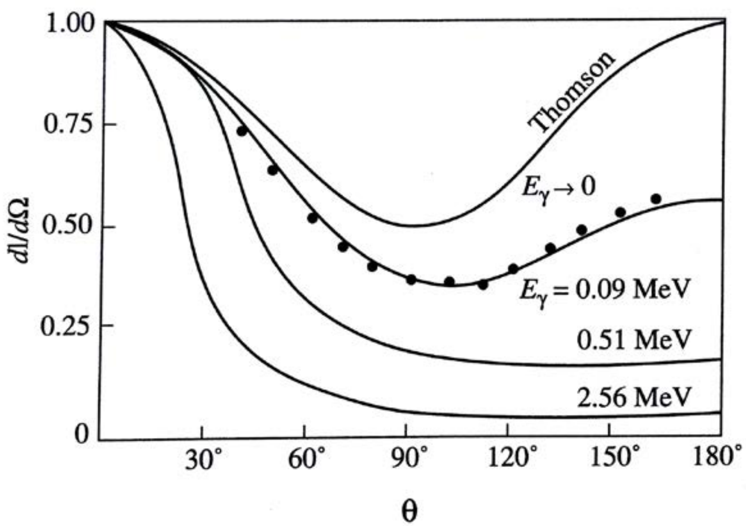


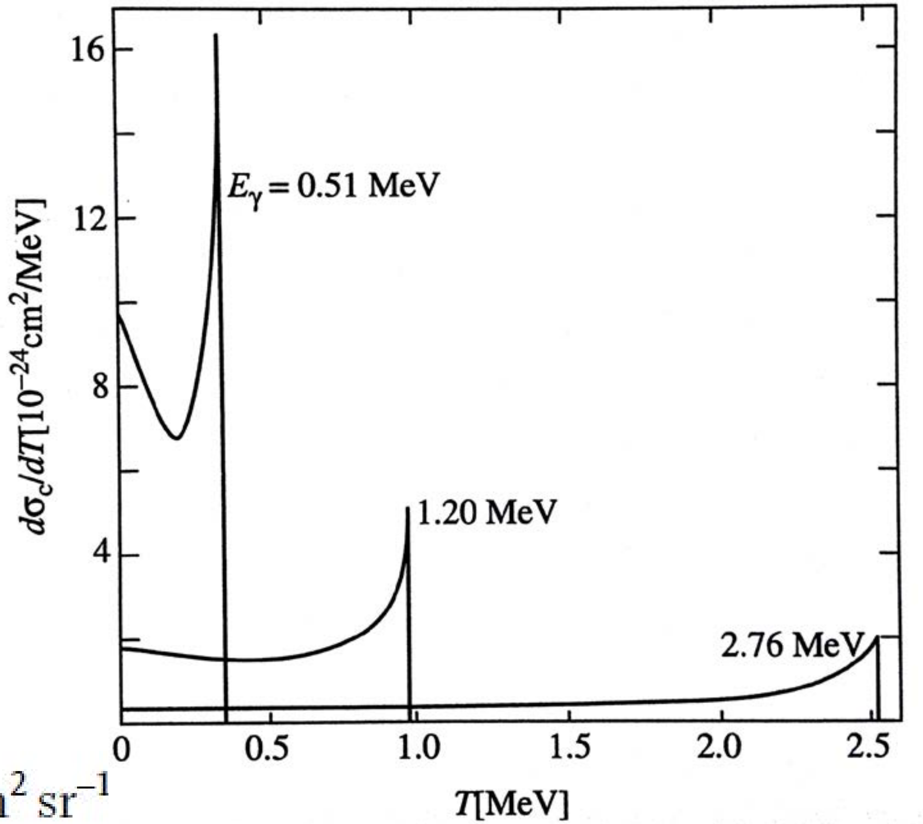
$$\frac{d_e\sigma}{d\Omega} = \frac{k_0^2 e^4}{2m^2c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2\theta\right) \text{ m}^2\text{ sr}^{-1}$$

