

22.02 Intro to Applied Nuclear Physics

Mid-Term Exam

Thursday March 17, 2011

Solution

Problem 1: Short Questions

24 points

These short questions require only **short answers** (but even for yes/no questions give a brief explanation)

1) What information about a quantum system can you obtain from the wavefunction?

Solution:

The wavefunction contains all possible information about the state of a quantum system. Thus from the wavefunction it is possible to extract information about the probability of measurement outcomes for any observable (thus also the expectation values of the observables). The probability distribution function of the system position (obtained from the absolute value square of the wavefunction) is just one of the .

2) If we measure the kinetic energy of a quantum particle and immediately after we measure its momentum, is the result of the second measurement random?

Solution:

No: if the momentum measurement gave an outcome $p = \hbar k$, then the kinetic energy is $T = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}$. This is because the momentum measurement projects the wavefunction of the system into a momentum eigenfunction (wavefunction collapse) and this momentum eigenfunction is also an eigenfunction of the kinetic energy since the two operators commute.

3) What does the Coulomb term in the Semi-empirical mass formula describe?

Solution:

The Coulomb term describes a decrease in the binding energy due to the Coulomb repulsion among protons in the nucleus. Thus for a given mass number A, it is less favorable to have a large number of protons. The Coulomb term dependence on A and Z is found from a simple model of the nucleus as a spherical charge.

4) A particle is in the quantum state, $\psi(y) = Ae^{-i\pi y}$.

a) What are the possible results of a momentum measurement?

b) What are the probabilities of each possible momentum measurement?

c) What physical situation is represented by this quantum state?

Solution:

a) The outcomes of a momentum measurement are the momentum operator eigenvalues, which span all real numbers. Given the system's state however, only one result has non-zero probability, since the state is an eigenfunction of the momentum operator. The corresponding eigenvalue is $p = -\hbar\pi$.

b) Since there is only one non-zero probability, the probability of finding $p = -\hbar\pi$ is equal to one.

c) This quantum state represent a free, unbound, system (i.e. a system that does not feel the effects of any potential). The system is better represented by a flux of particle with wavenumber k traveling in the $-y$ direction.

5) When is the wavefunction describing a quantum system an energy eigenfunction?

Solution:

When the wavefunction is stationary or time-independent. In that case it has to satisfy the time-independent Schrödinger equation which is the energy eigenvalue equation. Another case is when the wavefunction has collapsed into an energy eigenfunction just after the measurement of the energy (it will then remain in that state, since it will be a stationary, time-independent state).

6) Which one of the following statements (if any) is correct, based on the properties of the angular momentum and its eigenfunctions?

a) A particle is in the angular momentum eigenstate, $\psi_{l,m_z}(\vartheta, \varphi) = |l=3, m_z=-4\rangle$.

b) A particle is in the angular momentum eigenstate, $\psi_{l,m_x,m_z}(\vartheta, \varphi) = |l=4, m_x=3, m_z=-2\rangle$.

c) A particle is in the angular momentum eigenstate, $\psi_{l,m_x}(\vartheta, \varphi) = |l=4, m_x=3\rangle$.

Solution:

We know that the eigenvalues of L^2 and the components L_x, L_z, L_y have to satisfy the relationship: $-l \leq m \leq l$. Thus the first statement is incorrect.

The second statement is also not possible because L_x and L_z do not commute, thus there cannot be a common eigenfunction of the two operators with a fixed m_x and m_z . The last statement is instead a valid eigenfunction of the angular momentum operator.

