22.01 Fall 2021, Quiz 2

November 22, 2021

Quiz Instructions: Answers can be given symbolically or graphically, no calculation is necessary aside from mental order-of-magnitude math. ***No devices, or anything else allowed, except for one calculator and one double-sided, 8.5 x 11 inch or A4 sheet of paper.*** Define any intermediate variables or symbols which you need to complete the problems. Generous partial credit will be given for correct methodology, even if the solution is not given.

Please upload a PDF of your answers to the Canvas site at any time in the 24 hour exam period. You can type your answers, draw them on the computer, use paper and take pictures with your phone, or anything else you like.

Many useful formulas are included for you on the last page of the exam.

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1 (70 points) Short Answers, 10 points each

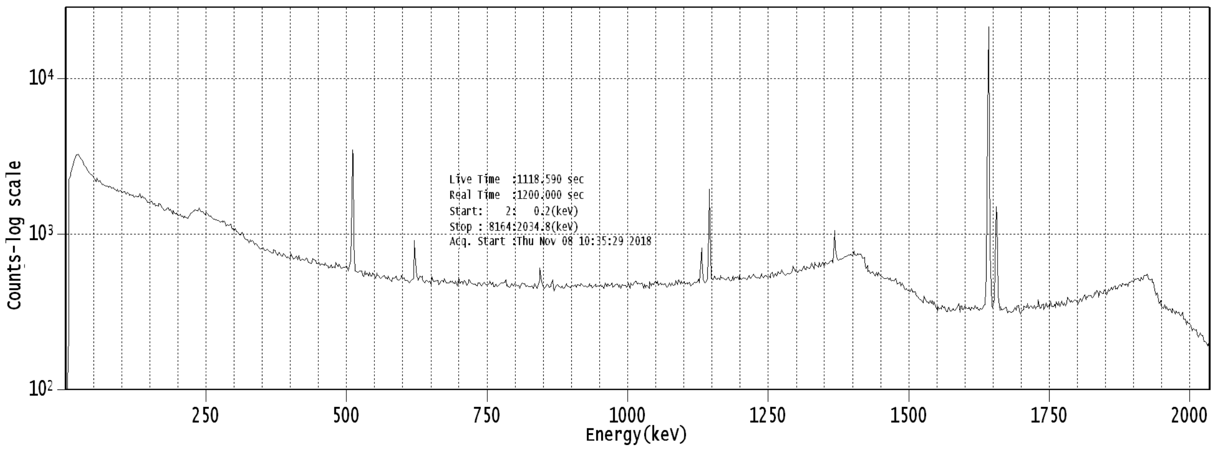
Each of these problems can be solved with one sentence, one equation, or one graph.

1.1 Which will cause more nuclear damage during ion irradiation of pure iron: 1 kg of protons, 1 kg of carbon ions, or 1 kg of iron ions? Why do you say so?

1.2 Using the two-group neutron criticality relation, predict what will happen to k_{eff} of a pure ^{239}Pu sphere which, after falling back to Earth upon a failed launch and heating up during re-entry, falls into an ice-cold region of the ocean. Just arrows on terms and justifications will do.

$$k_{eff} = \frac{\left(\bar{\nu}_f \overline{\Sigma}_{f_f} + \bar{\nu}_{th} \overline{\Sigma}_{f_{th}} \left[\frac{\overline{\Sigma}_{s_{f \rightarrow th}}}{\overline{D}_{th} B_g^2 + \overline{\Sigma}_{a_{th}}}\right]\right)}{\overline{\Sigma}_{s_{f \rightarrow th}} + \overline{\Sigma}_{a_f} + \overline{D}_f B_g^2}$$

1.3 For the following photon spectrum, explain the physical origin(s) of each visible feature in terms of photon interactions.



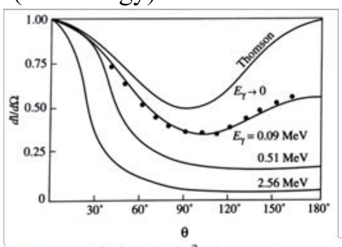
1.4 For the photon spectrum above, continue drawing what you would expect the spectrum to look like between 2000-3000 keV, based on what you can infer about the spectrum from the 0-2000 keV data.

