

STRUCTURAL DESIGN NOTES
TOPIC D
DESIGN RULES

1. JUDGING STRESS ACCEPTABILITY: RULES OF THE ASME CODES

1.1 Need for Judgments

The calculations of topic C permit us to estimate pressure vessel stresses on the basis of simplified assumptions. More detailed calculations (e.g., by using a finite element computer code) would permit estimation of stresses on more realistic bases. However, in either case, a design process is not complete until we judge the acceptability of the calculated stresses. These notes (topic D) provide an overview of some design rules of the ASME code that relate to such judgments.

1.2 Basis for Judgments

The acceptability of a given numerical stress value depends on an interrelated set of considerations:

- the nature (or classification) of the stress;
- vessel material property information;
- expected operating conditions throughout the vessel lifetime; and
- the degree of quality assurance employed.

The dependence on stress classification is perhaps the most surprising on this list. For illustration, consider a calculated pressure stress of 200 MPa and a calculated thermal stress of 200 MPa. The thermal stress will usually be judged to be much less severe than the same value of pressure stress. So we must know how a stress is classified before judging its severity.

The fact that there is a dependence on quality assurance also has counter intuitive features. The term quality assurance encompasses design analyses, material testing, fabrication controls, and inservice inspections. For example, the ASME code Section III provides requirements for nuclear components and extensive quality assurance actions are mandated. A factor of safety is employed that is characterized by $FS = 3$. On the other hand, the ASME code section IV for heating boilers gives $FS = 5$. It seems intuitive that we would want to be more conservative in the design of nuclear components and hence would use a larger rather than a lower factor of safety. The key, of course, is that fewer quality assurance actions are mandated for the heating boilers and that a larger value for FS must therefore be used. The remainder of Topic D is based on ASME code section III and on related information.

1.3 The ASME Codes

ASME codes and standards have a history that can be traced to 1884. The intent at that time was to establish test codes to be used in equipment specification. Boiler safety was a prime consideration. The ASME boiler test code of 1884 has evolved into the ASME boiler and pressure vessel code of interest to us here. In particular, section III of that code, nuclear components, deals with rules relevant to reactor vessel design and construction.

The ASME codes are written to be legally enforceable documents (invoked by a national, state, or local government and enforced using third party inspectors, e.g., insurance company employees). The codes cover many features and alternatives that will not be considered here. The items considered here must be viewed as a very small sample of the full code provisions. The sample chosen is perhaps oversimplified but it does illustrate methods of judging stress acceptability.

1.4 Additional Details

The summary of §1.1 through §1.3 includes information taken from Refs. 1 to 3 and from Ref. 4, §6.19 and §7.4. Additional details can be found in these references.

As indicated above, a simplified view of code requirements is presented in the sections that follow. This view has been assembled chiefly from Refs. 5 to 8. These references contain more complete information on the topics of interest.

2. MATERIAL CHARACTERIZATION

2.1 Strength

The strength of a pressure vessel material is defined by choosing a single value of allowable design stress (S_m) **Error! Switch argument not specified.** for each design temperature. S_m **Error! Switch argument not specified.** is chosen to be the minimum of the following four quantities:

- 1/3 of the specified minimum room temperature ultimate strength (S_u) **Error! Switch argument not specified.**, where specified minimum indicates the value required for raw material acceptance;
- 1/3 of the value of S_u **Error! Switch argument not specified.** at design temperature;
- 2/3 of the specified minimum room temperature yield stress (S_y) **Error! Switch argument not specified.**; and
- 2/3 of the value of S_y **Error! Switch argument not specified.** at design temperature.

The safety factor corresponding to this list for choosing S_m **Error! Switch argument not specified.** is designated by $FS = 3$ (i.e., the ratio of S_u **Error! Switch argument not specified.** to S_m **Error! Switch argument not specified.** must be at least 3). See Ref. 4, p. 494 for a corresponding list for $FS = 4$.

2.2 Design Fatigue Curve

Consider that many strain-controlled fatigue tests have been performed using specimens constructed of the pressure vessel material. Each of these tests gives a failure point characterized by (strain range = ϵ) **Error! Switch argument not specified.** and by cycles to failure = N). These test results are converted to a design fatigue curve by use of the following steps:

- Convert each (ϵ , N) **Error! Switch argument not specified.** data point to a (S_a , N) **Error! Switch argument not specified.** data point by:

$$S_a = \frac{\epsilon E}{2} \quad (1)$$

and where E = Young's modulus.