Note: many of you tried to answer this question from considerations regarding the relationship $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$. If we apply this equation to the state $|l=3, m_z=-4\rangle$, this gives:

$$\hbar^2 3(3+1) |l=3, m_z=-4\rangle = (\hat{L}_x^2 + \hat{L}_y^2) |l=3, m_z=-4\rangle + \hbar^2 4^2 |l=3, m_z=-4\rangle$$

Changing the order of the equation we obtain it implies that $|l=3, m_z=-4\rangle$ is an eigenfunction of the operator $\hat{L}_x^2 + \hat{L}_y^2$, with eigenvalue $-4\hbar^2$:

$$(\hat{L}_x^2 + \hat{L}_y^2) |l=3, m_z=-4\rangle = \hbar^2(3(3+1) - 4^2) |l=3, m_z=-4\rangle = -4\hbar^2 |l=3, m_z=-4\rangle$$

However the operator $(\hat{L}_x^2 + \hat{L}_y^2)$ should have positive eigenvalues: Thus the state $|l=3, m_z=-4\rangle$ cannot be an eigenfunction of \hat{L}^2, \hat{L}_z .

Note that the equations above are operator equations, or, when applied to functions, equations involving functions. They are not directly equations involving the *eigenvalues alone*. Thus I cannot state the implication:

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 \quad \rightarrow \quad \hbar^2 l(l+1) = \hbar^2 m_x^2 + \hbar^2 m_y^2 + \hbar^2 m_z^2$$

The only thing I can state is if I consider the expectation value of the operators wrt a valid eigenfunction:

$$\begin{aligned} \langle l, m_z | \hat{L}^2 | l, m_z \rangle &= \langle l, m_z | \hat{L}_x^2 | l, m_z \rangle + \langle l, m_z | \hat{L}_y^2 | l, m_z \rangle + \langle l, m_z | \hat{L}_z^2 | l, m_z \rangle \\ &\rightarrow \hbar^2 l(l+1) = \langle l, m_z | \hat{L}_x^2 | l, m_z \rangle + \langle l, m_z | \hat{L}_y^2 | l, m_z \rangle + \hbar^2 m_z^2 \end{aligned}$$

$|l, m_z\rangle$ is not an eigenfunction of \hat{L}_x and \hat{L}_y so the previous equation is not correct.

7) When is a quantum system “bound”? Give a condition in terms of the system energy E and potential energy V .

Solution:

A quantum system is bound in a given region of space when its energy $E > V(x)$ in that region of space, but $E < V(x)$ everywhere else, such as in a potential well (with no possibility to ever escape via tunneling). If the well is not infinite, the wavefunction penetrates outside the potential well region (as a decaying exponential): even when this happens, the state is still considered bound.

8) Is the Q-value of a nuclear reaction (such as alpha-decay) the only factor that determines if the reaction does happen spontaneously?

Solution:

No. Although the Q-value for some reactions might be favorable, they still don't happen if there is a large potential barrier (e.g. the Coulomb barrier) that lead to a negligible tunneling probability.

Problem 2: Rotations and angular momentum

26 points

Note: This problem only required very simple answers based on what you know about commutators and operator properties (the only thing you needed to know about angular momentum is that the various component do not commute). Since many of you found it hard, I am giving below very detailed calculations. This was not needed nor required of you (each answer could have taken 3 lines) but hopefully it will help the understanding.

a) (2 points) Consider classical rotations in a 3D Euclidean space. We define $R_{\vec{n}}(\vartheta)$ the operator describing a rotation around the axis \vec{n} by an angle ϑ . Do the operators $R_x(\vartheta)$ and $R_z(\varphi)$ commute? Do the operators $R_x(\vartheta)$ and $R_y(\varphi)$ commute? (a yes/no answer is enough)

Solution:

$R_z(\vartheta)$ and $R_z(\varphi)$ commute (as any two rotations along the same axis) while $R_x(\vartheta)$ and $R_y(\varphi)$ do not commute (as any two rotations along two different axes).

b) Now we consider rotations in quantum mechanics. We write rotations as the operators $\hat{R}_{\vec{n}}(\vartheta)$.

For small angles ϑ we can write these rotations using the angular momentum operator as $\hat{R}_{\vec{n}}(\vartheta) = 1 - i\frac{\vartheta}{\hbar}\hat{L}_n$ (for example $\hat{R}_x(\vartheta) = 1 - i\frac{\vartheta}{\hbar}\hat{L}_x$). Do rotations in quantum mechanics commute?