Angular Differential Cross Section: The Klein-Nishina Formula

Yip, pp. 207-209

This leads directly to the Compton Edge shape



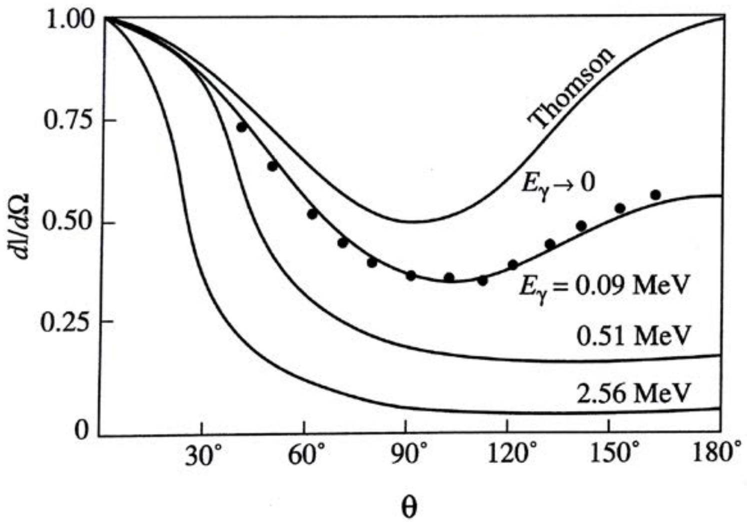


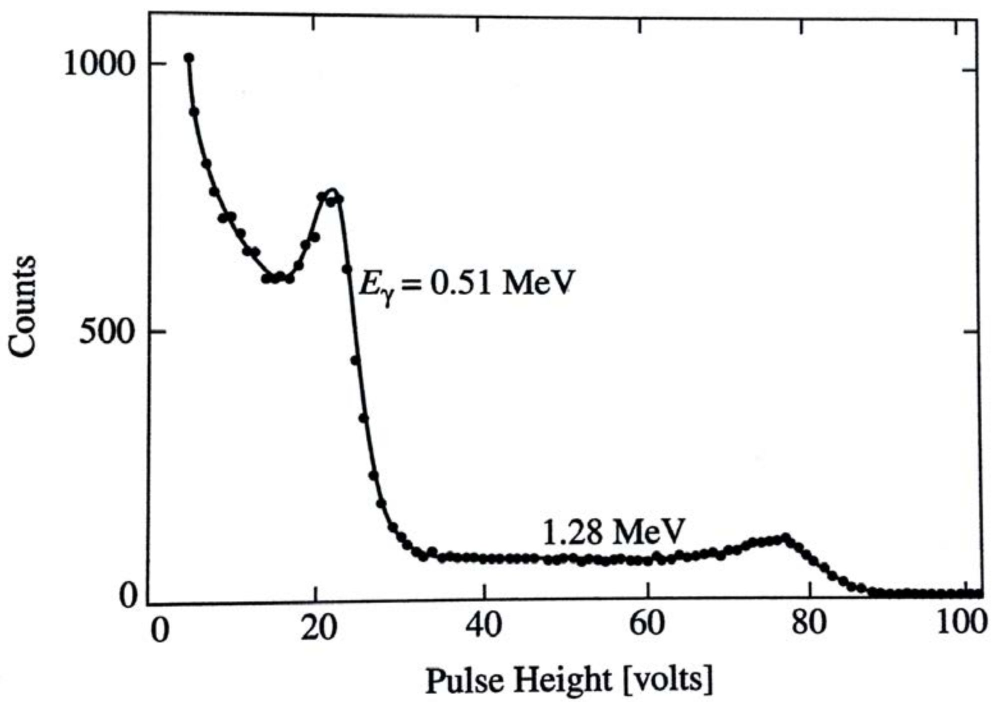
$$\frac{d_e\sigma}{d\Omega} = \frac{k_0^2 e^4}{2m^2c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'}+\frac{\nu'}{\nu}-\sin^2\theta\right) \text{ m}^2\text{ sr}^{-1}$$

