22.01, Problem Set 5

Complete all the assigned problems, and do make sure to show your intermediate work.

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1 (60 points) Skill Building Problems

1.1 (12 points) Conceptual Questions

Learning Objective: Cement your intuitive understanding about peculiarities of photon interactions.

1. At which photon energies and atomic numbers are each of the *four* main photon interactions the most prominent? Which of these effects can be completely neglected at which energies? (Consider the Photoelectric effect, Compton scattering, pair production, and photofission). Look at the JANIS cross sections for photofission for a couple heavy elements to help you answer.
2. Which electron energy shell transitions (give the numbers or labels of the levels involved) are responsible for the discontinuities in the mass attenuation coefficient of photons in lead, and at which energies? Which of the three photon interaction methods is responsible for these discontinuities? Confirm that your estimates of the shell levels are correct, by looking up their energy transitions in the NIST x-ray energy transition database.
3. Why does the pair production cross section become non-zero abruptly at energies above $\hbar\omega = 2m_ec^2$?
4. Explain why it is more likely to see a single-escape pair production peak from a large NaI detector, while it is more likely to see a double-escape pair production peak from a small semiconductor detector.

1.2 (12 points) Compton Scattering Calculations

Learning Objective: Use the Klein-Nishina cross section and Compton scattering formulas to calculate photon scattering quantities.

1. Calculate the Compton wavelength shift ($\Delta\lambda$), the kinetic energy of the Compton electron (T_{e^-}), and the energy of the scattered photon ($E' = h\nu'$) for the following situations:
   (a) 10 keV photons scattered at angles of $\theta = \frac{\pi}{3}$, $\theta = \frac{\pi}{2}$, $\theta = \pi$
   (b) 100 keV photons scattered at angles of $\theta = \frac{\pi}{3}$, $\theta = \frac{\pi}{2}$, $\theta = \pi$
   (c) 1 MeV photons scattered at angles of $\theta = \frac{\pi}{3}$, $\theta = \frac{\pi}{2}$, $\theta = \pi$
2. What patterns do you notice in your calculations as functions of energy and angle?

3. Using the Klein-Nishina angular differential cross section for Compton scattering, at which of the three angles is Compton scattering most likely for each listed energy?

$$\frac{d\sigma_C}{d\Omega} = \frac{k_0^2 e^4}{2m_e^2 c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - sin^2\theta\right) \tag{1}$$

1.3 (24 points) Mass Attenuation Coefficients

Learning Objective: Solve a realistic x-ray shielding problem and explain features in mass-attenuation coefficient graphs.

1. Choose a lead apron thickness to shield dental patients from 90% of the x-rays emitted from a pure 40 kV x-ray source. Do you have to consider any additional x-rays produced in the lead, and if so, account for this in your calculations.

2. Explain the qualitative differences in the attenuation coefficients of lithium and tungsten in a quantitative manner, at the following energies: $E_\gamma < 100$keV, $E_\gamma = 1$MeV, $E_\gamma = 100$MeV. By this, we mean compare relative values of the relevant scattering cross sections, and explain any discrepancies between these and the relative values of the attenuation coefficients.

3. What is the origin of the discontinuities in the attenuation coefficient for tungsten? Why is there more than one step change within close proximity at some places?

4. For which energies is the *attenuation coefficient* in water higher than that in air? What about the *mass attenuation coefficient*?

1.4 (12 points) Spectra from Scratch

Learning Objective: Predict a gamma spectrum to test your knowledge of gamma ray interactions in different types of detectors.

Draw the expected *photon (gamma plus x-ray)* spectrum expected for the decay of ^{19}O, for (a) the case of a large, insensitive NaI detector and (b) the case of a small, sensitive HPGe detector. Label the location of every expected peak's energy (give the numerical value and a symbolic calculation), and describe the mechanism responsible for each. ***Only consider gamma rays with at least a 50% intensity.***

2 (40 points) Noodle-Scratchers (*team write-ups OK, as long as everyone typesets their own*)

2.1 Unknown Gamma Spectral Identification *(Answer not given... yet)*

Learning Objective: Identify and quantify an unknown gamma spectrum resulting from neutron activation analysis (NAA).

Using the NAA spectra obtained in the MIT nuclear reactor and HPGe detector, and the table of potential isotopes for "Longs" analysis on the Canvas site, identify/label this gamma spectrum resulting from NAA of a local apple seed, and calculate the amounts of observed isotopes in parts per million weight (ppmw) of each isotope knowing that the seed weighed 89.78 mg. You will want to use external databases to help you search, such as the Gamma Search database and the KAERI tables. Label every significant peak you find on the spectrum itself (you must sort out what is a "true peak"), and note the mechanistic origins of each. Also label locations where you *would have expected peaks,* and explain why you don't see them. Assume this sample was in the MIT nuclear reactor for a period of one hour with a thermal neutron flux of $\Phi = 10^{14}\ \frac{\mathrm{n}}{\mathrm{cm^2 s}}$ ($\mathrm{E_{neutrons}} = 0.025\,\mathrm{eV}$), and that one day elapsed between activation and counting.

To solve this problem in steps, you'll need to perform the following:

1. Identify the peaks in the gamma spectrum as we did together in class.

2. For each significant peak, convert from a number of counts (the raw data, Net Peak Area) to radioactivity in Bq.
3. Compute how much of that isotope must have existed immediately after removal from the MIT reactor, noting it had been decaying for one day.
4. Work backwards to figure out how much of the original isotope had to exist to explain your quantity in (3).
   (a) You can model this as a series decay problem, where the original isotope (let's say ^{23}Na for example) underwent neutron capture (n, γ) to become ^{24}Na. This time, instead of knowing N_{10} of ^{23}Na at the start, you know the quantity $\mathrm{N}_2\,(\mathrm{t} = 3,600\,\mathrm{sec})$ instead.
   (b) Here your "effective λ" values should include both their natural decay (for N_2 only (here ^{24}Na)) and their burning, or artificial decay (for all isotopes).
   (c) Also account for burning of isotope N_2 (here ^{24}Na) while it's still in the reactor. Use JANIS to find $(n, absorption)$ cross sections where you can, and assume a value of 1 barn when the data is not available. Use the most recent ENDF library for your cross sections.
5. Finally, now that you can solve for N_{10} of each isotope, convert from number of atoms to weight, and divide by the apple seed mass to get concentration in ppmw.

I strongly suggest implementing this numerically, so once you've got a formula, you can just put each isotope's net counts, decay constant, and cross sections in a formula or spreadsheet to compute things for you.

2.2 Compton Scattering Derivation Practice (answer given)

Learning Objective: This problem will develop your ability to derive quantities from a pure physical model, using the two-particle kinematics derivation from class as an example.

Using photon energy and momentum relations, derive the formula for the Compton wavelength shift $(\Delta\lambda)$ as a function of θ. Use the fact that the momentum of an electron can be expressed as $cp = \sqrt{T\left(T + 2m_e c^2\right)}$ ***(NOTE: There are two possible approaches: (1) Conserve x-momentum, y-momentum, and total energy*** $\left(T + m_e c^2\right)$***, and (2) conserve total momentum and kinetic/photon energies. You just have to show one.)***

- ***Answer:*** $\Delta\lambda = \frac{h}{m_e c}\left(1 - cos\left(\theta\right)\right)$

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