

## Coherent and Incoherent Spin Scattering Radius

Let assume an operator for the coherent and incoherent spin as

$$\tilde{a} = \tilde{A} + \tilde{B} \vec{I} \cdot \vec{i}$$

$$\vec{J} = \vec{I} + \vec{i}$$

$$\vec{J}^2 = \vec{I}^2 + \vec{i}^2 + 2\vec{I} \cdot \vec{i}$$

$$\vec{I} \cdot \vec{i} = \frac{1}{2} [\vec{J}^2 - \vec{I}^2 - \vec{i}^2]$$

$$\tilde{a}|\xi\rangle = \tilde{A}|\xi\rangle + \tilde{B} \vec{I} \cdot \vec{i} |\xi\rangle$$

$|\xi\rangle \rightarrow$  *spin state eigenvector*

$$\tilde{a}|\xi\rangle = \tilde{A}|\xi\rangle + \tilde{B}\frac{1}{2}[\vec{J}^2 - \vec{I}^2 - \vec{i}^2]|\xi\rangle$$

$$a = A + B\frac{1}{2}[J(J+1) - I(I+1) - i(i+1)]$$

*For  $i = 1/2 \Rightarrow J = I - 1/2$  and  $J = I + 1/2$*

*a) For  $J = I - 1/2$*

*we have*

$$a^- = A + \frac{1}{2}B[(I - 1/2)(I + 1/4) - I^2 - I - 3/4]$$

*or*

$$a^- = A - \frac{1}{2}B(I + 1)$$

*b) For  $J = I + 1/2$*

*we have*

$$a^+ = A + \frac{1}{2}B[(I + 1/2)(I + 3/2) - I^2 - I - 3/4]$$

*or*

$$a^+ = A + \frac{1}{2}BI$$

*A and B as a function of  $a^-$  and  $a^+$*

*From*

$$a^+ = A + \frac{1}{2}BI$$

*and*

$$a^- = A - \frac{1}{2}B(I + 1)$$

*A and B become*

$$A = \frac{I+1}{2I+1} a^+ + \frac{I}{2I+1} a^-$$

*and*

$$B = \frac{2}{2I+1} (a^+ - a^-)$$

If  $a^+ = a^- = a$  then  $B = 0$  and  $A = a$ . This indicates that no spin coherent scattering exists. Hence,  $A$  relates to the coherent and  $B$  to the incoherent scattering.

$\sim^2$

The expected values for the operator  $a$  can be derived as:

$$\langle a^2 \rangle = \langle \xi | \tilde{a} \tilde{a}^* | \xi \rangle$$

$$\langle a^2 \rangle = A^2 + AB \langle \vec{I} \cdot \vec{i} \rangle + BA \langle \vec{I} \cdot \vec{i} \rangle + B^2 \langle (\vec{I} \cdot \vec{i})^2 \rangle$$

The cross terms are zero since no correlation exists between the neutron spin and the nucleus spin, i.e.,  $\langle \vec{I} \cdot \vec{i} \rangle = 0$ .

$$\langle a^2 \rangle = A^2 + B^2 \langle (\vec{I} \cdot \vec{i})^2 \rangle$$

$$\langle (\vec{I} \cdot \vec{i})^2 \rangle = \langle (I_x i_x)^2 + (I_y i_y)^2 + (I_z i_z)^2 \rangle$$

$$\langle (i_x)^2 \rangle = \langle (i_y)^2 \rangle = \langle (i_z)^2 \rangle = 1/4$$

$$\langle (\vec{I} \cdot \vec{i})^2 \rangle = \frac{1}{4} \langle \vec{I}^2 \rangle$$

*or*

$$\langle (\vec{I} \cdot \vec{i})^2 \rangle = \frac{1}{4} I(I + 1)$$

$$\langle a^2 \rangle = A^2 + B^2 \frac{1}{4} I(I+1)$$

*With A and B as*

$$A = \frac{I+1}{2I+1} a^+ + \frac{I}{2I+1} a^-$$

*and*

$$B = \frac{2}{2I+1} (a^+ - a^-)$$

$$\langle a^2 \rangle = \left[ \frac{I+1}{2I+1} a^+ + \frac{I}{2I+1} a^- \right]^2 + \frac{I(I+1)}{(2I+1)^2} (a^+ - a^-)^2$$

$$a_{coh} = \frac{I+1}{2I+1} a^+ + \frac{I}{2I+1} a^-$$

$$a_{inch} = \frac{[I(I+1)]^{1/2}}{2I+1} (a^+ - a^-)$$

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