



# Reactor Physics – Design Parameters for PWRs

**22.39 Elements of Reactor Design, Operations, and Safety**

**Lecture 5**

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## General Aspects

- **Design Objective**
  - **Safe, reliable, economical operation at the rated power level over the core lifetime.**
- **Interaction with other disciplines**
  - **Thermal considerations determine core size and geometry.**
  - **The length of time a fuel element can utilized is determined by its ability to withstand radiation damage and thermal/mechanical stresses.**
- **The design process is iterative drawing heavily on past experience.**



## Principal Design Functions Involving Reactor Physics

- **Core criticality and power distribution**
  - Are space and time dependent because of fuel burnup and isotope production over the core life
  - Depend on core enrichment, moderator-to-fuel ratio, core geometry, location and type of reactivity control, fuel element design
- **Reactivity and control analysis (safety)**
  - Must control excess reactivity in initial fuel loading
  - Allocate this reactivity to movable control rods, soluble neutron poisons in the coolant (“chemical shim”; boron), and “burnable poisons” or “mechanical shim” (gadolinium, borosilicate glass).
  - Describe short-term reactivity changes (and reactor kinetic behavior); reactivity coefficients.
- **Depletion analysis (economic performance)**
  - Monitor fuel composition and reactivity as a function of energy removal



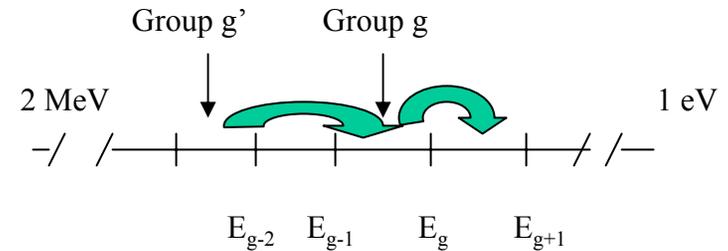
# The Big Picture

## The Multigroup Diffusion Equations:

$$\frac{1}{v_g} \frac{\partial \phi_g}{\partial t} - \nabla \cdot \mathbf{D}_g \nabla \phi_g + \Sigma_{tg} \phi_g(\underline{\mathbf{r}}, t) =$$

$$= \sum_{g'=1}^G \Sigma_{sg'g} \phi_{g'} + \chi_g \sum_{g'=1}^G v_{g'} \Sigma_{fg'} \phi_{g'} + S_g^{\text{ext}}$$

$g = 1, 2, \dots, G$



## Group constants:

$$\Sigma_{tg} \equiv \frac{\int_{E_g}^{E_{g-1}} dE \Sigma_t(\mathbf{E}) \phi(\underline{\mathbf{r}}, \mathbf{E}, t)}{\int_{E_g}^{E_{g-1}} dE \phi(\underline{\mathbf{r}}, \mathbf{E}, t)}$$



# Power Distribution

- **Problem:** The group constants depend on the flux itself.
- **Criticality:** Set  $\frac{1}{v_g} \frac{\partial \phi_g}{\partial t}$  and  $S_g^{\text{ext}}$  equal to zero.
- **Perform two multigroup calculations:**
  - Use a library code to obtain cross sections and average them by treating spatial and time dependence very crudely. Calculate the intragroup fluxes relying on models of neutron slowing down and

thermalization, e.g., assuming that  $\phi(E) \propto \frac{1}{E}$  for energies between 1

eV and  $10^5$  eV,  $\phi(E) \propto \chi(E)$  in the high-energy range, and proportional to the Maxwellian distribution for thermal energies.

- This *fine spectrum calculation* may involve as many as 1000 groups.
- These intragroup fluxes are, then, used to calculate the group constants for a *coarse group calculation* with spatial dependence. Iteration is possible.
- **LWRs:** Usually three fast groups and one thermal.
- **Fast Reactors:** As many as 20 groups.