The term S_a 13 designates an alternating stress amplitude. S_a 14 is half of the observed stress range when the strain range is small (very high cycle fatigue). When, however, the strain range is larger (e.g., low cycle fatigue), the value for S_a 15 is artificially much larger than the actual observed stress amplitude. S_a 16 must be considered to be a convenient substitute for strain range in a strain controlled fatigue test and is usually not appropriate for expressing an actual stress.

(S_a versus N)

- Obtain a curve which is a best fit of the 17 data points. This best fit curve is obtained by some equivalent of choosing a and b in the following equation (see also Ref. 4, pp. 252-253 and Ref. 7, pp. 10-11).
$$S_a = 4\sqrt{N} \left[100 - a \right]^{-b}$$

where a can be interpreted as the percent reduction in area in a uniaxial tensile test ($N = 1/4$); and where b can be interpreted as the endurance limit (S_e) 18 for the material.

- Adjust the best fit curve for effects of operating cycles that have a non-zero mean strain. That is, the mean strain for fatigue test cycles is zero. However, in many applications, the mean strain is not zero (e.g., because of residual plastic strains developed during manufacturing or earlier in the operating history). In these cases, the stress amplitude S_a 19 may give an incomplete indication of cyclic damage. The best fit S_a 20 curve is therefore modified for some materials by lowering it at high cycle extremes to adjust for "mean stress." Techniques for obtaining this (adjusted best fit stress) are described elsewhere (Ref. 4, p. 255 and Ref. 7, pp. 12-15). Fatigue calculations based on the present notes can be performed on a simplified basis which excludes the mean stress adjustment.
- Obtain a design curve to adjust for data scatter and for other uncertainties. This is done by considering two shifted curves. The first is obtained by using stress equal to one-half of (adjusted best fit stress) at each value of (best fit cycles). The second is obtained by using cycles equal to one-twentieth of (best fit cycles) at each value of (adjusted best fit stress). The final design curve (S_a versus N) 21 is taken to be the lower of the two shifted curves (see also Ref. 4, p. 254).

The product of these steps is a design fatigue curve that is used in sections that follow as a conservative representation of cyclic damage effects.

3. STRESS HOPPERS (STRESS CLASSIFICATION)

3.1 Hopper Terminology

We have now given methods of describing the material by an allowable design stress $(S_m)_{22}$ and by a design fatigue curve $(S_a \text{ versus } N)_{23}$. The next question is: what design criterion should we use to judge the acceptability of a calculated stress? This can be answered by one of the following equivalent questions:

- what is the nature of the stress?
- what is the stress classification?
- what "hopper" is appropriate?

The term hopper in this section refers to the following definition (from "New College Standard Dictionary," Funk & Wagnalls, New York, 1947); hopper = "a tank or boxlike receptacle for holding water, grain, sugar, etc., which may be emptied by opening its bottom." The term hopper is intended to evoke the picture of gathering all calculated stresses for a given point in the vessel, placing each in the correct hopper, and emptying each hopper separately to assemble input for evaluations.

3.2 Hopper Names

All calculated stresses for a given operating condition and for a given point in the vessel are gathered and placed in one of the following five hoppers (or, in one of the following classifications):

P_{m24} = general primary membrane stress. This stress is produced by mechanical loads (e.g., pressure). Primary indicates a load controlled quantity. That is, the ratio of stress to load does not change if yielding occurs upon load increase; also, deformations are not self-limiting. The stress is uniformly distributed over the shell thickness and is statically equivalent to the shell membrane force (normal force per unit center surface length). Stress concentration and discontinuity effects are excluded from the P_{m25} hopper.

P_{L26} = local primary membrane stress. This stress is produced by mechanical loads (e.g., pressure). The stress is again uniformly distributed over the shell thickness and is again statically equivalent to the shell membrane force. The term local implies that high stresses are limited in spatial extent (e.g., an extent smaller than $0.5 \sqrt{(Rt)}$ ²⁷ in the axial direction). Discontinuity effects provide the major contribution to the P_{L28} hopper. Note that the results of discontinuity effects are secondary stresses. Therefore the "primary" designation for this

hopper is at least partially a misnomer. Stress concentration effects are excluded from the P_{L29} hopper. P_{m30} contributions are also included here.

P_{b31} = primary bending stress. This stress is produced by mechanical loads (e.g., pressure). Primary again indicates a load controlled quantity (based again upon behavior if yielding occurs upon load increase). The stress is distributed so that it is linearly proportional to distance from the shell center surface and is statically equivalent to a shell bending moment (per unit center surface length). Stress concentration and discontinuity effects are excluded from the P_{b32} hopper.

