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JACK HARE: So today, we're going to be talking about Langmuir probes. Now, I'll freely admit I find Langmuir probes a little bit confusing. I would say that you need to know a lot about sheath theory to properly understand the Langmuir probe. And to do that in a single class is very, very challenging.

So if you look in Hutchinson's textbook, he takes a significant amount of time on these probes. He does that because he really likes probes, and that's very reasonable. Everyone has their little biases. I'm not going to spend that much time. And what I've done in particular is I've tried to rework some of the material in Hutchinson's book in a way that makes more sense to me.

Now, it may not make more sense to you, so this is why I strongly encourage you to go have a look at Hutchinson's book as well and maybe interpolate between those two. And I also included in the syllabus a reference to the book by Lieberman and Lichtenberg, which is very good on sheath theory. It's a book on low temperature plasmas where sheaths are very common. So if you are a little bit weak on sheath physics and you want to review that, I recommend that book.

Now, one thing I will say is that everyone uses different notation. And in an attempt to make things clearer, I've also used a third and different notation from that used in Hutchinson and Lieberman and Lichtenberg. So good luck with that.

I think mine is less confusing than Hutchinson's, but of course it's that classic thing about there are too many standards. And now I've invented a new standard. So just keep an eye on all these quantities. I'll try and explain what they are. If you're not sure about any of them, please shout out.

So the Langmuir probe-- it's a very simple probe. It's simple because all we take is a conductor, some sort rod of metal, and stick it into our plasma like this. And then we measure the potential. I'm going to be calling v 0 here. This is the potential on our probe.

And so if we think about what this looks like, we can draw a little probe. Maybe it's a little rod of metal like this. And it's surrounded by plasma like this. And we know from our intro to plasma physics, whenever we see a conductor in a plasma, there's going to be something forming around it. So if this is the plasma and this is the probe, then this is a region that we call the sheath.

And so we're going to have to understand how sheaths work. But before we get fully going with sheaths, we're just going to give an intuitive explanation of why we're going to get a sheath. And then I'm going to give you a very brief overview of the sheath results without doing any actual formal derivation of them because it will take too long.

So at the instant that we stick this probe into the plasma, it's going to be immersed in a bath of electrons and ions. And so there are going to be electrons and ions striking this conductor. And we're going to assume that when they strike this conductor, they stick to it. So effectively, it starts to gain charge.

So the first thing we want to know is what is the flux of particles? So this is a flux capital gamma of particles of species, J, that is striking against this probe here. And we're going to make an approximation that this probe is a planar surface. That means we pick up a factor of 1/4, just in geometry. And that is going to be 1/4 of the number of probe particles times by the average speed that the probe particles are going at.

So this is the average particle flux. I haven't said anything about charge yet. I just want to note here, I'm going to be using u for velocity, and the reason is that there's an awful lot of v's that crop up. And I think using a lowercase v for velocity and an uppercase V for potential is going to cause a huge amount of confusion. So this is the first place where I diverge from Hutchinson. So just keep that in mind.

AUDIENCE: Where does the 1/4 come from?

JACK HARE: The 1/4 comes from geometry. If you go look this up in some standard statistical mechanics textbook, you'll find out it's because you've got a load of particles moving in random directions, and you've got a plane here. And so it's the number of particles that have a velocity, which is directed through the plane that you're talking about here. So it's a geometric factor. I haven't derived it.

AUDIENCE: Ok. Thanks.

JACK HARE: Cool. And so that means that the total current that this probe is instantaneously collecting as soon as we stick it into the plasma is going to be equal to the particle charge times these two fluxes. So we're going to have-- I'm going to put a minus e at the front here. We're going to have an area which represents the surface area of the probe. We're going to have our 1/4 that we have for both species, and then we're going to have a term that looks like ni ui minus ne ue here.

So that's the current that we're instantaneously collecting. Now, it turns out, of course, that the average velocity of the electrons is going to be much, much bigger than the average velocity of the ions in general, and that is just a fact because the electron mass is much, much less than the ion mass. And so if they've got the similar temperatures, the electrons are going to be moving faster, which means that at least as soon as we stick the probe into the plasma, this current is going to be completely dominated by the electrons.

So it's going to be 1/4 eA ne electron density, ue, like that. And almost all of the particles which are colliding with the probe are going to be electrons. So this means that the probe charge is negative because it's collecting loads of electrons. And it keeps going until all the electrons are repelled or until a vast majority of the electrons are repelled. And it will keep going until we have no current.