1.5 **Set up, but do not solve, a system of equations which quantifies the amount of ^{60}Co one would have as a function of time, by starting with a mass M_{59} of ^{59}Co in a nuclear reactor with a neutron flux Φ. Define any symbols you need to complete the equation. How do you determine the optimum time to harvest your ^{60}Co?**

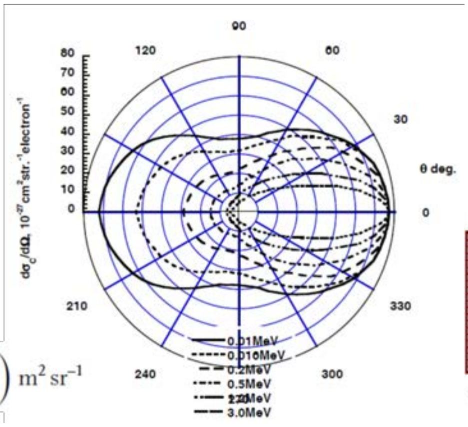
1.6 **Draw an approximate sketch of the energy deposition as a function of depth, resulting from a proton beam entering a human skull and stopping at a tumor at the center of the brain. Model this as a two-layer system (bone-brain).**

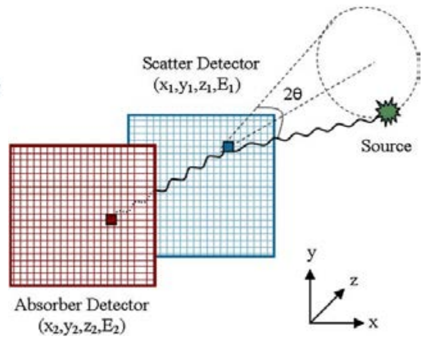
1.7 **A *Compton Camera Swarm* (which I think I just made up) consists of lots of drone-mounted detectors looking for coincident photon detections, using the relationship between Compton scattering energy and angle. Assuming the drone swarm starts as a sphere at the origin (of our arbitrary coordinate system), and it thinks it detects a photon coming somewhere from the +z direction, draw the ideal shape in which the swarm should rearrange itself to maximize the likelihood of further Compton scatter detections coming from the same direction. Use the Klein-Nishina angular photon cross section formula to guide your answer.**

A simplified version… and the Thomson scattering (low energy) limit



$$\frac{d_e\sigma}{d\Omega} = \frac{k_0^2 e^4}{2m^2c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2\theta\right) \text{ m}^2\,\text{sr}^{-1}$$





2 **(15 points) Design a person-sized detector/shield combination, which is excellent at detecting the flux of high energy (fast) neutrons, amidst a radiation field consisting of high and low energy neutrons, electrons, and photons. Also take into account how to shield the detector operator from all forms of radiation present, who would be standing behind the detector. Assume the radiation source is in front of the detector, and the operator is standing behind it as a shield. You can use any combination of materials which you would like.**

(15 points) Next, make it quantitative: Assume that you want to shield the operator from 99% of all the radiation which would be present in front of the detector, assuming the operator is standing behind it. Assume fluxes of $\Phi_{n_{high-E}}$, $\Phi_{n_{low-E}}$, $\Phi_{e^-_{high-E}}$, $\Phi_{e^-_{low-E}}$, $\Phi_{\gamma_{high-E}}$, $\Phi_{\gamma_{low-E}}$. Set up, but do not solve, the equations which would allow you to determine the thicknesses of the detector layers to ensure at least 99% shielding from each and every form of radiation.

Useful Formulas

$$\Delta\lambda = \frac{h}{m_e c}(1 - cos\theta) \qquad T_{e^-} = h\nu - h\nu' = h\nu \frac{1 - cos\theta}{\frac{m_e c^2}{h\nu} + 1 - cos\theta}$$