Q = secondary stress. This stress is produced by mechanical loads or by differential thermal expansion. Secondary indicates that a strain controlled situation exists, that local yielding can accommodate the effects of excessive stresses, that the deformations are self-limiting, and that the ratio of stress to load can decrease upon load increase. The stress is distributed linearly in the shell thickness direction and is statically equivalent to the sum of a shell membrane force and a shell bending moment. Stress concentration, P_b , and P_{L33} effects are excluded from the Q hopper. Discontinuity effects (in excess of P_{L34}) are included.

F = peak stress. F includes stress concentration effects and also includes the difference between the calculated stress distribution and the linear distributions of the other hoppers.

Each entry to each of these hoppers must be in terms of nine components of a stress tensor (i.e., considering symmetry, three normal stresses and three shear stress) expressed using some common coordinate system (e.g., axial, hoop, and radial).

4. DESIGN CRITERIA (CODE LIMITS)

In principle, we must consider hopper contributions for all operating conditions and for all points in the vessel. In practice, only a fairly small number of operating condition, point combinations are likely to be limiting. These combinations are first judged by loading criteria appropriate to the first four hoppers (P_m , P_L , P_b , and Q)³⁵. Then fatigue damage criteria are used for the peak stress contents of the fifth hopper (F).

4.1 Loading Criteria

The stresses for each combination of (operating condition, point in the vessel) are judged by the following loading design criteria:

$$P_m \leq S_m ; (3a)$$

$$P_L \leq 1.5S_m \quad (3b)$$

$$(P_L + P_b) \leq 1.5S_m \text{ ; and} \quad (3c)$$

$$(P_L + P_b + Q) \leq 3S_m \text{ .} \quad (3d)$$

Although the contents of other hoppers can contribute, each of these four equations provides the code limit for one hopper (that is, Eq. 3a for hopper P_m³⁶, 3b for P_L³⁷, 3c for P_b³⁸, and 3d for Q). It can be seen that, as previewed in §1.2, the acceptability of a given numerical stress value does depend markedly on the nature of the stress, with acceptable values ranging from (S_m) to (3S_m).³⁹

Equations 3a through 3d are more complicated than is apparent. The left hand side of each equation indicates a tensor with nine components. In each case in which more than one hopper is involved, a summation of tensor components with like subscripts is implied. In addition, the comparison implied by each equation requires that the left hand side to be converted to a single scalar value. This conversion is performed by using the Tresca (maximum shear stress) theory. That is, three principal stresses ($\sigma_1, \sigma_2, \sigma_3$)⁴⁰ are obtained that correspond to the nine component stress tensor. Then the Tresca stress is:

$$\text{Maximum} \{ |\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1| \} \quad (4)$$

This Tresca stress is the single scalar value that can be used to characterize the left hand side and to decide if the inequality is satisfied.

4.2 Fatigue Damage

The peak stress hopper (F) contents are handled differently. They are used in a fatigue damage evaluation. That is, a hypothesized time history of loading is established for the entire vessel lifetime. Then, in principle, for each point in the vessel, the stress tensor (all nine components) must be followed as a function of time and must be compared to the design fatigue curve of §2.2. In practice, only a few points must be examined. In addition, a time history can be chosen that is simpler than the expected operating history and can still provide a conservative basis for fatigue design. Details of the fatigue damage calculations are as follows:

- The term loading includes both mechanical loads (e.g., pressure) and temperature distributions.
- The stress tensor that must be followed is the sum $(P_L + P_b + Q + T)$ ₄₁.
- Represent the loading history as the sum of individual loading cycles. The i^{th} ₄₂ loading cycle ($i = 1, 2, \dots, I$) repeats N_i ₄₃ times during the vessel lifetime. During each repetition, all stresses for the i^{th} ₄₄ loading cycle have the same time behavior.
- Find a "maximum stress range" for the i^{th} ₄₅ loading cycle. First, subtract the stress at an arbitrary time (t) in the cycle from the stress at some reference time (t^*) ₄₆ to obtain a stress difference $(\Delta \sigma_{ij})$ ₄₇:

$$\Delta \sigma_{ij} = \sigma_{ij}(t) - \sigma_{ij}(t^*) \quad (5)$$

Second, obtain a single scalar value to characterize this stress difference by converting the stress components of $(\Delta \sigma_{ij})$ ₄₈ to a Tresca stress (using the techniques of Eq. 4). Finally, try various combinations of t and t^* to obtain a maximum value of the Tresca stress for this cycle. Designate half this maximum value as S_{ai} , the alternating stress amplitude that characterizes the i^{th} loading cycle.