Solution:

As rotation along different axes are written in terms of angular momentum operators that do not commute, we expect them not to commute. We can prove this by calculating the commutator:

$$[R_a(\vartheta), R_b(\varphi)] = [1 - i\frac{\vartheta}{\hbar}L_a, 1 - i\frac{\varphi}{\hbar}L_b]$$

The commutator of a sum, $[A + B, C + D]$ is calculated by expanding out each term: $[A + B, C + D] = [A, C] + [A, D] + [B, C] + [B, D]$ (as you can verify from the definition of commutator). Then

$$[R_a(\vartheta), R_b(\varphi)] = [1, 1] + [1, -i\frac{\varphi}{\hbar}L_b] + [-i\frac{\vartheta}{\hbar}L_a, 1] + [-i\frac{\vartheta}{\hbar}L_a, -i\frac{\varphi}{\hbar}L_b] = -\frac{\vartheta\varphi}{\hbar^2}[L_a, L_b]$$

where we used the fact that a scalar always commutes with everything. Because of the commutation properties of the angular momentum operator, we know that if $a = b$ (e.g., $a = b = x$) we have $[L_x, L_x] = 0$ and the rotation commute. However, if $a \neq b$ then the two rotation do not commute. This is similar to the classical result.

c) Calculate the difference between making first a rotation $R_y(\varphi)$ followed by a rotation $R_x(\vartheta)$ and making first $R_x(\vartheta)$ and then $R_y(\varphi)$. Can you express this difference as a rotation?

Solution:

This difference is nothing else than the commutator $[R_x(\vartheta), R_y(\varphi)] = R_x(\vartheta)R_y(\varphi) - R_y(\varphi)R_x(\vartheta)$. Given what we calculate above, thus is given by:

$$[R_x(\vartheta), R_y(\varphi)] = -\frac{\vartheta\varphi}{\hbar^2}[L_x, L_y] = -i\frac{\vartheta\varphi}{\hbar}L_z$$

Note that this can be written as a rotation around the z axis, $[R_x(\vartheta), R_y(\varphi)] = R_z(\vartheta\varphi) - 1$.

We could also have calculated explicitly the two cases:

$$\begin{aligned} \text{1st case: } R_x(\vartheta)[R_y(\varphi)[\psi]] &= (1 - i\frac{\vartheta}{\hbar}L_x)[(1 - i\frac{\varphi}{\hbar}L_y)[\psi]] = (1 - i\frac{\vartheta}{\hbar}L_x)[\psi - i\frac{\varphi}{\hbar}L_y[\psi]] = \\ &= \psi - i\frac{\varphi}{\hbar}L_y[\psi] - i\frac{\vartheta}{\hbar}L_x[\psi] - i\frac{\vartheta}{\hbar}L_x[-i\frac{\varphi}{\hbar}L_y[\psi]] = \psi - i\frac{1}{\hbar}(\varphi L_y[\psi] + \vartheta\hbar L_x[\psi]) - \frac{\vartheta\varphi}{\hbar^2}L_x[L_y[\psi]] \\ \text{2nd case: } R_y(\varphi)[R_x(\vartheta)[\psi]] &= (1 - i\frac{\varphi}{\hbar}L_y)[(1 - i\frac{\vartheta}{\hbar}L_x)[\psi]] = (1 - i\frac{\varphi}{\hbar}L_y)[\psi - i\frac{\vartheta}{\hbar}L_x[\psi]] = \\ &= \psi - i\frac{\vartheta}{\hbar}L_x[\psi] - i\frac{\varphi}{\hbar}L_y[\psi] - i\frac{\varphi}{\hbar}L_y[-i\frac{\vartheta}{\hbar}L_x[\psi]] = \psi - i\frac{1}{\hbar}(\varphi L_y[\psi] + \vartheta\hbar L_x[\psi]) - \frac{\vartheta\varphi}{\hbar^2}L_y[L_x[\psi]] \end{aligned}$$

The two expressions are the same *except* for the terms $\propto \frac{\vartheta\varphi}{\hbar^2}$ where L_x, L_y appear in different order:

$$\text{1st case} - \text{2nd case} = -\frac{\vartheta\varphi}{\hbar^2}(L_x[L_y[\psi]] - L_y[L_x[\psi]]) = -\frac{\vartheta\varphi}{\hbar^2}[L_x, L_y][\psi] = -i\frac{\vartheta\varphi}{\hbar}L_z[\psi]$$

thus we obtain the same result.

d) A quantum system is in a state ψ such that it is left unchanged by a rotation along x: $\hat{R}_x(\vartheta)\psi = \psi$. Is ψ an eigenfunction of \hat{L}_x ?

