22.01 Fall 2019, Problem Set 2

Complete all the assigned problems, and do make sure to show your intermediate work.

Part I

Skill-Building Problems (60 points, individual write-ups)

1 Liquid-Drop Nuclear Models

Learning Objective: This question tests your intuitive knowledge of each of the terms in a semi-empirical equation, so you can express in your own words why each term was created.

For these questions, consider the liquid drop model of nuclear mass, which states that the mass of a nucleus can be empirically calculated as in Eq. 4.10 (p. 59) of *Nuclear Radiation Interactions* by S. Yip:

$$BE\left(A,Z\right) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z\left(Z-1\right)}{A^{\frac{1}{3}}} - a_a \frac{\left(N-Z\right)^2}{A} + \delta; \quad \delta = \begin{Bmatrix} \frac{a_p}{\sqrt{A}}\ even-even \\ 0\ even-odd \\ -\frac{a_p}{\sqrt{A}}\ odd-odd \end{Bmatrix} \tag{1}$$

1. Explain the origin of each additive term in this expression. Pay particular attention to the exponents in each one, and explain why they are what they are.
2. Why does the δ term in this expression change sign for odd/even nuclei?
3. Modify Equation 1 to empirically calculate the total rest mass of a given nucleus.

2 Radioactivity Calculations

Learning Objectives: These problems drill your usage of the activity and specific activity equations, so that you can quickly perform such calculations in the future.

For these problems, consider the the isotopes of scandium ranging from ${}^{41}_{21}$Sc to ${}^{49}_{21}$Sc. It is *highly* suggested to use a spreadsheet like Excel or Gnumeric to help complete these problems, to avoid lots of tedious writing and calculator punching. If you do, then be sure to upload your spreadsheet of calculations to the LMOD site. Otherwise we can't give you partial credit!

1. Calculate the radioactivity of one gram of each isotope in Becquerels.

2. Calculate the specific activity of each isotope in $\frac{Ci}{g}$.

3. Graph the radioactivity of nine grams of Sc, assuming the quantity contains one gram of each isotope at $t = 0$. You may have to use a log-linear or a log-log plot to visualize your answer.

Pro Tip: If you want to quickly make really beautiful graphs, try using Desmos. I'll be using it throughout the semester. You can even copy/paste LaTeX equations directly into it!

3 Allowable Nuclear Reactions

Learning Objective: Calculate and use answers from the Q-equation to determine whether nuclear reactions are allowed, and discover that not all allowed reactions are actually observed.

For these problems, determine whether the following reactions would be allowed, and answer the additional questions.

1. Which of the following decay methods are energetically allowable from the *ground state* of ^{213}Bi? Back up your reasoning with an energetic argument (Q-value).

   (a) Alpha decay

   (b) Beta decay

   (c) Positron decay

   (d) Electron capture

   (e) Isomeric transition

   (f) Spontaneous fission

      i. Can you make up a nuclear reaction where this particular one is energetically allowable? In other words, can you find a pair of "fission products" that would make this reaction exothermic?

      ii. Why do you think it's never actually observed?

2. For the reactions which are allowed, write the full nuclear reaction in each case, and draw a graph of the energy spectrum you would expect to see from each released form of radiation, including secondary ejections of particles or photons.

Part II

Noodle-Scratchers (40 points, team write-ups)

4 Recasting the Semi-Empirical Mass Formula (with answer)

Learning Objective: This problem will expand your ability to interpret how to derive and simplify an expression to relate one variable to another, with a certain goal in mind (here maximizing nuclear stability). You will have to choose the quantity which best describes nuclear stability.

Using the liquid drop model of nuclear stability, derive an expression relating the most stable number of total nucleons (A) for a given nucleus with Z protons. How does your prediction graphically compare with the isotopes of scandium (Sc)?

Answer:

$$0 = -\frac{a_s}{3} A^{\frac{-4}{3}} - \frac{4a_c}{3} Z\left(Z-1\right) A^{\frac{-7}{3}} + 4Za_a A^{-2} - 8Z^2 a_a A^{-3} \quad (2)$$

5 Predicting the Island of Stability (open-ended)

Learning Objective: This problem tests your ability to create a new, semi-empirical mathematical term to best fit a set of data. It is often useful to develop these empirical correlations in the absence of first-principles or mechanistic information.

Does the semi-empirical liquid drop model of nuclear stability predict the "island of stability" containing the super-heavy elements (SHEs)? If so, graphically or mathematically explain how. If not, read through the article from Physics Today (posted on the LMOD site) and develop a missing mathematical term to the semi-empirical mass formula, which would account for the more stable SHEs. Justify your extra term by *quantitatively* checking the improved fit with the known elements.