Angular Differential Cross Section: The Klein-Nishina Formula

Yip, pp. 207-209

This leads directly to the Compton Edge shape, which is easily measured



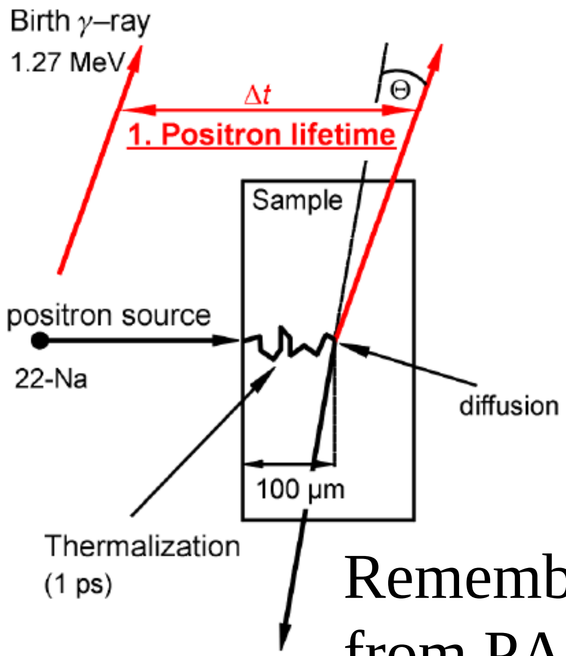


$$\frac{d_e\sigma}{d\Omega} = \frac{k_0^2 e^4}{2m^2c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2\theta\right) \text{m}^2\,\text{sr}^{-1}$$