Solution:

ψ is an eigenfunction of \hat{R}_x with eigenvalue 1. Because R_x and L_x commute, ψ must be an eigenfunction of \hat{L}_x as well.

We can verify this by using the definition of \hat{R}_x :

$$\hat{R}_x(\vartheta)\psi = \psi \quad \rightarrow \quad (1 - i\frac{\vartheta}{\hbar}L_x)\psi = \psi \quad \rightarrow \quad -i\frac{\vartheta}{\hbar}L_x\psi = 0\psi$$

Thus ψ is an eigenfunction of L_x with eigenvalue 0, since $L_x\psi = 0\psi$.

e) We studied in class that the eigenvalues of the angular momentum operator along x , \hat{L}_x , are $\hbar m_x$ with integers $m_x = -l, -l + 1, \dots, l$. Consider a quantum state $\psi = \frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1$, where φ_m is the normalized eigenfunction of \hat{L}_x corresponding to the eigenvalue $\hbar m_x$.

What is the probability of finding $\hat{L}_x = 0$ in a measurement? What is $\langle \hat{L}_x \rangle$?

Solution:

The probability of obtaining a particular eigenvalue m_i in a measurement is given by $P_i = |\langle \psi | \varphi_i \rangle|^2$, where φ_i is the eigenfunction corresponding to the eigenvalue m_i . Thus the probability of finding the zero eigenvalue of L_x , $\hbar \times 0$ is $|\langle \psi | \varphi_0 \rangle|^2 = \frac{1}{4}$.

Similarly, we can find the probabilities of $m_x = -2$ and 1 to be $\frac{1}{6}$ and $\frac{7}{12}$ respectively. Then the expectation value (or average) is simply:

$$\langle L_x \rangle = -2\hbar\frac{1}{6} + 0\hbar\frac{1}{4} + \hbar\frac{7}{12} = \frac{\hbar}{4}$$

Note that the inner product can be calculated very easily since we know that for normalized eigenfunctions the inner product $\langle \varphi_i | \varphi_j \rangle$ is zero for two different eigenfunctions (they are orthogonal) and one if $i = j$:

$$\langle \psi | \varphi_0 \rangle = \langle \frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1 | \varphi_0 \rangle = \frac{1}{\sqrt{6}}\langle \varphi_{-2} | \varphi_0 \rangle + \frac{1}{2}\langle \varphi_0 | \varphi_0 \rangle + \sqrt{\frac{7}{12}}\langle \varphi_1 | \varphi_0 \rangle = \frac{1}{\sqrt{6}}\times 0 + \frac{1}{2}\times 1 + \sqrt{\frac{7}{12}}\times 0 = \frac{1}{2}$$

Similarly we can calculate $\langle \psi | \varphi_{-2} \rangle = \frac{1}{\sqrt{6}}$ and $\langle \psi | \varphi_1 \rangle = \sqrt{\frac{7}{12}}$. From ψ we thus have that $P(L_x = -2\hbar) = P_{-2} = \frac{1}{6}$, $P(L_x = 0\hbar) = P_0 = \frac{1}{4}$, $P(L_x = \hbar) = P_1 = \frac{7}{12}$ (all the other P_i are zero) and the expectation value of L_x (or its average) is $\langle \hat{L}_x \rangle = \sum_i m_i P_i \hbar = -2\hbar\frac{1}{6} + 0\hbar\frac{1}{4} + \hbar\frac{7}{12} = \frac{\hbar}{4}$. We can as well use the usual definition of expectation value:

$$\langle L_x \rangle = \langle \psi | L_x | \psi \rangle = \left\langle \frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1 \left| L_x \right| \frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1 \right\rangle$$

or using the explicit definition of inner product in terms of an integral:

$$\langle L_x \rangle = \int \psi^* L_x[\psi] d^3r = \int (\frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1)^* L_x[\frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1] d^3r$$