And by the time that the probe has reached a situation where there's no total current, we say that the probe has reached a potential, which we call the floating potential. And it's called the floating potential because if we just stick the probe in the plasma, we don't try to bias it at all. And we just let its potential adjust until it reaches this i equals 0 level. It will reach some potential. We just let it float to that potential. It's a floating potential.

But the key thing here is that this is not the same as the plasma potential. So the plasma itself is going to be at some other potential, which we will derive later on. And so the floating potential, we're going to call Vf, and the plasma potential we're going to call Vp here. And this difference in potentials is due to the fact that we have perturbed the plasma.

We have stuck some probe into it, which has changed the nature of the plasma. It's created this sheath. It's caused all sorts of electrons and ions to fly around, charges to fly around. So there's no good reason for it to have the same potential as the plasma at all. OK. Any questions about this?

OK. We're going to go on. So what we want to do now our probe has reached a floating potential is actually change that potential and measure what current we draw. We know that, for the floating potential, we get 0 current. But if we bias the probe in some way, we'll get different currents. Nigel, I see your hand.

AUDIENCE: Yeah. I wanted to ask, what ground exactly are we referencing the potential measurements to? Is it just any random ground or is it anything specific?

JACK HARE: I can give you a short answer, and I probably won't give you the longer answer. The short answer is, as you know, the potential is just relative. We're only interested in relative potentials. And it makes sense in this to use the potential of the plasma very far away as our reference, v equals 0.

AUDIENCE: OK.

JACK HARE: And that's what we're going to do here. But of course, mathematically, we could choose any other potential, and that's absolutely fine. Another useful ground if you have a probe stuck inside, for example, a laboratory plasma, another potential-- you could use as a reference, but we're not going to-- would be the potential that the vacuum vessel takes. And another potential you could use if you are a Langmuir probe mounted to a satellite flying through space will be the potential of the satellite itself.

I'm just going to mute 218 because it's very sensitive microphones in there. Thank you very much. But please feel free to unmute if you want to ask a question. OK. Any other questions before we keep going? OK, good stuff.

So as I was saying, we are now going to bias our probe, which is effectively changing the probe potential, V0, and measure the current that we draw. And depending on where we bias this probe, we're going to end up in different regions. So we're going to have here as an overview, a qualitative picture. And we're going to go and do the calculations for the quantitative picture later on.

So our qualitative picture, we have an IV curve, where we have I vertically like this and V along here. We're going to identify our 0 current with the floating potential, as we've already done. We're going to say that somewhere above the floating potential is the plasma potential. And it turns out that we end up with an IV curve that looks like this.

And there are a few important quantities on here. There's the electron saturation current. And I'll just draw this a little bit better down here for the ions because it showed some asymptote. There is the ion saturation current down here.

And I'll explain where all of these things came from. I just want you to have this drawing in mind while I start explaining it. So one place we know what the current would be, or two places we know what the current would be, are at these specific potentials here. So at the plasma potential, the current is going to be entirely due to electrons which are streaming into the probe. Because if we're up at the plasma potential, we know that there will be no ions in general.

So we're just going to end up with this electron saturation current, 1/4, times the electron charge times the probe area times the density times by the random electron velocity. We showed that earlier. So this is obviously quite a large current. Another place that we know, as I said, is at the floating point here, where the current is just going to be equal to 0.

OK. But then there are three other regimes that I want to talk about, and I'm going to call them A, B, and C here. So let's start with A. So A is when we have our probe potential greater than the plasma potential here. We're going to attract all the electrons. We're going to repel all the ions.

And so we're just going to end up with basically the same current. So I is going to be roughly equal to the electron saturation current. Now, in general, the current actually generally starts to rise. And it still increases with voltage weekly, for reasons that we're not really going to go into here. But it's quite difficult to get the mathematics for this correct.

We don't tend to work in this regime. We don't tend to bias the probe such we're drawing the electron saturation current, and we'll talk a bit about that later on. And so we're not going to go into too much detail about why that happens. But that's why it's that way on the camera.

There's also the regime B here. This is where V0 is less than the plasma potential but greater than the floating potential. And here, we start getting into a region where some electrons are repelled and some ions are attracted. And that's because the probe is charging up more negative, and so it wants to repel things here. We will go into trying to find an analytical formula for exactly what this curve looks like, but just qualitatively, this is why we have a decrease in the current here.

And then finally, we have this regime, C. And in this regime here, we have a potential on the probe which is less than the floating potential. And we're going to repel all the electrons and we're going to gather all the ions. And we will then be drawing the ion saturation current, which is 1/4 ea ni ui bar.

