

Thursday, October 9<sup>th</sup>, 2008, 9:30 – 11:00 a.m.

## OPEN BOOK

## QUIZ 1 (solutions)

**Problem 1 (55%) – Heat up of the waste canisters following a ceiling collapse at the Yucca Mountain spent fuel repository**

The energy equation for the (air + structures) control volume is:

$$\frac{\partial E}{\partial t} = \dot{Q}_{decay} \quad \Rightarrow \quad E_2 - E_1 = \dot{Q}_{decay} t_2 \quad (1)$$

where E is the total energy of the control volume,  $\dot{Q}_{decay} = 200$  kW and  $t_2$  is the final time. Solving Eq. (1) for  $t_2$  and expanding the term  $E_2 - E_1$ , one gets:

$$t_2 = \frac{E_2 - E_1}{\dot{Q}_{decay}} = \frac{M_a c_{va} (T_2 - T_{a1}) + M_w [u_{w2}(T_2, P_{w2}) - u_{w1}] + M_s c_s (T_2 - T_{s1})}{\dot{Q}_{decay}} \quad (2)$$

where  $T_2 = 200^\circ\text{C}$ ,  $c_{va} = 719$  J/kg-K,  $R_a = 286$  J/kg-K,  $T_{a1} = 50^\circ\text{C}$ ,  $c_s = 470$  J/kg-K,  $M_s = 6 \times 10^5$  kg,  $T_{s1} = 100^\circ\text{C}$ . The mass and the initial internal energy of water in air can be found from knowledge of its partial pressure ( $P_{w1} = \phi P_{sat}(T_{a1}) = 10$  kPa, with  $\phi = 0.8$ ) and temperature ( $T_{w1} = T_{a1} = 50^\circ\text{C} = 323$  K), and the total volume it occupies ( $V_a = 3500$  m<sup>3</sup>)

$$M_w = \frac{V_a}{v_w(P_{w1}, T_{a1})} \approx 235 \text{ kg} \quad v_{w1} = v_w(P_{w1}, T_{a1}) = 14.9 \text{ m}^3 / \text{kg}$$

$$u_{w1} = u_w(P_{w1}, T_{a1}) = 2443 \text{ kJ/kg}$$

The mass of air in the chamber is:

$$M_a = \frac{[P_1 - P_{w1}] V_a}{R_a T_{a1}} \approx 3447 \text{ kg}$$

where  $P_1 = 101$  kPa. To find  $u_{w2}$ , we first note that the volume occupied by the water in air in the final state is still  $V_a$ . Also, the final state of the water in air is superheated. Thus:

$$14.9 \text{ m}^3 / \text{kg} = \frac{M_w}{V_a} = v_{w1} = v_{w2}(T_2, P_{w2})$$

Then from the steam tables, the pressure of water in air at the final state is  $P_{w2} = 15$  kPa, and  $u_{w2} = 2661$  kJ/kg. Finally, from Eq. (2) we get  $t_2 \approx 1.43 \times 10^5$  s = 39 hours.

ii) The final pressure in the chamber,  $P_2$ , is simply the sum of the partial pressures of water and air:

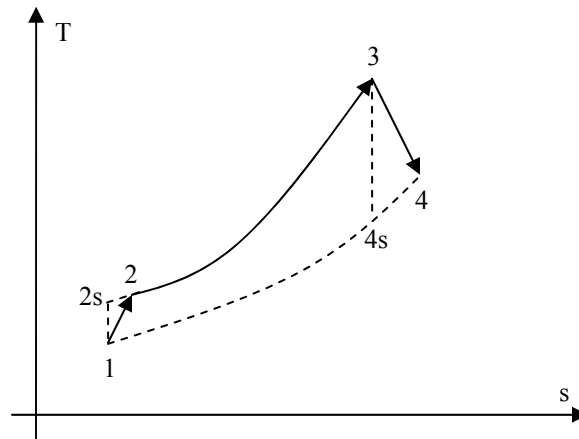
$$P_2 = P_{w2} + M_a R_a T_2 / V_a \approx 148 \text{ kPa}$$

iii) If there were liquid water in the chamber at the initial state, it would increase the thermal capacity of the system (thus acting as an effective 'heat sink'), and therefore the time required to reach 200°C would increase.

iv) It would not be advisable to flood the tunnels because the waste canisters would corrode faster and, importantly, the fission products released over time would be more easily transported through the ground.

## Problem 2 (45%) – Aircraft nuclear propulsion

i) The T-s diagram for the cycle is shown in the figure below.



**T-s diagram for the nuclear engine cycle**

ii) The mass flow rate,  $\dot{m}$ , is:

$$\dot{m} = \rho_1 VA \approx 159.2 \text{ kg/s}$$

Where  $V=180 \text{ m/s}$ ,  $A=0.8 \text{ m}^2$  and the air density at the intake,  $\rho_1$ , can be calculated from the perfect gas equation of state as:

$$\rho_1 = \frac{P_1}{R_a T_1} \approx 1.1 \text{ kg/m}^3$$

with  $P_1=80 \text{ kPa}$  and  $T_1=253 \text{ K}$ .

iii)

*Compressor*

$T_{2s} = T_1 \left( \frac{P_{2s}}{P_1} \right)^{(\gamma-1)/\gamma} = T_1 r_p^{(\gamma-1)/\gamma} \approx 376 \text{ K}$  where  $r_p = P_1/P_{2s}=4$ . From the definition of isentropic efficiency of the compressor ( $\eta_c=0.9$ ) it is possible to calculate  $T_2$  as

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} \approx 389.6 \text{ K}$$

The compressor power is then:

$$\dot{W}_T = \dot{m} c_p (T_2 - T_1) \approx 21.86 \text{ MW}$$

### Reactor

The reactor power is:

$$\dot{Q} = \dot{m}c_p(T_3 - T_2)$$

Therefore, the reactor outlet temperature,  $T_3$ , is:

$$T_3 = T_2 + \frac{\dot{m}c_p}{\dot{Q}} \approx 577.1 \text{ K}$$

$T_3$  is also the maximum air temperature in the engine.

### Turbine

The temperature of point 4s is  $T_{4s} = T_3 \left( \frac{P_{4s}}{P_3} \right)^{(\gamma-1)/\gamma} = T_3 r_p^{(1-\gamma)/\gamma} \approx 388.4 \text{ K}$  with  $r_p = P_3/P_{4s} = 4$ . From

the definition of isentropic efficiency of the turbine ( $\eta_T = 0.9$ ) it is possible to calculate  $T_4$  as

$$T_4 = T_3 - \eta_T(T_3 - T_{4s}) \approx 407.2 \text{ K}$$

The turbine power is then:

$$\dot{W}_T = \dot{m}c_p(T_3 - T_4) \approx 27.18 \text{ MW}$$

Finally, the net power developed by each engine is:

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_C \approx 5.32 \text{ MW}$$

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