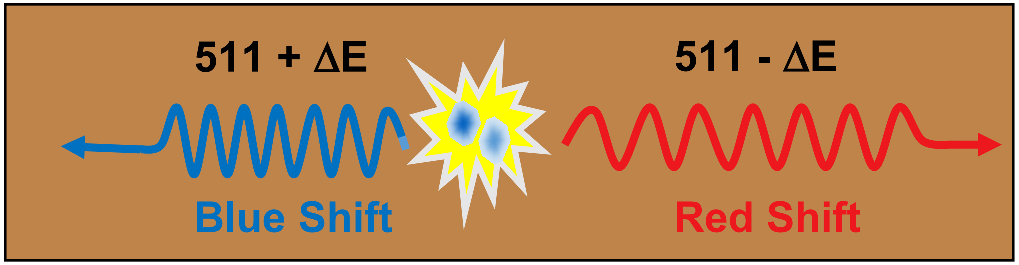
Pair Production

Photon interacts with the nucleus, creates a positron/electron pair

$$h\nu = 2mc^2 + T_+ + T_-$$



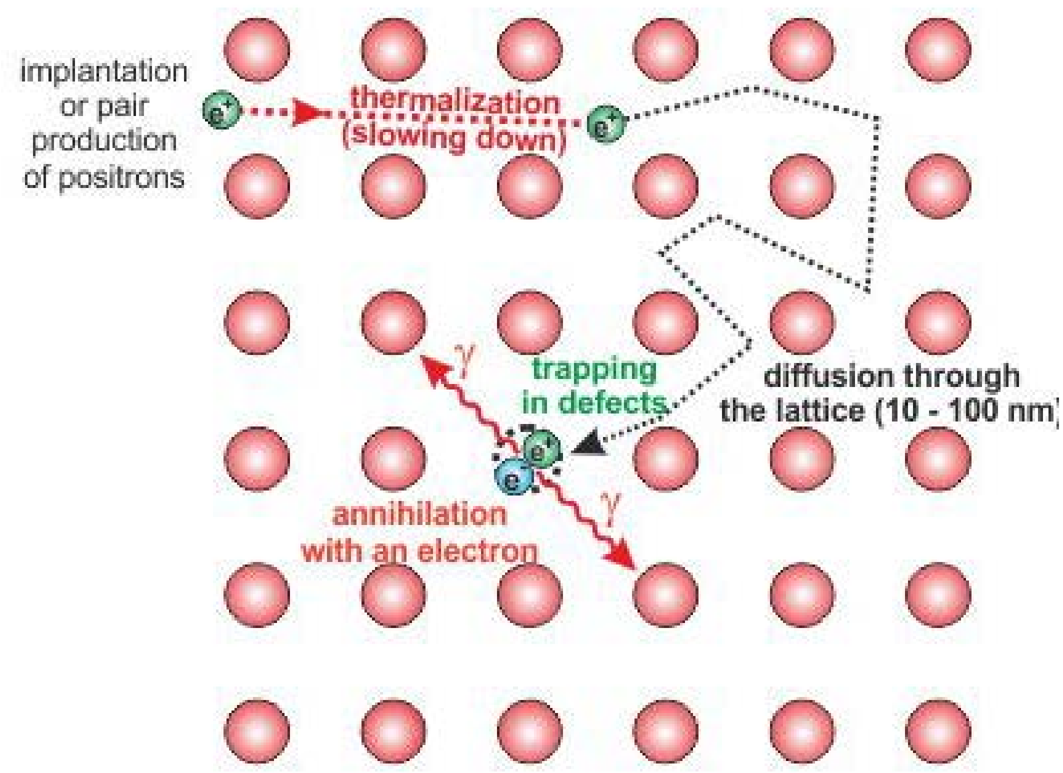
Remember this from PAS?



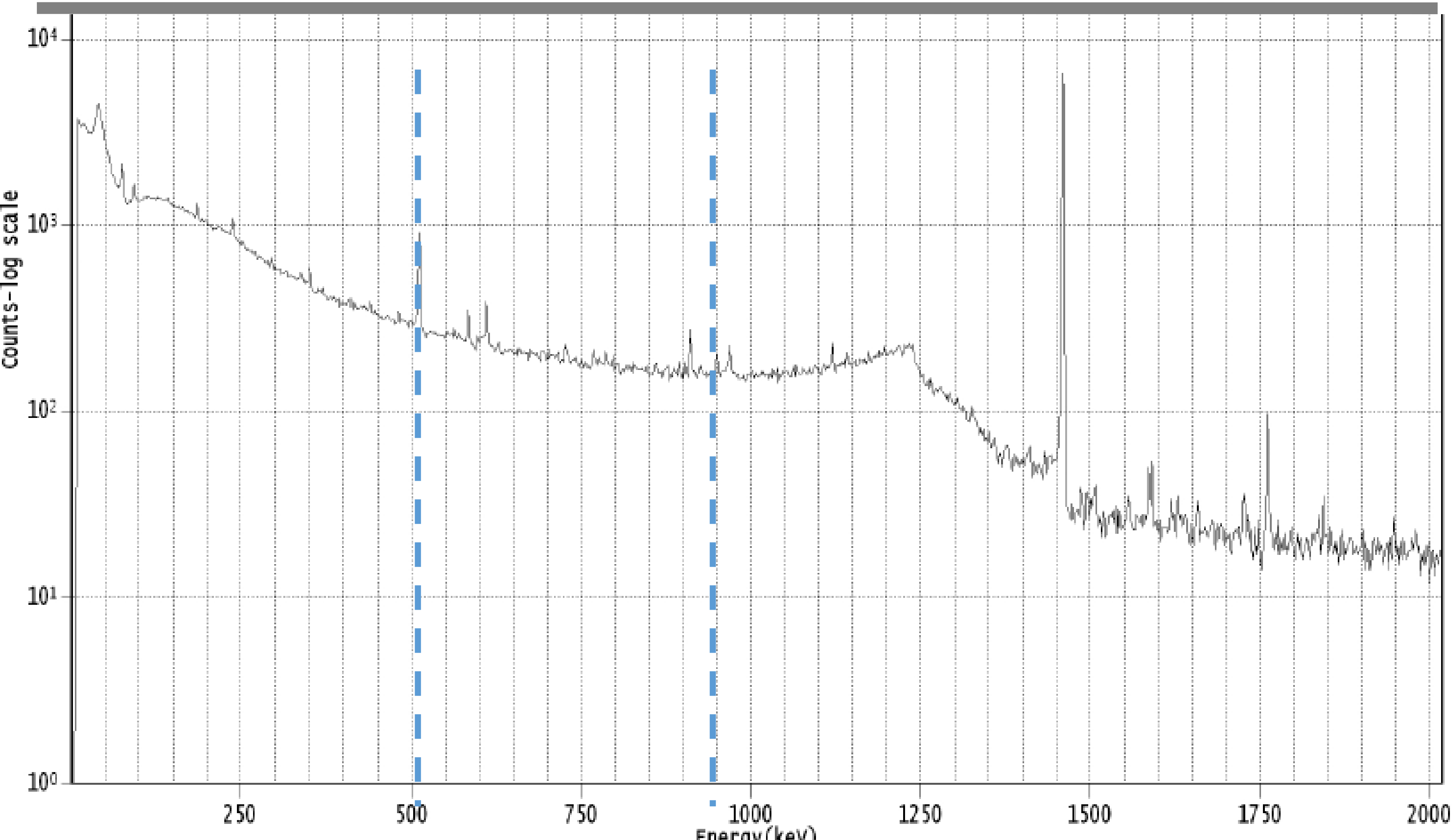
Doppler shift, ΔE
Is proportional to electron momentum, p_L