So this is just the counterpart for the electron saturation current. And of course, it's smaller by the square root of the mass ratio or something like that, probably. So it's a fair bit smaller than the electron saturation current here. Now, the problem with this picture, and the reason why we can't be more quantitative is at the moment, all of our solutions-- so ise and isi-- they're functions of things like ne, ue, and ni, ui. And they're functions of these at the probe surface.

That's not particularly useful because what we'd like to do is relate these quantities to the parameters inside the plasma-- things like the density and the temperature and things like that. And so these measurements of the properties of the plasma right at the probe surface aren't particularly useful. We need to have some theory that links-- excuse me.

We need a theory to link the probe quantities to the plasma quantities. And that theory is unfortunately sheath theory. So we're going to have a go at deriving quantitatively some of the quantities here that I've just sketched very qualitatively. Before we go on, please ask any questions.

So who can tell me what the length scale for this sheath is? What's an important length scale in sheath theory? Yeah, just please shout it out.

AUDIENCE: Debye length?

JACK HARE: Yeah, Debye length. I never quite remember how to spell Debye. I may have put an extra E in there. Maybe this one doesn't exist. OK. So this is often written as lambda Debye. It's equal to, in SI at least, epsilon 0 te over e squared ne to the 1/2.

It's an important length scale for sheaths. But as you probably know, it's not in, for example, MHD, or other simpler, fluid pictures of the plasma. It's just important to sometimes realize the limitations of some of the other models we use.

So you can't model how a Langmuir probe works using MHD. It just doesn't work. And what is important about the Debye length? What length scale does it represent? What's a physical intuition we have for it?

AUDIENCE: Distance over which voltages are shielded.

JACK HARE: Yeah. I guess we can say within a few lambda Debye, the probe perturbation vanishes. So although we've stuck this probe in, and it's going to be at a very different potential to the plasma, the plasma won't see it over a few lambda Debye or so.

And if we take some cold plasma-- so let's say it's got a temperature of about one electron volt. It's got a density of around 10 to the 17 per meter cubed. This will give us a Debye length on the order of 20 micrometers here. And so for any reasonable probe which we could construct which has some characteristic length scale here of A, this Debye length is going to be very, very, very small.

So we can say that lambda Debye divided by A-- lambda Debye is much less than A. And this leads us to being able to do in most of our sheath theory what we call a quasi planar assumption. So we don't have to worry about the shape of the probe. We just worry about some infinite plane. And so we can treat this like a one-dimensional situation.

You will see the sheath equations are complicated enough without doing it in 1D. So this is a nice simplification to be able to do. So the sheath equations to solve this properly-- well, you need to start doing things involving Gauss's law and the Poisson equation.

And it turns out that when you start putting this properly, these equations are non-linear. You're going to need a Boltzmann factor inside here as well-- minus eV over Te and stuff like that. So these equations look very non-linear. They've got exponentials in. They've got gradients in, all sorts of stuff like that. And that means they're very complicated. They're still complicated with big assumptions.

So one assumption that we almost always make in sheath theory is that the ions are cold. They've got a temperature of 0. And obviously, that may not be very well justified in your plasma. But it allows you to make a lot of progress. This actually is relatively well justified in the sorts of low temperature plasmas you tend to stick Langmuir probes in. But I'm sure that you can think of a plasma where the ion temperature is not 0 or isn't much less than the electron temperature.

But what I'm saying is that even if we put in these very big simplifying assumptions, the sheath equations are still very complicated. And so what I'm not going to do is give you a complete derivation of the sheath equation. This is what I did last time I taught the course two years ago, and I've never seen a class lose interest more quickly. It was a complete disaster.

So I'm going to give you a very high level overview. I'm going to tell you the results of it. And I understand that might be unsatisfactory. And so I encourage you to go look at Hutchinson's book or Lieberman and Lichtenberg and learn a little bit more about it if you want to.

At the end of the day, I think these Langmuir probes are a tool. We should understand how they work in some limitations, but we just want to know what the answer is. So we're going to plow on a little bit. Any questions on any of this before we keep going?

OK. So this is my one-page brief overview of sheath physics. What I want you to bear in mind is we are trying to get-- what we want out of this are the electron density and the electron velocity at the probe and the ion density and the ion velocity at the probe, in terms of the density far away from the probe, which we'll call as infinity, and things like the temperature of the plasma here. So this is what we're going after. So whenever I write down an equation, think to yourself, did that help or did it not help?

OK. So I am going to draw a little sketch, which is going to take me a little while to get right here. On the right-hand side of our domain, we're going to have a wall, which is our quasi-planar probe here. And we're going to have a coordinate pointing from left to right, which is the x-coordinate here.