## Two-Group Criticality Calculation (Bare Homogeneous Reactor)

$$-\nabla \cdot D_1 \nabla \varphi_1 + \Sigma_{R1} \varphi_1 = \frac{1}{k} [\nu_1 \Sigma_{f1} \varphi_1 + \nu_2 \Sigma_{f2} \varphi_2]$$

$$-\nabla \cdot D_2 \nabla \varphi_2 + \Sigma_{a2} \varphi_2 = \Sigma_{s12} \varphi_1$$

$$\Sigma_{R1} \equiv \Sigma_{a1} + \Sigma_{s12}$$

Assuming that  $\varphi_1(\underline{\mathbf{r}}) = \varphi_1 \psi(\underline{\mathbf{r}})$        $\varphi_2(\underline{\mathbf{r}}) = \varphi_2 \psi(\underline{\mathbf{r}})$

$$\nabla^2 \psi(\underline{\mathbf{r}}) + \mathbf{B}^2 \psi(\underline{\mathbf{r}}) = 0, \quad \psi(\underline{\mathbf{r}}_s) = 0$$

We get the  
(effective)  
multiplication  
factor

$$\mathbf{k} = \frac{\nu_1 \Sigma_{f1}}{\Sigma_{R1} + \mathbf{D}_1 \mathbf{B}^2} + \frac{\Sigma_{s12}}{(\Sigma_{R1} + \mathbf{D}_1 \mathbf{B}^2)} \frac{\nu_2 \Sigma_{f2}}{(\Sigma_{a2} + \mathbf{D}_2 \mathbf{B}^2)}$$

**For criticality:  $\mathbf{k} = 1$**



## More on the Multiplication Factor

$$k = \left[ \frac{\nu_1 \Sigma_{f1}}{\Sigma_{R1} + D_1 B^2} \right] + \left[ \frac{\Sigma_{s12}}{(\Sigma_{R1} + D_1 B^2)} \frac{\nu_2 \Sigma_{f2}}{(\Sigma_{a2} + D_2 B^2)} \right]$$

$k_1$ : due to fissions in the fast group

$k_2$ : due to fissions in the thermal group

$$k_2 = \frac{\left( \frac{\Sigma_{s12}}{\Sigma_{R1}} \right) \nu_2 \left( \frac{\Sigma_{f2}}{\Sigma_{a2}} \right)}{\left( 1 + \frac{D_1}{\Sigma_{R1}} B^2 \right) \left( 1 + \frac{D_2}{\Sigma_{a2}} B^2 \right)} = \frac{p}{(1 + L_1^2 B^2)} \frac{\eta_2 f_2}{(1 + L_2^2 B^2)} = \eta_2 f_2 p P_{NL1} P_{NL2}$$



## The Six-Factor Formula

$$k = \eta_2 f_2 p \varepsilon P_{NL1} P_{NL2} = (\eta_{th} f_{th} p \varepsilon) P_{NL1} P_{NL2} = k_{\infty} P_{NL1} P_{NL2}$$

$$\eta_2 \equiv \nu_2 \left( \frac{\Sigma_{f2}}{\Sigma_{a2}^F} \right) \quad \text{Average number of neutrons produced per thermal neutron absorbed in the fuel}$$

$$f_2 \equiv \left( \frac{\Sigma_{a2}^F}{\Sigma_{a2}} \right) \quad \text{Thermal utilization: conditional probability of absorption of a thermal neutron in the fuel}$$

$$L_1^2 \equiv \frac{D_1}{\Sigma_{R1}} \quad \text{Diffusion area for fast neutrons}$$

$$P_{NL1} \equiv \frac{1}{(1 + L_1^2 B^2)} \quad \text{Nonleakage probability for fast neutrons}$$

$$p \equiv \left( \frac{\Sigma_{s12}}{\Sigma_{R1}} \right) \quad \text{Resonance escape probability}$$

$$\varepsilon \equiv \left( 1 + \frac{k_1}{k_2} \right) = \left[ 1 + \left( \frac{\nu_1 \Sigma_{f1}}{\nu_2 \Sigma_{f2}} \right) \left( \frac{\Sigma_{a2} + D_2 B^2}{\Sigma_{s12}} \right) \right] \quad \text{Fast fission factor}$$