What Happens to the Positron?

https://www.hzdr.de/db/Cms?pNid=3581

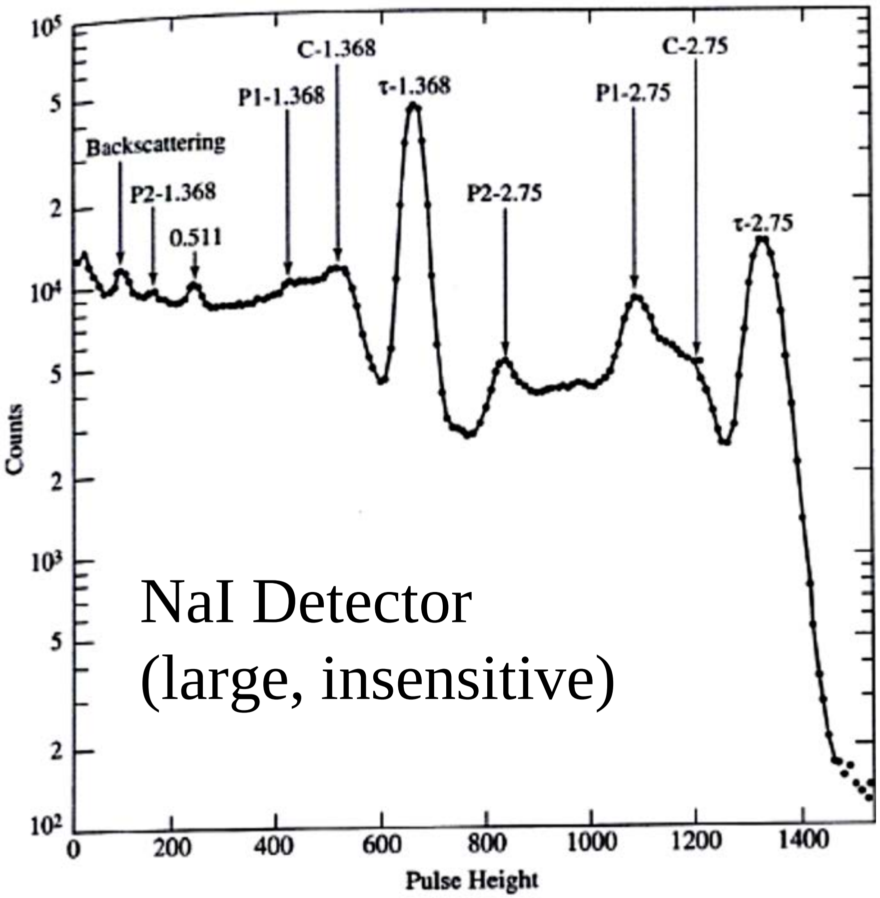


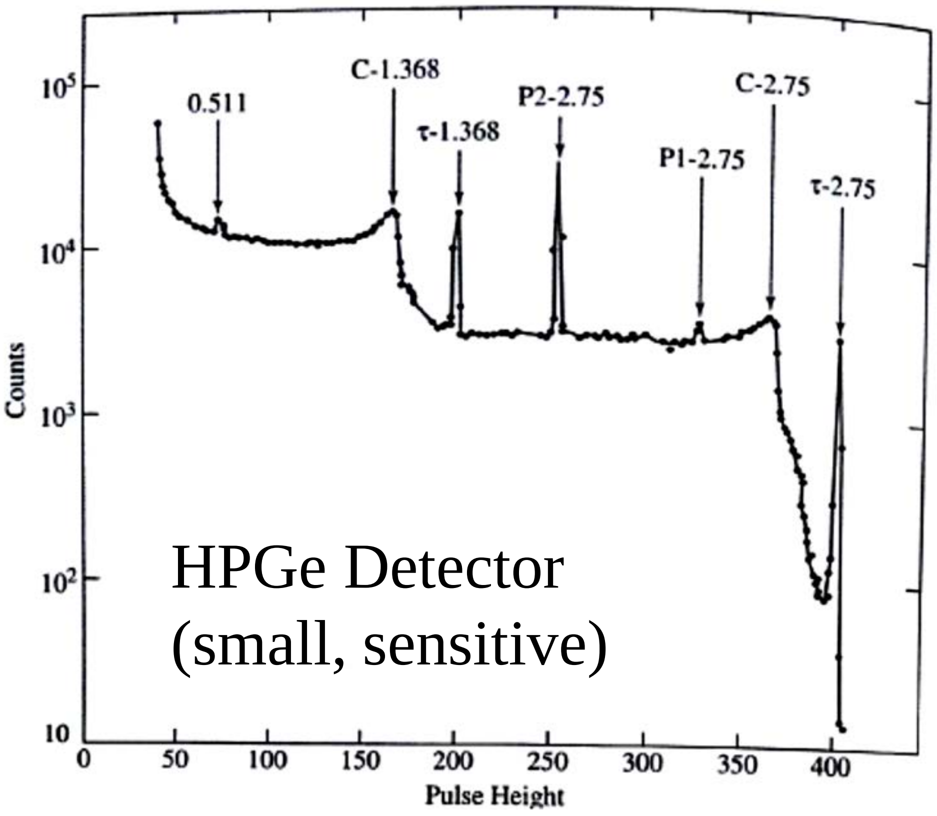
What Evidence Do We Have?