And there are a few important places on this x-coordinate. There's infinity, which is very far away in the unperturbed plasma. There is a point that I'm going to label s here, and there's a point that I'm going to label 0. So 0 is the probe. This region between s and 0 is the sheath. This is the imaginatively named presheath. and this is the plasma.

So again, what we want to do is take measurements at the probe and link them back to the properties in the bulk plasma without worrying about what this sheath is doing. But what the sheath is doing is very, very important, so we need to work that out first. So first of all, I'm going to sketch the voltage. And so this voltage is going to be flat in the plasma, and it's going to have a value of 0.

And we are just defining the voltage in the plasma as 0 because that's our reference voltage here. The voltage is going to drop off ever so slightly in the presheath here. And I'm really exaggerating, but it's going to drop off a little bit in the presheath. And then it's going to drop down to whatever potential it is at the probe here, which is the bias point that we put our probe at. And we call that V0, and we call the potential at the edge of the sheath Vs V sheath.

And I could write V infinity here, but again, we've decided that's going to be 0. So we're going to measure everything with respect to that. And you'll see in the picture I've drawn here, everything is negative with respect to that.

OK. The other thing I can draw-- and you can imagine this is a separate axis down below here-- is the density. So out in the plasma, we have a density of ne, and that's equal to ni, and that's equal to n infinity. So I've invoked quasi neutrality inside my plasma here. There's no change in the potential, so there's no change in the electric field, and so there's no difference in the number of charge carriers.

And I'm assuming that z equals 1 here. This is a hydrogenic plasma. Just for simplicity, of course, I can put z back in if you want to make life more complicated. Then in the presheath region, this is a little bit subtle. We are saying, yeah, the potential is changing, but it's not changing that much. It's quasi-neutral-ish.

So the density is going to-- turns out it drops a little bit. But we're still going to say ne is roughly ni, but it's now no longer n infinity. Again, we can do that more rigorously if you look in the book. For now, we're just going to go with it. And then finally, this density splits so that we get the ion density and the electron density.

And this is happening because in this region where the potential is changing very fast, we have strong electric fields. We no longer have quasi neutrality. We actually have electrons being repelled so there's fewer of them. And we have plenty of ions here. And so this picture is obviously applying in the region down where our probe is biased low enough that we're repelling electrons and we're gathering ions. And I'll justify that in a moment.

So this sheath forms. Let me just think about this. So we only get a sheath when our V0 at the probe is less than minus Te over 2e. E So that's T subscript e. I'll try and write that a little bit more clearly.

This is the first result that is completely non-obvious. I have not derived for this. This is just true. So we might think, where are we going to get some of these potentials in our system? What does our system have? Well, the only thing it's really got is a temperature.

And we often measure the temperature in electron volts, so you might be thinking to yourself, a-ha. Maybe the voltages that are going to show up in this solution are going to do with the temperature of our plasma. And you're right, but the factor of 2, that's not at all obvious. It's just you can work it out.

And so we only get a sheath forming when we bias the probe sufficiently negative, and that is because the voltage at the sheath-- so this Vs here is exactly minus Te over 2e. And so if you bias your probe above that potential, you don't get a sheath forming. All of this sheath physics falls apart here.

And later on, we're going to show that Vf-- let me think how to write this properly. We're going to show that V0 minus Vf is about 3Te or so. So this means that our probe has to be biased somewhere around about Vf, or at least significantly lower than the plasma potential here. So we're going to be operating near the floating point.

And there's other good reasons why we want to operate near that floating potential as well, but skipping ahead. That's just a preview there. So that's the first fact that we now know, is that our sheath is going to be at minus Te over 2e like that. The next thing we're going to say is that the ions are accelerated across the sheath.

So although we know the potential at the sheath, the potential at the probe is still arbitrary, so this isn't particularly useful. But we can make use of the continuity equation to actually make some rather powerful arguments here. So continuity is going to say that the flux of ions which are hitting the probe, which is the quantity that we're measuring with our probe. So this is ni at 0, ui at 0. That is simply going to be equal to the flux of ions which is crossing the sheath boundary here.

So ni like this and ui like this. This actually turns out to be very, very powerful because it means we don't need to know what the density and the velocity are at the probe itself. We just need to know what they are at the sheath. And the reason that's useful is because we can write the density at the sheath here as roughly equal to the electron density of the sheath.

Remember, we're in this quasi-neutral regime down to here. And so at this point, we're saying, eh, the potential drop is small enough that it will cause a neutral. So we can still use quasi neutrality to write the ion density in terms of electron density. And the electrons we're going to treat with a Boltzmann factor here.