## Comments

- $\eta_{th}$   $^{233}\text{U}$ : 2.29,  $^{235}\text{U}$ : 2.07,  $^{239}\text{Pu}$ : 2.15, natural uranium: 1.34, enriched uranium: 1.79. Increases initially as Pu is produced from  $^{238}\text{U}$ , decreases later as fission products are produced
- $f_{th}$  About 0.9 for natural uranium. Larger as absorptions in nonfuel material decrease.
- $\rho$  About 0.70 for homogeneous mixtures, 0.9 for heterogeneous mixtures, increases as the ratio of moderating atoms to fuel atoms becomes large.
- $\epsilon$  About 1.05 for natural uranium.
- $L_1$  Water: 0.052 m, heavy water: 0.114 m, graphite: 0.192 m
- $L_2$  Water: 0.027 m, heavy water: 1.0 m, graphite: 0.54 m
- $B^2$  Typically less than  $10 \text{ m}^{-2}$ , therefore  $P_{NL1} > 0.97$  and  $P_{NL2} > 0.99$  for  $\text{H}_2\text{O}$



# Time Dependence

- **To study the time-dependence of the flux we have to solve the multigroup equations in slide 3 augmented to include the equations for delayed neutrons.**
- **There are two time scales:**
  - **Short-term changes (seconds) due to temperature effects and external deliberate changes**
  - **Long-term changes (hours or more) due to fuel depletion and fission-product buildup.**



## Point Kinetics

- **Recall (slide 6):** 
$$\begin{pmatrix} \phi_1(\underline{\mathbf{r}}) \\ \phi_2(\underline{\mathbf{r}}) \end{pmatrix} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \psi(\underline{\mathbf{r}}) \quad \nabla^2 \psi(\underline{\mathbf{r}}) + \mathbf{B}^2 \psi(\underline{\mathbf{r}}) = \mathbf{0}, \quad \psi(\underline{\mathbf{r}}_s) = \mathbf{0}$$

- **Local perturbations, e.g., by moving the control rods, leads to a readjustment of  $\psi(\underline{\mathbf{r}})$  that is usually slight and happens in a few milliseconds. Then, the readjusted shape rises or falls “as a whole” depending on whether the perturbation increased or decreased  $k$ .**
- **Point kinetics allows us to investigate the level (or average) flux assuming that the shape does not change appreciably.**
- **We average over all energy groups and write the neutron density as**

$$\mathbf{n}(\underline{\mathbf{r}}, t) = \mathbf{n}(t) \psi(\underline{\mathbf{r}})$$

- **$\mathbf{n}(t)$  is the total neutron density or the total power**

$$\text{Power}(t) = \sum_{g'=1}^G \int_V dV w_{fg'} \Sigma_{fg'} \phi_{g'}(\underline{\mathbf{r}}, t) \propto \mathbf{n}(t)$$

- **Using this equation in the space- and time-dependent equations and including delayed neutrons leads to**



## Point Kinetics Equations

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_1^6 \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \quad i = 1, 2, \dots, 6$$

$$\rho(t) \equiv \frac{k(t) - 1}{k(t)} \quad \text{Reactivity}$$

$$\Lambda \equiv \frac{\ell}{k(t)} \cong \ell \equiv \frac{1}{v \Sigma_a (1 + L^2 B^2)}$$

**Mean generation time between birth of a neutron and absorption inducing fission**

$\ell$

**Prompt neutron lifetime between birth of a neutron and absorption;  
10<sup>-3</sup> to 10<sup>-4</sup> for thermal reactors; 10<sup>-7</sup> for fast reactors**



## Delayed Neutron Precursors

Decay Constants and Yields of Delayed-Neutron Precursors in Thermal Fission of Uranium-235

$t_{1/2}$ (s)	$\lambda_i$ (s <sup>-1</sup> )	$\beta_i$	$\beta_i/\lambda_i$ (s)
55.7	0.0124	0.000215	0.0173
22.7	0.0305	0.00142	0.0466
6.22	0.111	0.00127	0.0114
2.30	0.301	0.00257	0.0085
0.61	1.1	0.00075	0.0007
0.23	3.0	<u>0.00027</u>	<u>0.0001</u>
Total		0.0065	0.084

