22.01 Problem Set 4

Complete all the assigned problems, and do make sure to show your intermediate work.

Part I

Skill-Building Problems (60 points, individual write-ups)

Learning Objectives: Test your intuitive knowledge of series radioactive decay, microscopic cross sections, and finish up the preparation for Quiz 1.

1. Without re-deriving the equations, modify the analytical solutions to the first two isotopes in the series decay equations to account for their being "burned" in a nuclear reactor, by a neutron flux $\Phi \left[\frac{n}{cm^2 s}\right]$, during decay. Note that the non-reactor analytical forms are as follows:

$$N_1(t) = N_{10} e^{-\lambda_1 t} N_2(t) = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}\left(e^{\lambda_1 t} - e^{-\lambda_2 t}\right) \tag{1}$$

1 RTG Operation

Learning Objective: Develop decay diagrams from scratch for a real, physical situation, and quantitatively decide how an RTG produces power from which decay(s).

In these problems, consider the decay of ^{126}Sn, an isotope which could be (but isn't) used in radioisotope thermoelectric generators (RTGs).

1. Write all the possible types of decay reactions for ^{126}Sn and its unstable daughter products, and state which decay processes (and competing processes) may be possible for each general type of reaction. You don't have to address every single energy level! Just group them into categories.
2. Draw a complete energy level diagram of ^{126}Sn decay to its eventual stable daughter nucleus.
3. It is clear that ^{126}Sn and its daughter isotopes release a few types of radiation at a few different energies. Which type(s) of radiation do you expect to be responsible for producing the most heat generation in an RTG, and why?

2 Waste Burning Reactors

Learning Objective: Develop series decay equations from scratch to model a real, physical situation.

Transatomic Power Inc. proposed a molten salt fueled reactor which could burn long-lived nuclear waste from fission reactors. One of these long-lived fission products, ^{129}I, has a half life of 15,700,000 years and will remain a major danger by beta decay in buried nuclear waste. If this reactor could efficiently burn most of this waste within about 10 years, it would be a great boost to reducing the issue of nuclear waste.

1. Write balanced nuclear reactions describing what happens to the following isotopes in the reactor: (1) $\mathbf{^{129}I}$, (2) $\mathbf{^{130}I}$, (3) $\mathbf{^{129}Xe}$, (4) $\mathbf{^{130}Xe}$.

2. Set up, but do not solve, differential equations describing the nuclear reactions above. Define any needed symbols. Account for the neutronic activation of one generation of daughter isotopes in the reactor.

3 Food Radioactivity

Learning Objective: Using real gamma spectra, determine the relative radioactivity of some foods one might eat without thinking.

Using the gamma spectra and count rates from our food lab (you can find the files on Canvas), determine the specific radioactivity in Bq/kg of 11 kg of gummy worms and 5 kg of dried wasabi peas. Which is more radioactive, and why do you think it is so? You'll have to account for:

1. Detector live time

2. Background spectrum

3. Detector efficiency at each energy

Use the Lund University Alpha/Gamma Decay Searching Tool to identify what your gamma peaks may be.

Part II

Noodle Scratchers (40 points, team write-up)

4 RTG Operation (20 points, final answer given)

Learning Objective: Here you will put your knowledge of the models for series radioactive decay to the test quantitatively, to arrive at a numerical answer for a realistic space mission design problem.

In these problems, consider the decay of ^{126}Sn, an isotope which could be (but isn't) used in radioisotope thermoelectric generators (RTGs).

1. **Calculate the initial mass in kg** of pure ^{126}Sn one would need to provide to deliver at least 1 kW of thermal power to a spacecraft after 25 years. Assume that the power comes only from the betas (the gamma rays escape). You will have to find the average beta decay energy released from the decay of each isotope, taking the probability of each beta decay into account. Why is ^{126}Sn such a terrible RTG isotope from a nuclear point of view?
Answer: 13,985 kg

5 Waste Burning Reactors (20 points, final answer not given)

Learning Objectives: Look up and use the correct, fundamental nuclear data to answer a specific question regarding long-lived radioactive waste, graphically guess the solution to the problem, and make a couple of sweeping assumptions to more easily solve the problem.

Transatomic Power Inc. proposed a molten salt fueled reactor which could burn long-lived nuclear waste from fission reactors. One of these long-lived fission products, ^{129}I, has a half life of 15,700,000 years and will remain a major danger by beta decay in buried nuclear waste. If this reactor could efficiently burn most of this waste within about 10 years, it would be a great boost to reducing the issue of nuclear waste.

1. Draw the approximate, to-scale solution to this system of equations, assuming a thermal neutron flux of $\mathbf{10^{14}\ \frac{n}{cm^2 s}}$, and a neutron energy of 0.025 eV. You will have to choose and look up the correct cross sections to use in this part on the JANIS website. NOTE: You don't need to actually solve the system analytically to complete this part!

2. Does the reactor burn most of the $\mathbf{^{129}I}$ in ten years? If so, explain as quantitatively as you can in symbolic terms. If not, how would you change the flux of the reactor to make it behave as promised?