So it's going to have the density that they would have out in the bulk plasma, but reduced by a factor of exponential e times the sheath over Te here. And if we plug the potential of the sheath in here, we'll get n at infinity, exponential of minus 1/2. And if you're wondering what exponential of minus 1/2 is, it's about 0.61.

I'm going to leave it as exponential, as minus 1/2 in all of these calculations, just so you don't ask, where did 0.61 come from? But it is just 0.61, so it's about a half. I'm just going to keep pushing on with this and then I'll take questions, because some of them may become clear. The next thing we're going to do is conservation of energy.

And we're going to assume that our ions start off with a velocity of 0. So let me rewrite that. I'll move this over here. I say that my ion velocity at infinity is equal to 0. So this is the cold ion approximation I talked about earlier. We're going to be accelerating these ions up.

And so then we can say that the kinetic energy of the ions at the sheath-- so 1/2 mi ui of s squared is just going to be equal to the change in potential energy here, which is e times the sheath potential. Remember, we're referencing V at infinity is 0, so there should be a minus V at infinity here. But that disappears. Yeah, Nigel. I see your hand.

AUDIENCE: Are those dashes bullet points or is that a minus 1/2?

JACK HARE: Those are all bullet points. I apologize.

AUDIENCE: OK. Thanks.

JACK HARE: Yeah, no worries. It's a good question. Ta-da. Any remaining minuses are your problem. OK? Good. Any other questions while we're paused?

OK. So from this, we can then infer the velocity at the sheath is going to be equal to the square root of Te over mI to the 1/2, where, again, I have just plugged in our sheath potential into here.

OK. This is rather good. We're almost there. The final thing we might want to know is what is the velocity of the electrons at the probe surface? So this would be ne of 0, and that would be equal to n infinity exponential minus e V0 over Te. So that's going to depend.

In this case, for the electrons, we're going to make it depend exactly on the probe potential for the ions we made use of this trick using the continuity equation to avoid actually knowing what their density was at the probe. Because it's clearly no longer equal to the electrons, and so we can't use the electron Boltzmann equation of state in order to get it.

So we're going to put this together on the next slide, but I just want to point out right now, I think we have now written all of the quantities for the electrons and ions at the probe, in terms of quantities in the bulk of the plasma. So this is extremely powerful because we can now use that relation. And I saw a comment in the chat here, which I'm just going to read.

We're assuming T is constant through the sheath. We are assuming that T is constant through the sheath. How is this apparent or valid? For example, keeping pressure constant. I believe the electric field will contribute to the pressure balance here, and so you don't have to worry about just the thermal pressure being involved. We've got electric fields involved here as well. OK. Any other questions?

OK. So now if we have our probe-- so this is for V0 less than Te over 2e, which is the condition that all the sheath theory requires. And just remember, V0 here is V of the probe. Hutchinson starts using Vp here to mean the voltage of the probe, but then you might also think that's the voltage of the plasma, and I find that very confusing. So I'm writing it as V0 here.

But we can now say that the current that we draw-- so I drawn is-- and we can split it up into contributions from the ion current. So this is just going to be equal to minus e times the area of the ions are passing through, ni of 0, ui of 0. And we can just replace that with ni of s, ui of s, using continuity. And we have that on the previous page.

And I just want to point out, when we previously wrote down this equation, we had a factor of 1/4. That 1/4 has disappeared because, in fact, we have no random motion of our ions anymore. We set their initial temperature equal to 0. And so this is now all directed. So our geometric factor is not irrelevant because now all the ions are going in the same direction.

So previously, it was 1/4 to account for random motion. Now it's just 1 because they're directed like this. And that means we can write down the ion current as minus the exponential of minus 1/2, which is just 0.61. The area times the electron charge times the electron ion density of infinity times the incoming velocity, which is Te over mi to the power of 1/2.

I just want to make a quick point that I'm not going to labor too much. This area here is technically the area of the sheath. However, that is roughly the area of the probe with corrections on the order of lambda B over A.

And we found that lambda tends to be very, very small. Lambda over A tends to be very, very small, so we don't really worry about those corrections. But if you end up in a regime where that's not true, you might want to worry about this a little bit more. So that's the ion current. Any questions on that?

The electron current now is going to be a factor of 1/4 coming back because the electrons are randomly moving. Some of them are being repelled, but in general, we're just dealing with a random Maxwellian distribution function. And then we have these factors of e and A. And then we have the density at the probe and the velocity at the probe here.

This A is genuinely the area of the probe because we're working with the x-coordinate set to 0 at the probe. We worked out what the density was of a Boltzmann factor. The thing that we didn't have is this. And this is just going to be Maxwellian. And so we use the standard result from a Maxwellian temperature here.