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} = I_0 e^{-\Sigma x} \qquad \frac{d\sigma_C}{d\Omega} = \frac{k_0^2 e^4}{2m_e^2 c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - sin^2\theta\right)$$

$$-\frac{dT}{dx} = \frac{4\pi k_0^2 N Z_1^2 Z_2 e_c^4}{m_e v^2} ln\left(\frac{2m_e v^2}{\bar{I}}\right) = \frac{2\pi k_0^2 N Z_1^2 Z_2 e_c^4}{E_i}\frac{m_{ion}}{m_e} ln\left(\frac{\gamma_e E_i}{\bar{I}}\right);\ \gamma_e = \frac{4m_e M}{(M + m_e)^2}$$

$$\frac{-\left(\frac{dT}{dx}\right)_{ioniz.}}{-\left(\frac{dT}{dx}\right)_{nucl.}} = \frac{2M}{m_e Z}\frac{ln\left(\frac{\gamma_e E_i}{\bar{I}}\right)}{ln\left(\frac{\gamma E_i}{E_d}\right)}; \qquad \gamma = \frac{4mM}{(m+M)^2} \qquad \frac{-\left(\frac{dT}{dx}\right)_{rad.}}{-\left(\frac{dT}{dx}\right)_{ioniz.}} = \left(\frac{m_e}{M}\right)^2\left(\frac{ZE_i}{1400 m_e c^2}\right) \qquad Range = \int_0^{E_i} -\left(\frac{dT}{dx}\right)^{-1}$$

$$\frac{1}{v}\frac{d\phi(\mathbf{r}, E, \mathbf{\Omega}, t)}{dt} = \frac{\chi(E)}{4\pi}\int_V\int_{E'}\int_{\Omega'} d^3r dE' \boldsymbol{d\Omega'} \nu(E')\Sigma_f(E')\phi(\mathbf{r}, E', \mathbf{\Omega'}, t)$$

$$+S_0(\mathbf{r}, E, \mathbf{\Omega}, t) + \int_V\int_{E'}\int_{\Omega'} d^3r dE d\mathbf{\Omega'}\Sigma_s(E')\phi(\mathbf{r}, E', \mathbf{\Omega'}, t) F(E' \to E, \mathbf{\Omega'} \to \mathbf{\Omega})$$

$$-\int_V d^3r dE d\mathbf{\Omega}\Sigma_t(E)\phi(\mathbf{r}, E, \mathbf{\Omega}, t) - \int_V d^3r dE \boldsymbol{d\Omega}\mathbf{\Omega}\cdot\boldsymbol{\nabla}\phi(\mathbf{r}, E, \mathbf{\Omega}, t)$$

$$\bar{\Sigma} = \frac{\int_0^{E_{max}}\Sigma(E)\Phi(E)dE}{\int_0^{E_{max}}\Phi(E)dE} \qquad B^2_{material} = \frac{\nu\Sigma_f - \Sigma_a}{D} \qquad B_g^2 = \left(\frac{\pi}{a_{ex}}\right)^2 \ or\ \left(\left(\frac{\nu_1}{R_{ex}}\right)^2 + \left(\frac{\pi}{H}\right)^2\right)\ or\ \left(\frac{\pi}{R_{ex}}\right)^2$$

$$\Sigma_a\Phi - D\nabla^2\Phi = \frac{\nu\Sigma_f\Phi}{k_{eff}} \implies \Sigma_a\Phi - DB^2\Phi = \frac{\nu\Sigma_f\Phi}{k_{eff}} \implies k_{eff} = \frac{\nu\Sigma_f}{\Sigma_a + DB^2}$$

$$\frac{-\nabla^2\Phi}{\Phi} = B^2_{geometry}; \quad ,\Phi(x) = A\,cos(B_g x) + G\,sin(B_g x)\ \ in\,Cartesian\,coordinates$$

$$k_{eff} = \frac{\left(\bar{\nu}_f\overline{\Sigma}_{f_f} + \bar{\nu}_{th}\overline{\Sigma}_{f_{th}}\left[\frac{\overline{\Sigma}_{s_{f\to th}}}{\overline{D}_{th}B_g^2 + \overline{\Sigma}_{a_{th}}}\right]\right)}{\overline{\Sigma}_{s_{f\to th}} + \overline{\Sigma}_{a_f} + \overline{D}_f B_g^2} \qquad Q = 0 = T_f\left(1 + \frac{1}{A}\right) - T_i\left(1 - \frac{1}{A}\right) - \frac{2}{A}\sqrt{T_i T_f}cos\theta \quad (1)$$