Glasstone & Sesonske,  
*Nuclear Reactor Engineering*,  
Chapman & Hall, 1994



## Reactivity Feedback

- The reactivity depends on the neutron density (or power level) itself.
- This is due to the fact that  $k$  depends on macroscopic cross sections, which themselves involve the atomic number densities of the materials:

$$\Sigma(\underline{r}, t) = N(\underline{r}, t) \bullet \sigma(\underline{r}, t)$$

- The atomic density depends on the power level because:
  - Material densities depend on temperature, which, in turn, depends on the power distribution
  - The buildup of poisons and burnup of fuel (long-term effect).

• We write

$$\rho(t) = \delta\rho_{\text{ext}}(t) + \delta\rho_f[\mathbf{P}]$$

$\delta\rho_{\text{ext}}(t)$       External reactivity from some reference power level  $P_0$  for which  $\rho$  is zero.

$\delta\rho_f[\mathbf{P}]$       Change in reactivity due to inherent feedback mechanisms.



# Power Coefficient of Reactivity

Average temperature of region j

$$\alpha_P \equiv \frac{d\rho}{dP} = \sum_j \left( \frac{\partial \rho}{\partial T_j} \right) \left( \frac{\partial T_j}{\partial P} \right) = \sum_j \alpha_T^j \left( \frac{\partial T_j}{\partial P} \right)$$

Temperature coefficient of reactivity for region j

To be determined by T-H analysis

**Safety requirement:  $\alpha_P < 0$**

$$\Delta\rho_{PD} \equiv \int_0^{FP} dP \alpha_P$$

**Power Defect:** Total reactivity change from hot zero-power state to hot full-power state; compensated by control-rod insertion and soluble boron. It is less than about 0.05.



## Dominant Coefficients of Reactivity

$$\alpha_P = \alpha_T^F \left( \frac{\delta \mathbf{T}_F}{\delta \mathbf{P}} \right) + \alpha_T^M \left( \frac{\delta \mathbf{T}_M}{\delta \mathbf{P}} \right)$$

$$\alpha_T^F \equiv \frac{\partial \rho}{\partial \mathbf{T}_F} = \frac{1}{\mathbf{k}^2} \frac{\partial \mathbf{k}}{\partial \mathbf{T}_F} \cong \frac{1}{\mathbf{k}} \frac{\partial \mathbf{k}}{\partial \mathbf{T}_F}$$

Doppler broadening of resonance absorption decreases  $\rho$ .

For PWRs:  $(-4 \text{ to } -1) \times 10^{-5} \Delta\rho/^\circ\text{K}$  or  $(-4 \text{ to } -1) \text{ pcm (per cent mille)}/^\circ\text{K}$ .

Moderator: Thermal expansion leads to loss of neutron moderation and a corresponding decrease in  $\rho$ . The decrease in the density of poison atoms leads to reactivity increase. Overall effect should be negative.

For PWRs:  $(-50 \text{ to } -8) \text{ pcm}/^\circ\text{K}$ .



## General Design Criteria 27 and 28

- *Criterion 27--Combined reactivity control systems capability.* The reactivity control systems shall be designed to have a combined capability, in conjunction with poison addition by the emergency core cooling system, of reliably controlling reactivity changes to assure that under postulated accident conditions and with appropriate margin for stuck rods the capability to cool the core is maintained.
- *Criterion 28--Reactivity limits.* The reactivity control systems shall be designed with appropriate limits on the potential amount and rate of reactivity increase to assure that the effects of postulated reactivity accidents can neither (1) result in damage to the reactor coolant pressure boundary greater than limited local yielding nor (2) sufficiently disturb the core, its support structures or other reactor pressure vessel internals to impair significantly the capability to cool the core. These postulated reactivity accidents shall include consideration of rod ejection (unless prevented by positive means), rod dropout, steam line rupture, changes in reactor coolant temperature and pressure, and cold water addition.