So this all comes out as 1/4 times e n at infinity exponential of e V0 over Te. And then there's this factor of the Maxwellian, which is 2 times 2Te over pi me to the 1/2. So it's slightly ugly, but that's the average or the mean velocity in a Maxwellian distribution here.

And then we can put this together. We can say that the total current that our probe is now drawing is equal to the ion current plus the electron current. And this is Hutchinson's equation. Gosh. I didn't update my notes, so I don't have it. But where is Hutchinson when you need him?

I don't have it straight away, unfortunately here. My memory of it is it's equation 3.2.23. And perhaps someone could check that.

AUDIENCE: I think it's 2.29.

JACK HARE: 3.2.29?

AUDIENCE: Yeah.

JACK HARE: Thank you very much. I appreciate it. OK. And it's this long thing. So the reason I'm giving you the equation citation is if I make a mistake copying this down, you should go refer to Hutchinson instead.

So what I've got is n infinity, electron charge, area of the probe, square root of electron temperature of the ion mass. And all of this is now times a component to do with the electrons, which is 1/2 times 2 mi over pi me to the 1/2 exponential e V0 over Te minus the ion contribution, which is exponential of minus 1/2. So this is e minus, and this is the ions here.

So this looks a little bit complicated, but let's have a little think about it. This ratio, mi over me is large. That just depends on fundamental quantities. But this exponential here is going to be small because we know that V0 is less than Vs. So it's less than minus Te over 2e.

So this quantity here is going to be less than 0.6. And although this is large, it's large like 40 or so. And we can imagine if we decrease the voltage on the probe even more, the small thing is going to be smaller than the large thing is large. And we can see how the electron current could be completely extinguished and we're left with just the ion current. Yes, Sean.

AUDIENCE: In Hutchinson's version, he has the ion term multiplied by the sheath area times the probe area. Are you just saying that that's about 1?

JACK HARE: Yeah, exactly. So he puts in this ratio. And like I said, it is important. But it's one of the things I'm glossing over in this class. But you're quite right. That's there. I just said, area of the sheath or the area of the probe equal to unity.

AUDIENCE: Got it, thanks.

JACK HARE: Cool. Any other questions? This is our big result. But we're now going to go use this to learn about what happens to Langmuir probes. Nigel?

AUDIENCE: Have we discussed how you find the area of the sheath? Or is that just a theoretical?

JACK HARE: So what I've said here is that the area of the sheath is basically the area of the probe.

AUDIENCE: Yeah, I mean beyond that.

JACK HARE: We haven't discussed that. There's a long discussion in Hutchinson about it. Effectively, there are corrections on the order of the Debye length to it, which may or may not be important. And it turns out if you bias the probe very negative, deep into the ion saturation region, the sheath actually continues to grow. And so you continue to gather more and more ion current.

And that's one of the reasons it's actually very hard to measure the ion saturation current because it's not like a flat asymptote. It's just a slowly growing function. So if you ever sit down and work with Langmuir probe data, any of you, you'll find it's incredibly frustrating because all the stuff in the textbooks doesn't really work.

And last time we did this, we did have some Langmuir probe data to analyze. But you know, I thought you guys would prefer to have full problem sets in 5, and so I cut it this time. If anyone's like, give me that Langmuir probe data, I'll happily send it across. You can have a play with it. It's from Dionysus, which is Kevin Waller's machine across the street in NW14 or whatever.

OK. So let's just check that this long equation matches what we thought we were going to get. So if we put v zero much, much less than minus t over 2e, we're going to get no electron currents. And that's due to this exponential here is just going to go to 0. And so I is going to be roughly equal to the ion saturation current.

And the ion saturation current, we can now see, is the density at infinity e times the area of the probe Te over mi to the 1/2 exponential minus 1/2. So that's cool because we know all of those parameters, apart from the density and the temperature. And those are things we want to measure. So we can't work out from the ion saturation current exactly the density and the temperature, but we know the density times the square root of the temperature. So maybe if we can find out the temperature from somewhere else, we can crack this whole thing open.

I keep clicking on the wrong monitor because I've got basically the same thing shown on three monitors. So now I've moved on to my next set of slides, but before we go on to interpreting the probes and actually measuring density and temperature, does anyone have any further questions on this key result?

All right. So now we're going to go on to interpretation of probes. So let's have a little drawing of our probe. It's going to have some tip like this. That tip is going to be surrounded by some insulator, and then that, in turn, is going to be surrounded by some sort of shield.

