22.01 Fall 2021, Quiz 1

October 18, 2021

Quiz Instructions: Answers can be given symbolically or graphically, no calculation is necessary aside from order-of-magnitude math.

No devices, or anything else allowed, except for one calculator (graphing is OK, or calculator apps on your phone/tablet - I'm trusting you!) and one double-sided, 8.5 x 11 inch or A4 sheet of paper, or electronic equivalent.

Define any intermediate variables or symbols which you need to complete the problems. Generous partial credit will be given for correct methodology, even if the solution is not given.

You will have 24 hours of your choosing within a 48 hour window to complete the exam. The exam will be available starting 14:00 Tuesday Oct. 19, until 14:00 Thursday Oct. 21.

Please upload a PDF of your answers to the Canvas site at any time in the 48 hour exam window. You can type your answers, draw them on the computer, use paper and take pictures with your phone, or anything else you like.

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1 (70 points) Short Answers, 10 points each

Each of these problems can be solved with one sentence, one equation, or one graph.

1.1 **In which region would you look on the Table of Nuclides to find isotopes most likely to undergo positron decay? Choose from the following, and explain your reasoning:** ***slightly neutron-rich ($N > Z$), very neutron-rich ($N \gg Z$), slightly proton-rich ($Z > N$), very proton-rich ($Z \gg N$), heavy (high A), light (low A).***

1.2 **Estimate, to an order of magnitude, how much heavier/lighter the meta-state of Technetium (^{99m}Tc) is compared to its ground state ^{99}Tc. The meta-state energy of ^{99m}Tc is at 0.143 MeV. Call it 0.1 MeV for simplicity.**

1.3 **Under what physical/mathematical condition(s) could an isotope theoretically decay by direct emission of a ^{14}C nucleus?**

1.4 **Why is "inelastic scattering (n, n')" not truly a scattering reaction? Use aspects of the generalized Q-equation (see Eq. 2) for your answer.**

1.5 **Why shouldn't we round nuclear masses/binding energies when identifying gamma peaks on a high-purity Germanium detector (HPGe) spectrum, like that of our Chernobyl honey? Assume these detectors have energy resolutions of ~0.5 keV.**

1.6 **Fukushima released about 25 PBq of radiation (2.5×10^{19} Bq), which is a HUGE amount and on par with that released from Chernobyl. By what math/logic would you explain to someone that fish in the ocean are still safe to eat?**

1.7 **Under what criterion could one theoretically observe alpha particle emission at an energy of $E_\alpha = Q$?**

2 **(30 points): Set up (do not solve) equations for, and sketch an approximate graph of, the amount of each of three isotopes (N_1, N_2, N_3) in a series decay chain inside a nuclear reactor with neutron flux Φ, where $\boldsymbol{N_3}$ is stable, and $\Phi\sigma_1 = \Phi\sigma_2 = \Phi\sigma_3 = \lambda_1 = \lambda_2$. Pay attention to amounts and rates of change of the isotopes at key points ($t = 0$, $t = \infty$, places where $\frac{dN_i}{dt} = 0$). Why can't you calculate it so easily from Equation 5 below, and why doesn't it *appear* that the total mass of the isotopes is conserved on this graph?**

Bonus Question (15 points) - The sun has a power of roughly 4×10^{26} W, powered by a chain of hydrogen fusion reactions, simplified as follows

$$6\,{}^1_1H \Longrightarrow 2\,{}^1_1H + {}^4_2He + 2\beta^+ + 2\nu + 2\gamma; \qquad Q = 20\,MeV \tag{1}$$

Estimate, to an order of magnitude, how many neutrinos enter you per second coming from the sun. Assume you subtend a solid angle of 2×10^{-24} Sr (steradians) from the center of the sun.

Useful Formulas

$$Q = T_3\left(1 + \frac{M_3}{M_4}\right) - T_1\left(1 - \frac{M_1}{M_4}\right) - \frac{2}{M_4}\sqrt{M_1 M_3 T_1 T_3}\cos\theta; \qquad \alpha = \left(\frac{A-1}{A+1}\right)^2 \tag{2}$$

$$A = \lambda N; \qquad BE\,(A,Z) = \left(ZM_H + (A-Z)\,M_n - M\,(A,Z)\right)c^2 \tag{3}$$

$$BE\,(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c\frac{Z\,(Z-1)}{A^{\frac{1}{3}}} - a_a\frac{(N-Z)^2}{A} + \delta; \quad \delta = \left\{\begin{matrix} \frac{a_p}{\sqrt{A}}\ even-even \\ 0\ even-odd \\ -\frac{a_p}{\sqrt{A}}\ odd-odd \end{matrix}\right\} \tag{4}$$

$$N_2\,(t) = \frac{N_0\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \qquad M = A + \Delta \qquad 1\,amu = 931.49\,\frac{MeV}{c^2} \tag{5}$$