Now,

$$L_x[\frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1] = -2\hbar\frac{1}{\sqrt{6}}\varphi_{-2} + 0\hbar\frac{1}{2}\varphi_0 + \hbar\sqrt{\frac{7}{12}}\varphi_1$$

because the φ_i are eigenfunctions of L_x . Thus the integral is:

$$\begin{aligned} \langle L_x \rangle &= \int (\frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1)^* (-2\hbar\frac{1}{\sqrt{6}}\varphi_{-2} + 0\hbar\frac{1}{2}\varphi_0 + \hbar\sqrt{\frac{7}{12}}\varphi_1) d^3r = \\ &= \int (-2\hbar\frac{1}{6}\varphi_{-2}^*\varphi_{-2}) d^3r + \int (-2\hbar\frac{1}{\sqrt{6}}\frac{1}{2}\varphi_0^*\varphi_{-2}) d^3r + \dots + \int \hbar\frac{7}{12}\varphi_1^*\varphi_1 d^3r \end{aligned}$$

where I expanded out (but not written explicitly) all the products. Since the φ_i are eigenfunctions, we know that they are orthonormal, thus,

$$\int \varphi_i^* \varphi_i d^3r = 1, \quad \int \varphi_i^* \varphi_j d^3r = 0$$

Then the only terms remaining from the above integral are:

$$\langle L_x \rangle = -2\hbar\frac{1}{6} + 0\hbar\frac{1}{4} + \hbar\frac{7}{12} = \frac{\hbar}{4}$$

Note that this expression is exactly the same we found by considering the probabilities:

$$\langle L_x \rangle = \sum_i m_i \hbar |\langle \psi | \varphi_i \rangle|^2 = \sum_i m_i \hbar |c_i|^2 = \sum_i m_i \hbar P_i$$

This result is of course very general as you have already seen in recitation: we can always write any wavefunction $\psi = \sum_i c_i \varphi_i$ where φ_i are eigenfunctions of the operator A we want to calculate the expectation value of (since the φ_i form a basis and $c_i = \langle \varphi_i | \psi \rangle$). Then

$$\langle A \rangle = \langle \psi | A | \psi \rangle = \sum_{ij} c_j^* c_i \langle \varphi_j | A | \varphi_i \rangle = \sum_{ij} c_j^* c_i \langle \varphi_j | a_i | \varphi_i \rangle = \sum_{ij} c_j^* c_i a_i \langle \varphi_j | \varphi_i \rangle = \sum_i |c_i|^2 a_i$$

Problem 3: Radioactive decay by proton emission

30 points

Useful quantities: Proton mass, $m_p c^2 = 938.272 \text{ MeV}$; $\hbar c = 197 \text{ MeV fm}$; $\frac{e^2}{\hbar c} = \frac{1}{137}$; $c = 3 \times 10^8 \text{ m/s}$; $R_0 = 1.2 \text{ fm}$.

a) (4 points) Consider the isotope Europium-131 (${}_{63}^{131}\text{Eu}$), with mass 121919.966 MeV. Given its A and Z numbers, do you expect this isotope to be stable?

Solution:

The ratio of the mass and proton number is $Z/A \approx 0.48$. We know that for heavy stable nuclei this ratio is instead ≈ 0.41 ($Z \approx A/2$ only for light nuclides). Thus we expect this isotope to be unstable and to decay by a process that will make it shed some protons. A possible decay channel for ${}_{63}^{131}\text{Eu}$ is proton emission. We want to analyze this

decay mode following the same theory we saw for alpha decay and in particular *estimate* the half-life of ${}_{63}^{131}\text{Eu}$. The following questions will guide you through the estimation.

b) (5 points) The mass of Samarium-130 is 120980.755 MeV. What is the Q -value for the reaction ${}_{63}^{131}\text{Eu} \rightarrow {}_{62}^{130}\text{Sm} + {}_1^1\text{H}$?