So this is a cutaway here. It's a cutaway of coaxial geometry insulator and tip. And if I draw this face on, it looks like, as I said, coaxial something like this. So you can have a lot of fun making Langmuir probes. But they are fundamentally little bits of cut-off coax cable.

And there's some reasons why you might want to use them made out of certain materials. So we tend to want to work near the floating potential. So we want to have the Vf-- our pro voltage be 0 close to Vf. And there's two reasons for that. The first of all, as we've seen, is that we actually have good theory here.

We have the sheath theory. We don't really know what the theory looks like in other places. Maybe we can go work it out, maybe we couldn't. So it's nice to work near the floating potential. The other reason is that it limits currents. And actually, this is a big problem because if you draw a large current, you get large heating. Or let's say this avoids heating.

Because if you're sticking your probe inside a plasma, it's already going to get hot. And if you start drawing a large amount of current, it's going to get even hotter. So you don't really want to work in the electron saturation current region where the current is huge. You want to work down where the current is basically 0 to avoid melting this.

Even then, you're going to be making the tip of this out of something like tungsten, which has a very high melting point. Your insulator is going to be some custom ceramic, and your shield will probably be tungsten as well. So you have to design these things very, very carefully not to melt.

Cool. So what we're going to do is we're going to sweep V0 near Vf. So you can imagine that we have control over the potential of our probe with some power supply, and we measure the current through that. So this little circuit is we've got our probe and our little plasma. And we've got some sort of resistor over which we measure the current, and we have some sort of bias that we're biasing the probe to. So this is I0, and this is V0 here.

And we're going to be near the floating potential. And if we go back to the equation that we had back here, and we set the current equal to 0-- so if we say I equals 0, we can rearrange this. And we can find out that the electron charge times the floating potential, which are operating here over the electron temperature, is going to be equal to 1/2 times the natural logarithm of 2 pi me over mi minus 1, all in brackets.

OK. So this is a vaguely complicated looking expression, but of course the only things that matter in here are the electron mass and the ion mass. And if you put those into something like hydrogen, you get about 3 here. And of course, it doesn't matter if it's not really hydrogen because we've got a natural logarithm here. So it changes slowly.

The point here is that this says that your floating potential, Vf-- e times Vf, at least is about 3 temperature units. So if your temperature is one electron volt, then your floating potential is about 3 volts. That sounds super useful, but it's not.

Why is it not useful? We've measured Vf by sweeping it there, and now from this, we've got the temperature. This is incredible.

AUDIENCE: Yeah, but doesn't that just cancel itself back out in that equation?

JACK HARE: I'm not sure it does that. Maybe you're thinking about this in the right way. The question would be Vf relative to what? We had this whole debate at the start. So if you measure Vf in your lab as 10 volts, but you don't know what V in the plasma is so you don't know whether the 10 volts that you're measuring-- what's that reference to? And that's reference to your building ground.

But you need to have Vf reference to the plasma potential. And you don't know the plasma potential. So this is actually pretty useless. So without the plasma, we have no reference for Vf. This is useless.

You could try and do it. If you remember, if I go all the way back in my notes to my little sketch here-- you could say, well, look. I can measure Vf. Down here is where the current goes to 0. And then I could measure the volts between Vf and Vp here, V plasma. And I'll define the plasma as when the electron saturation current happens-- so when this curve rolls over and starts to saturate, I'll call that V plasma.

And the difference here, that'll be delta Vf. And that'll be 3 times the electron temperature. So you can do it crudely like that. The trouble is that as I've sketched it here, this doesn't really roll off. It actually just keeps going up like that. And if you look at some real data, it's usually even worse than that.

So it's a super inaccurate way of measuring the temperature. But you could use this. It's fine. The other problem, of course, is you'd have to bias your probe into electron saturation region and risk it melting.

So good reasons why you wouldn't want to do this technique, but maybe you want to if you're desperate. But don't worry, there's still better things we can do. So again, we could use Vp at I equals the electron saturation current, but that's inaccurate.

So we don't tend to do that. What we do instead is an alternative technique where we look at this equation very hard and we think to ourselves, hm. How does I change with V0 here? So we can write down analytically dI dV0.

And maybe you want to do this around about the floating potential here, because that's where we want to operate where we've got all our nice sheath theory, where we've got our nice equations. And we find out that what we get out is e over Te times I minus the ion saturation current here. Now, when you do this, you also get a little term that looks like-- how does the ion saturation current change with the probe potential? And this is due to the fact that the area of the sheath is not exactly equal to the area of the probe. But for our purposes, we're going to assume that is negligible.