Understanding Detectors

Yip, p. 221-222





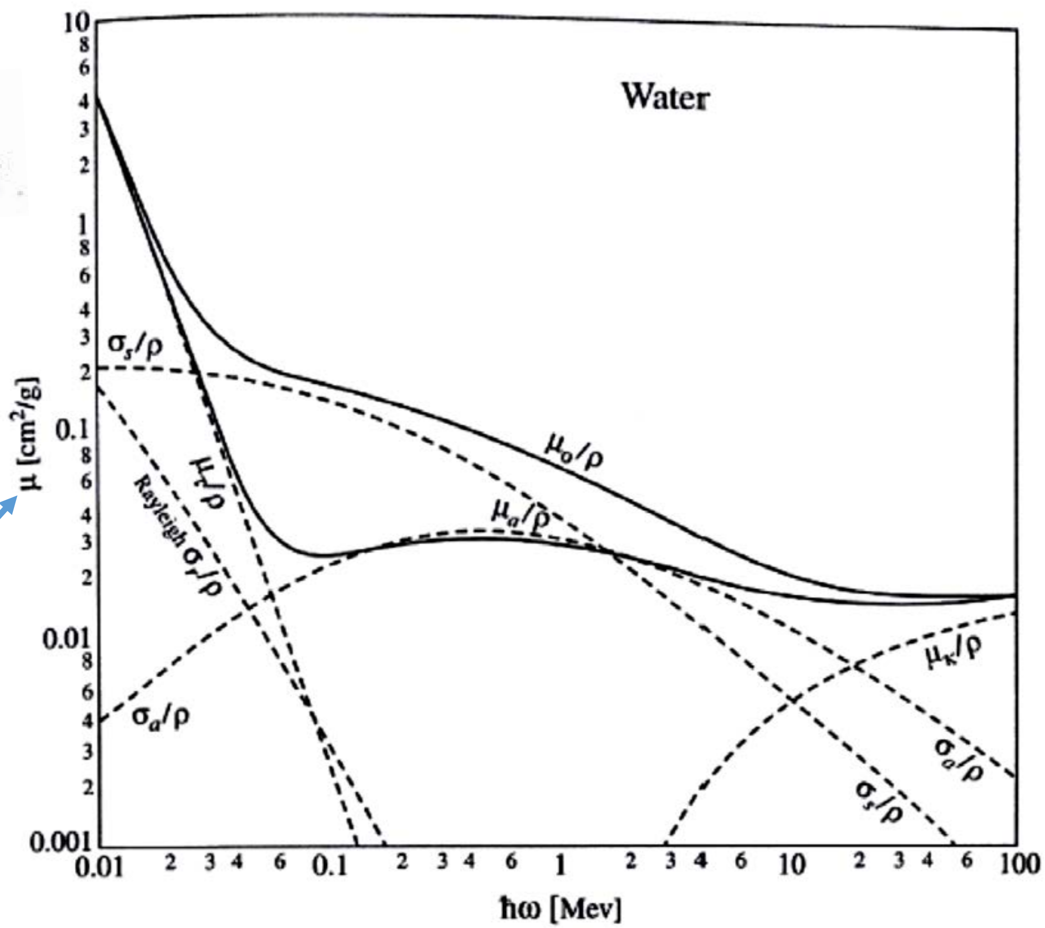
Mass Attenuation Coefficients

$$\mu = \mu_C + \mu_\tau + \mu_\kappa$$

C: Compton
τ: Photoelectric
κ: Pair production

Should be μ/ρ

$$I = I_0 e^{\left(\frac{\mu}{\rho}\right)\rho x}$$



Cross Sections for Photon Interactions

Yip, pp. 216-217

Photoelectric Effect:

$$\mu_\tau/\rho = (N_o/A)\,\sigma_\tau, \quad \sigma_\tau \sim Z^5/(\hbar\omega)^{7/2} \quad \textit{per atom} \qquad (10.44)$$

Compton Scattering:

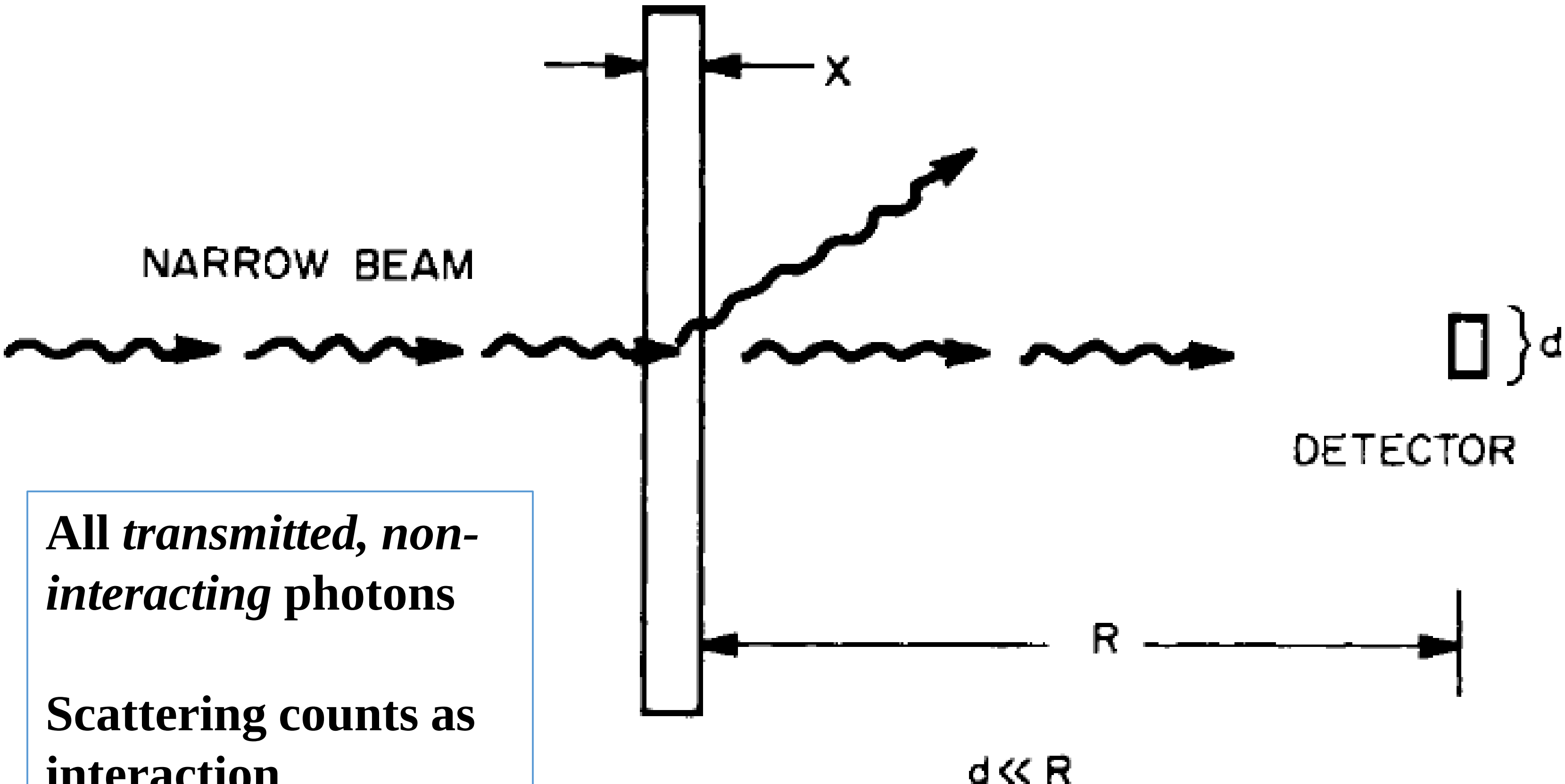
$$\mu_C/\rho = (N_o/A)\,Z\sigma_C, \quad \sigma_C \sim 1/\hbar\omega \quad \textit{per electron} \qquad (10.43)$$

Pair Production:

$$\mu_\kappa/\rho = (N_o/A)\,\sigma_\kappa, \quad \sigma_\kappa \sim Z^2 \ell n(2\hbar\omega/m_e c^2) \quad \textit{per atom} \qquad (10.45)$$

What Does μ Measure?

Turner, p. 188



All *transmitted, non-interacting* photons

Scattering counts as interaction

Comparative Mass Attenuations