## Standard Review Plan: 4.3 Nuclear Design 2

The areas concerning reactivity coefficients include:

The applicant's presentation of calculated nominal values for the reactivity coefficients such as the moderator coefficient, which involves primarily effects from density changes and takes the form of temperature, void, or density coefficients; the Doppler coefficient; and power coefficients.



## 10 CFR 50.68 Criticality Accident Requirements (1)

- Each licensee shall comply with the following requirements in lieu of maintaining a monitoring system capable of detecting a criticality as described in 10 CFR 70.24:
  - Plant procedures shall prohibit the handling and storage at any one time of more fuel assemblies than have been determined to be safely subcritical under the most adverse moderation conditions feasible by unborated water.
  - The estimated ratio of neutron production to neutron absorption and leakage ( $k$ -effective) of the fresh fuel in the fresh fuel storage racks shall be calculated assuming the racks are loaded with fuel of the maximum fuel assembly reactivity and flooded with unborated water and must not exceed 0.95, at a 95 percent probability, 95 percent confidence level. This evaluation need not be performed if administrative controls and/or design features prevent such flooding or if fresh fuel storage racks are not used.



## 10 CFR 50.68 Criticality Accident Requirements (2)

- If no credit for soluble boron is taken, the k-effective of the spent fuel storage racks loaded with fuel of the maximum fuel assembly reactivity must not exceed 0.95, at a 95 percent probability, 95 percent confidence level, if flooded with unborated water. If credit is taken for soluble boron, the k-effective of the spent fuel storage racks loaded with fuel of the maximum fuel assembly reactivity must not exceed 0.95, at a 95 percent probability, 95 percent confidence level, if flooded with borated water, and the k-effective must remain below 1.0 (subcritical), at a 95 percent probability, 95 percent confidence level, if flooded with unborated water.
- Radiation monitors are provided in storage and associated handling areas when fuel is present to detect excessive radiation levels and to initiate appropriate safety actions.
- The maximum nominal U-235 enrichment of the fresh fuel assemblies is limited to five (5.0) percent by weight.

# Core Composition Changes – Fission Product Poisoning

- Some fission products have large thermal absorption cross section. The poisoning effect is insignificant for fast reactors.
- Most important products:  $^{135}_{54}\text{Xe}$  with  $\sigma_a^X \cong 3 \times 10^6 \text{ b} = 3 \times 10^{-22} \text{ m}^{-2}$  and  $^{149}_{62}\text{Sm}$  with  $\sigma_a^S \cong 5 \times 10^4 \text{ b} = 3 \times 10^{-24} \text{ m}^{-2}$
- We measure the impact of a poison by calculating the reactivity decrease it causes.

$$\Delta\rho = \rho' - \rho = \frac{k' - 1}{k'} - \frac{k - 1}{k} = \frac{1}{k} \left( 1 - \frac{k}{k'} \right)$$

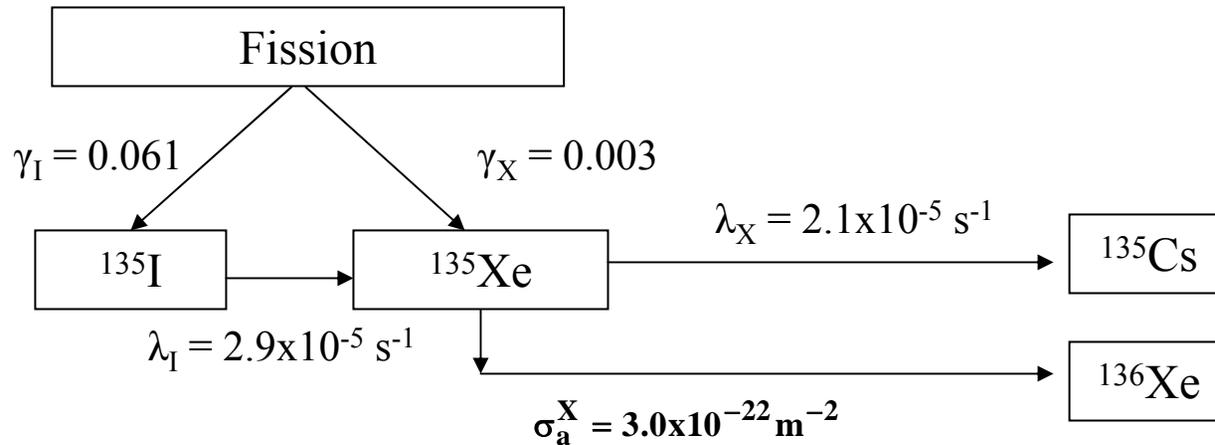
- The thermal utilization  $f \equiv \left( \frac{\Sigma_{a2}^F}{\Sigma_{a2}} \right)$  is the only factor that is appreciably affected by the poison.

