

22.101 Problem Set 1 Solutions 9/18

Problem 1

(i) When we say a thermal neutron has energy $k_B T$, we mean that there is a distribution of neutron energies owing to the temperature. This distribution is a Maxwellian and the most probable energy for a neutron in this distribution is $k_B T$. We choose this energy $k_B T$ as representative of a thermal neutron.

Thermal Neutron:

$$\begin{aligned}\text{Energy: } T &= 20^\circ\text{C} = 293 \text{ K} \\ k_B &= 1.38 \cdot 10^{-23} \text{ J/K} = 8.617 \cdot 10^{-5} \text{ eV/K} \\ \Rightarrow E &= k_B T = \mathbf{0.0253 \text{ eV}}\end{aligned}$$

$$\text{Speed: } E = 1/2 \cdot m v^2 \Rightarrow \mathbf{v = 2.2 \cdot 10^5 \text{ m/sec}}$$

$$\begin{aligned}\text{Wavelength: } \lambda &= h/p = h/(mv) \\ h &= \text{Planck's constant} = 6.626 \cdot 10^{-34} \text{ J}\cdot\text{sec} = 4.136 \cdot 10^{-15} \text{ eV}\cdot\text{sec} \\ \Rightarrow \lambda &= \mathbf{1.8 \text{ \AA}}\end{aligned}$$

$$\text{Frequency: } E = (h/2\pi) \cdot \omega \Rightarrow \mathbf{\omega = 3.84 \cdot 10^{13} \text{ rad/sec}}$$

Comparison with reactor:

Energy: The atoms in a reactor will exhibit the same energy distribution as the neutrons because they will be at the same temperature. Thus, the most probable energy for the atoms in a reactor will be the same as that for the neutrons, 0.0253 eV.

Speed: The uranium atoms are much more massive than the neutrons. For the same energy, the uranium atoms will be moving much more slowly. The speed corresponding to the most probable energy (incidentally, not the same as the most probable speed) for a uranium atom with a temperature of 20°C will be $1.43 \cdot 10^4$ cm/sec.

Interatomic distance: The distance between atoms of a solid is generally on the scale of angstroms. The distance between units of UO_2 is about 5.4 Å. This is the same order as the wavelength of a thermal neutron.

We might be interested in comparing the thermal neutron wavelength with typical atomic separations in a solid in order to assess the usefulness of thermal neutrons in studying the structure of the material. We find that the wavelength is on the same order as the atomic separations and so neutrons will diffract when passing through the material. We can use the diffraction pattern to make inferences of the structure of the atoms making up the material.

Additionally, the energy of the thermal neutrons is similar to the energy of the atoms. Results of scattering experiments with thermal neutrons in material will be able to shed light on dynamic properties of the material (e.g. normal modes of lattice vibrations, etc.).

ii) x-ray photon, $E=2 \text{ keV}$

$$\text{Frequency: } E = h\nu \Rightarrow \mathbf{\nu = 4.836 \cdot 10^{17} \text{ Hz}}$$

Wavelength: $\lambda = c/\nu$,
 $c = 3 \times 10^8 \text{ m/sec}$
 $\lambda = 6.2 \text{ \AA}$

We might be interested in comparing the x-ray wavelength with the thermal neutron wavelength in order to compare their relative ability to probe the structure of matter. Since they have comparable wavelengths, they will have the same resolution in scattering experiments. Since x-ray crystallography is useful for studying the atomic structure of materials, neutrons of similar wavelength (thermal) also ought to be useful.

Energy of electron with wavelength similar to thermal neutron:

Thermal neutron $\lambda = 1.8 \text{ \AA}$

Take electron with $\lambda = 1.8 \text{ \AA}$

$\lambda = h/p \Rightarrow p_{\text{electron}} = 3.68 \times 10^{-24} \text{ kg} \cdot \text{m/sec}$

$E = p^2/2m_{\text{electron}}$

$m_{\text{electron}} = 511 \text{ keV}/c^2 = 9.11 \times 10^{-31} \text{ kg}$

$\Rightarrow \mathbf{E = 46.53 \text{ eV}}$

iii) Coulomb (electrostatic force) in cgs units is: $F = q_1 \cdot q_2 / r^2$

The energy associated with this force will be the work it takes to bring the two protons to their current separation distance. This is the integral of the force from infinity to the distance of separation for these two particles:

$E = W = \int_{\infty}^r F \cdot dx = q_1 \cdot q_2 / r$

The charge on the two protons is the elementary charge unit e , so

$E = e^2 / r$

We have the classical electron radius $r_e = e^2 / m_e c^2$ so

$E = m_e c^2 r_e / r = \mathbf{1.44 \text{ MeV}}$

Since this energy is associated with only two protons, a heavy nucleus will be at least several times this amount i.e. several dozen MeV or more.

Problem 2

See Table of Nuclides for specific data.

Either mentioning the first three isotopes of hydrogen would be sufficient but including those with very short half-lives is acceptable.

Applications of Hydrogen:

Fission: The hydrogen in water is responsible for water's usefulness as a moderator in thermal reactors.

Fusion: The reaction of the hydrogen isotopes deuterium and tritium is critical for fusion reactors.

NST: The spin of the hydrogen-1 nucleus is often the subject of NMR spectroscopy and imaging.

Many other answers are certainly acceptable but these were the most common.

Problem 3

See Table of Nuclides for data. All cross sections that I mention below are taken at thermal energies.

Note that the absorption cross section is generally the sum of the capture cross section and fission cross section. Since hydrogen has a negligible fission cross section, it is acceptable to list the capture cross section as the absorption cross section.

There are several points that ought to be included in your paragraph for the last part of this question:

- A correct referencing of the requested data will show that H-1 has a much higher absorption cross section than H-2 and that U-235 will have a much higher fission cross section than U-238.
- Also, you will find that H-2 and U-235 have far lower natural abundance than H-1 and U-238, respectively.
- Heavy water (with H-2) is more desirable as a moderator and/or coolant for fission reactors than normal water because it will absorb fewer neutrons leading to a superior neutron economy in the reactor.
- A fission chain reaction can be self-sustaining with U-235 but not with U-238 so a higher U-235 isotopic fraction in fuel is desirable. Alternatively, you could point out that U-235 has a higher fission cross section than U-238 so a higher fraction of U-235 in the core will also lead to an improved neutron economy.
- Enrichment of isotopes is expensive.
- You can conclude that to improve the neutron economy of a thermal reactor, you can enrich your moderator in H-2 (Canadian approach with CANDU) and/or enrich your fuel in U-238 (US approach with LWR's).