- Enter the design fatigue curve with S_{ai} and extract N_{di} , the design allowable cycles for the i^{th} loading cycle.
- Complete a similar calculation for each loading cycle, $i = 1, 2, \dots, I$. Obtain a fatigue usage factor, U_F , by using the following linear damage rule (Miner's hypothesis, Ref. 4, pp. 256-258):

$$U_F = \sum_{i=1}^n \frac{\sigma_i}{\sigma_y} \quad (6)$$

- The code limit for fatigue damage is:

$$U_F \leq 1 \quad (7)$$

which also gives the means of judging the severity of stresses in the peak stress hopper (F).

5. PHYSICAL BASES FOR DESIGN RULES

Some of the code limits given in §4 can be interpreted on a physical basis by using an idealized material and simple loadings. An additional discussion of these interpretations is presented elsewhere (Ref. 7, pp. 6-10).

5.1 Idealized Material

These interpretations are given in terms of an idealized (elastic plus perfectly plastic) material. That is, the stress strain curve for this idealized material is:

$$\sigma = E \epsilon ; \quad \text{for } \sigma \leq \sigma_y ; \text{ and} \quad (8a)$$

$$\sigma = \sigma_y ; \quad \text{for } \sigma > \sigma_y ; \quad (8b)$$

where $\epsilon_y = \sigma_y / E$; and where σ_y = yield stress.

5.2 Primary Membrane Load

For a primary (i.e., load controlled) uniaxial tensile load, a specimen constructed of the idealized material fails by large deformation upon attempts to increase the stress beyond σ_y . Thus the limit load for primary membrane loading is $\sigma = \sigma_y$.

5.3 Primary Bending Load

Consider the primary (i.e., load controlled) pure bending of a rectangular beam constructed of the idealized material. The extreme fibers of the beam first reach the yield stress at a bending moment equal to M_y 53, where:

$$M_y = (bh^2/6) S_y \quad (9)$$

where b = the base of the rectangular cross section; and
where h = the height.

In this case, portions of the beam remain elastic and prevent large deformations as the bending moment increases above M_y 54. Beam elements remain elastic up to the limit moment M_L 55, where

$$M_L = (bh^2/4) S_y \quad (10)$$

Since $(M_L/M_y) = 1.5$ 56, the limit load stress (elastically calculated) for primary bending loading is $\sigma = 1.5 S_y$ 57.

5.4 Secondary Stresses

A different concept is invoked as the basis for secondary stress limits. That concept is "shakedown to elastic behavior" and can be illustrated by using strain controlled uniaxial loading. Consider that a uniaxial tensile specimen is subjected a strain cycle starting at $\epsilon = 0$ 58, and monotonically extended to $\epsilon = \epsilon_m$ 59. Subsequent cycles have the same strain behavior. Following the stress, strain behavior for the postulated loading and then trying various value of ϵ_m 60, we observe that shakedown to elastic behavior occurs for all $\epsilon_m \leq \epsilon_y$ 61. Therefore the shakedown limit stress (elastically calculated) for secondary loading is $\sigma = S_y$ 62.

5.5 Summary

The idealized results of §5.2 through §5.4 are compared to the code limits of §4.1 (as applied to the same loading):

Hopper Category
Code Limits

Idealized Results

P_m S_m	S_y
P_L $1.5 S_m$	$2 S_y$
P_b $1.5 S_m$	$1.5 S_y$
Q $3 S_m$	$2 S_y$

Comments are:

- A margin with respect to yield of at least 1.5 has previously been adopted in defining S_m 63 (§2.1). This margin applies directly to the P_m 64 and P_b 65 categories.
- The Q category uses a code limit that omits the same 1.5 yield margin. This omission is based on the judgment that a little plastic action during overloads can be tolerated. That is, exceeding primary stress limits is judged to be potentially much more damaging than exceeding secondary stress limits.
- A loading in the P_L 66 category to a large extent has the nature of a secondary stress. Therefore the use of 1.5 in the code limit for this category implies that an extra margin has been adopted here with respect to the idealized results.
- The strain hardening of actual materials (i.e., the increase of stress above the yield stress) provides additional margin that is not incorporated in the discussions based on an idealized material.

A final comment concerns the use of elastic calculations to obtain stresses and the use of strain controlled fatigue test results for evaluating fatigue behavior. These practices are justified partially on the basis of choosing temperatures of operation to prevent creep deformation. They are also justified on the basis of the primary and secondary limits of §4.1. Satisfying the limits assures that bulk regions of the vessel either have elastic behavior or shake down to elastic behavior. Local regions of plastic strain thus either disappear or are supported by surrounding bulk regions that have elastic behavior.

CAUTION

As indicated in §1.3, the view of the code presented herein is perhaps over simplified. The references, material supporting the references, and updates should be consulted to obtain a more thorough awareness of code provisions and bases.

REFERENCES

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