$$\Delta\rho = \frac{1}{k} \left( 1 - \frac{k}{k'} \right) \cong \left( 1 - \frac{f}{f'} \right) = 1 - \frac{\Sigma_a + \Sigma_{aP}}{\Sigma_a} = - \frac{\Sigma_{aP}}{\Sigma_a}$$

$$\Delta\rho = - \frac{\Sigma_{aP}}{\Sigma_a}$$



## Xenon Poisoning



$$\frac{\partial \mathbf{I}(\underline{\mathbf{r}}, t)}{\partial t} = \gamma_{\mathbf{I}} \Sigma_f \varphi(\underline{\mathbf{r}}, t) - \lambda_{\mathbf{I}} \mathbf{I}(\underline{\mathbf{r}}, t)$$

$$\frac{\partial \mathbf{X}(\underline{\mathbf{r}}, t)}{\partial t} = \gamma_{\mathbf{X}} \Sigma_f \varphi(\underline{\mathbf{r}}, t) + \lambda_{\mathbf{I}} \mathbf{I}(\underline{\mathbf{r}}, t) - \lambda_{\mathbf{X}} \mathbf{X}(\underline{\mathbf{r}}, t) - \sigma_a^{\mathbf{X}} \varphi(\underline{\mathbf{r}}, t) \mathbf{X}(\underline{\mathbf{r}}, t)$$

$$\mathbf{I}_{\infty} = \frac{\gamma_{\mathbf{I}} \Sigma_f \varphi_0}{\lambda_{\mathbf{I}}} \quad \mathbf{X}_{\infty} = \frac{(\gamma_{\mathbf{I}} + \gamma_{\mathbf{X}}) \Sigma_f \varphi_0}{\lambda_{\mathbf{X}} + \sigma_a^{\mathbf{X}} \varphi_0}$$

$$\Delta \rho = - \frac{\sigma_a^{\mathbf{X}} (\gamma_{\mathbf{I}} + \gamma_{\mathbf{X}}) \Sigma_f \varphi_0}{(\lambda_{\mathbf{X}} + \sigma_a^{\mathbf{X}} \varphi_0)} \quad \Delta \rho = -0.023 \quad \text{for } \varphi_0 = 10^{17} \text{ n/m}^2\text{s}$$