Solution:

The Q -value can be calculated from the mass differences:

$$Q = m_{\text{Eu}} - m_p - m_{\text{Sm}} = 0.939 \text{ MeV}$$

c) (6 points) Calculate the frequency $f = \frac{v}{R}$ for the proton to be at the edge of the Coulomb potential. Here R is the Samarium radius and v the proton speed when taking Q as the (classical) kinetic energy.

Solution:

From the Q calculated above we can obtain the velocity as $\frac{1}{2}m_p v^2 = Q$ or,

$$v = \sqrt{\frac{2Q}{m_p c^2}} c = 0.045c = 1.34 \times 10^{22} \text{ fm/s}$$

In this calculation I assumed that the proton has all the kinetic energy, while the daughter nuclide is still at rest. This is in any case a good approximation, given the masses. A more precise calculation can be obtained if we consider conservation of momentum, $m_{\text{Sm}} v_{\text{Sm}} + m_p v = 0$, to find:

$$v = \sqrt{\frac{2Q}{m_p c^2 (1 + m_p/m_{\text{Sm}})}} c = 0.045c = 1.34 \times 10^{22} \text{ fm/s}$$

(the result is the same to the second decimal place).

The nuclear radius is given by $R = R_0 A^{1/3} = 6.079 \text{ fm}$. Thus the frequency is $f = \frac{v}{R} = 2.21 \times 10^{21} \text{ s}^{-1}$.

Note: as in the alpha decay model, we could have worked in the center of mass frame and use the reduced mass and total radius in the calculation above. Since the proton is much smaller (in terms of mass and radius) than Samarium, the difference in the two calculations is negligible.

d) (5 points) What is the Coulomb potential at the distance R , $V_C(R)$? (this is the potential barrier height). What is the distance R_c at which the Coulomb potential is equal to the Q -value?

Solution:

The Coulomb potential is given by

$$V_C(R) = \frac{e^2 Z}{R} = \frac{e^2}{\hbar c} \frac{Z}{R} = \frac{1}{137} 197 \text{MeV fm} \frac{61}{6.079 \text{fm}} = 14.65 \text{MeV}$$

Note: because the fine constant $\frac{e^2}{\hbar c} = \frac{1}{137}$ was given in cgs units, the Coulomb potential could be calculated from it in cgs units. Since it is a dimensionless constant, I could have written as well $\frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$ in SI units and $V_C(R) = \frac{e^2 Z}{4\pi\epsilon_0 R}$.

To find the distance R_c we equate the Coulomb potential to the Q -value:

$$\frac{e^2 Z}{R_c} = Q \rightarrow R_c = \frac{e^2 Z}{Q} = R \frac{V_C(R)}{Q} = 94.84 \text{fm}$$

e) (4 points) To estimate the tunneling probability we replace the Coulomb barrier with a rectangular barrier of height $V_H = V_C(R)/2$ and length $L = (R_c - R)/2$ (see figure). What is the tunneling probability?

Solution:

Since we want to *estimate* the tunneling probability, we take the approximate expression $P_T = 4e^{-2\kappa L}$. We first need to calculate κ :

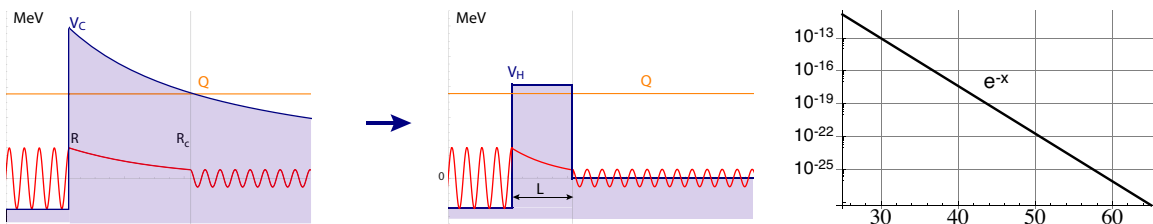
$$\kappa = \frac{\sqrt{2m(V_H - Q)}}{\hbar} = \frac{\sqrt{2m_p c^2 (V_C/2 - Q)}}{\hbar c} = 0.55 \text{fm}^{-1}$$