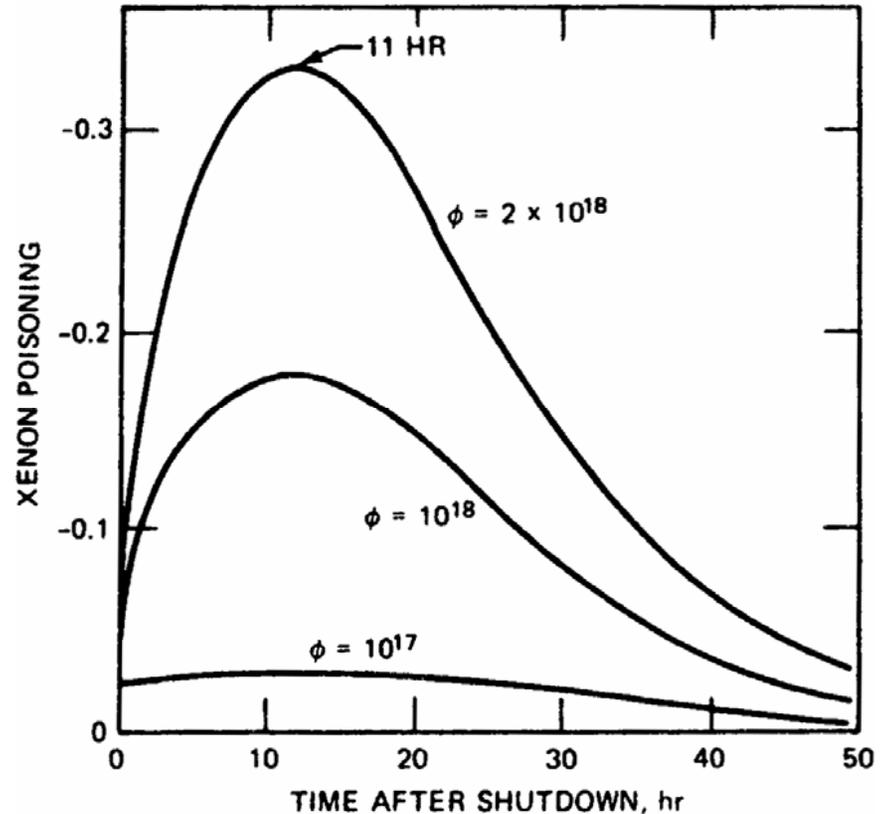
## Xenon and Reactor Shutdown

$$\frac{\partial I(\underline{r}, t)}{\partial t} = -\lambda_I I(\underline{r}, t)$$

$$\frac{\partial X(\underline{r}, t)}{\partial t} = \lambda_I I(\underline{r}, t) - \lambda_X X(\underline{r}, t)$$

A reactor operating at a flux of  $2 \times 10^{18}$  n/m<sup>2</sup>s will have a negative insertion of reactivity of about -0.33, a sizable amount.

Glasstone & Sesonske,  
*Nuclear Reactor Engineering*,  
Chapman & Hall, 1994





## Fuel Depletion (Burnup)

- During reactor operation, fuel ( $^{235}\text{U}$ ) is depleted and new fuel ( $^{239}\text{Pu}$ ) is produced.
- There is a net decrease of reactivity over time.
- In general,

$$N_{\mathbf{F}}(\underline{\mathbf{r}}, t) = N_{\mathbf{F}}(\underline{\mathbf{r}}, 0) \exp\left[-\sigma_{\mathbf{F}}^a \int_0^t \phi(\underline{\mathbf{r}}, s) ds\right] \equiv N_{\mathbf{F}}(\underline{\mathbf{r}}, 0) \exp\left[-\sigma_{\mathbf{F}}^a \Phi(\underline{\mathbf{r}}, t)\right]$$

$$\Phi(\underline{\mathbf{r}}, t) = \int_0^t \phi(\underline{\mathbf{r}}, s) ds \quad \text{is the neutron fluence}$$

- The dependence of the flux on the fuel density complicates the calculations.
- In lieu of the fluence, it is customary to use the thermal energy output per unit mass of fuel (burnup) in MWD/T (MW days per metric ton of uranium fuel).
- Current limit: 62 GWd/T. For a 1,000 MW PWR, burnup is about 15 GWd/T after one year.



## **REGULATORY GUIDE 1.77**

### **ASSUMPTIONS USED FOR EVALUATING A CONTROL ROD EJECTION ACCIDENT FOR PRESSURIZED WATER REACTORS**

- **“In general, failure consequences for  $UO_2$  have been insignificant below 300 cal/g for both irradiated and unirradiated fuel rods. Therefore, a calculated radial average energy density of 280 cal/g at any axial fuel location in any fuel rod as a result of a postulated rod ejection accident provides a conservative maximum limit to ensure that core damage will be minimal and that both short-term and, long-term core cooling capability will not be impaired.”**