We then have $2\kappa L = 48.58$. Reading out from the graphic, this corresponds to $P_T = 4e^{-2\kappa L} \approx 4 \times 10^{-21}$ (if instead we had a good scientific calculator we would get $P_T = 3.19 \times 10^{-21}$). Since the tunneling probability is very low, the approximation we took in considering $P_T = 4e^{-2\kappa L}$ instead of the exact expression is a good one.

f) Finally, give the decay rate λ and the half-life for the proton emission decay of ${}^{131}_{63}\text{Eu}$.

Solution:

The decay rate is obtained from the same semi-classical model we studied for alpha decay. Thus it is given by the product of the frequency at which the proton is at the potential barrier (or gets separated from the parent nuclide) times the probability of tunneling through the barrier. Thus the decay rate is given by $\lambda = f P_T = 7.05 \text{s}^{-1}$ and the half life is $t_{\frac{1}{2}} = \ln 2 / \lambda = 0.1 \text{s}$.



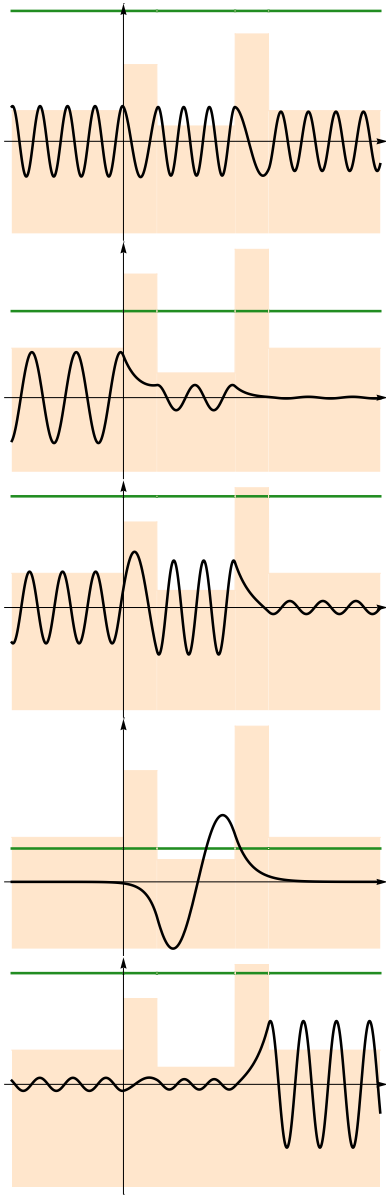
Problem 4: Match the potential

20 points

A quantum system in a 1D geometry is subjected to the potential energy as in the figures on the right, with 5 regions of different potential height. Match the 1D energy eigenfunctions on the left with the correct energy (if any) depicted on the right. Provide a **brief** explanation of the reasoning that lead you to each of your matchings.

(Notice: here I plot the real part of the eigenfunction).

Solution:



A-4: The wavefunction in A is always an oscillating function, so it corresponds to a traveling wave with always positive kinetic energy. This means that $E > V$ in all the five regions

B-1: The wavefunction shows exponential decay in two regions (II and IV), thus it must have negative kinetic energy there, or $E < V$.

C-2: Here the wavefunction is oscillating except in region 4, where it is an exponential decay. Thus we have $E < V_{IV}$

D-3: In this case, the energy of the system is greater than the potential only in region III. Thus the system is confined to that region: the system is bound. The solution will thus have an oscillating behavior in region III and just a small penetration in the outer regions as it decays exponentially to zero.

E-2: As in the C case, the wavefunction is oscillating in all the regions except in region 4, where it looks like an increasing exponential. This stationary solution corresponds to the same potential as above, but with different boundary conditions: the particle is now incoming from the right instead than the left as we mostly solved in class.

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