So we're just going to set that extra term equal to 0 here. And if you look at the first term, and you think about how to rearrange this, you can see that the electron temperature then can be written as e times I minus I sI, and that is divided by dI d0, like this. And so if we look again at our plot of our ID characteristic for this probe, where we have the floating potential here, and we have the plasma potential up here and the electron saturation and the iron saturation, we're working with the slope of the curve in this point here.

And what we want to do is a two-step process where, first of all, we fit the natural logarithm of I minus I Is versus V0, and that is going to give us the electron temperature. So you can probably just stare at the equation for electron temperature and see where that comes from. And then once you've got the electron temperature, we can measure the ion saturation as well.

And we know that the ion saturation current is equal to exponential of minus 1/2 e A0 n infinity Te over mi to the 1/2. We've just measured Te from this, so now we can measure density. So we need to measure the slope here, and we need to measure the ion saturation current. And we can do that just by sweeping V0 in this narrow range here from the ion saturation current up to just above the floating potential.

And we never draw very much current because the ion saturation current is very, very low. And that gives us out the temperature and the density, which is pretty remarkable for something that we've just stuck in the plasma that's just a little rod of metal. OK. So that's how you use a single Langmuir probe. Any questions on that?

AUDIENCE: Do we have a name for the voltage at which the ion current saturates?

JACK HARE: No, we don't.

AUDIENCE: OK.

JACK HARE: Yeah. I'm trying to think what it is. Effectively, we want to have this exponent be very small. So you could write something to do with this being less than 0.61, and it has to be-- I don't know. Let's say it's 10 times less. And then you could define some voltage at which it's 10 times less, and we could call that the ion saturation potential.

But I haven't seen that in the literature. But you could work out what it needs to be. So you could say, if I think I know what the temperature is, I know how much I have to bias my probe negative.

The other thing is just to bias your probe negative until it starts to roughly asymptote. But as I said before, it never actually asymptotes. What happens is it just keeps going down. So it's a real pain in the ass to measure this properly. OK, good. Daniel, I saw your hand first.

AUDIENCE: Yeah. So I thought I had an idea of this until a moment ago, and then realized maybe I don't. But where's the return current in this? Because you can put-- my intuition is you put a voltage on this and it'll do stuff initially. But if you wait long enough, it'll settle to some other charge where you've just charged the plasma slightly.

JACK HARE: Yeah. So the plasma is going to be drawing electrons back off from the vacuum chamber or from the vast infinity of the universe.

AUDIENCE: OK. So it is from just other places where electrons can come from.

JACK HARE: Now, you've got to remember that all the way around the vacuum chamber, there is a sheath. And that sheath obviously has balanced and electron currents. But the area there is so huge in the vacuum chamber that if this probe is locally drawing a few more electrons then the rest of the vacuum chamber can just push-- slightly fewer electrons, and it'll all work itself out. And as we know, quasi neutrality is pretty strictly enforced by plasma because the electric fields get big. So it's no trouble at all to get those electrons back from somewhere else, yeah.

AUDIENCE: OK, cool. That makes sense.

JACK HARE: Good question, yeah. And Nigel, I see your hand.

AUDIENCE: So the practice of actually doing this, besides just sweeping the ion saturation current regime near the floating potential-- you also need to then sweep positive enough to figure out what the plasma potential is? Is that correct?

JACK HARE: Oh, no, sorry. With the previous technique, we were talking about if you just want the temperature, then you would have to get the plasma potential. The beauty of this technique is you don't have to go anywhere close to the plasma potential. You're just sweeping.

You just need the slope at this point. So you need di, d, and d0 at V0 equals Vf. And so of course, to determine that slope, you just need to sweep far enough that you can fit a straight line and you're happy with that.

AUDIENCE: And so the benefit of this, like we were saying, is that we don't need to reach high currents, which could screw with theory?

JACK HARE: Yeah. So that's one thing. And the other thing is the theory-- yeah, so actually, sorry. You said it there. We don't want to reach high currents where our theory is invalid or high potentials where our theory is invalid. And we don't want to reach the high currents as high potentials, where we draw lots of current and melt our probe.

AUDIENCE: Got it. Thank you.

JACK HARE: The melting of the probe thing probably sounds a bit funny, but this is genuinely a huge problem, which is why I keep saying it. So we must operate at the lowest currents possible.

AUDIENCE: Oh, no. I work with bias probes.

JACK HARE: OK, great. Cool. Nice. Any other questions on this before we go on to some more advanced Langmuir probes?

AUDIENCE: Yeah.

JACK HARE: Go for it.

AUDIENCE: Sure. So how do you measure the ion saturation current if it keeps on going down, like you said?

JACK HARE: Yeah. So it drops for a bit-- sorry, you can't see my mouse on my screen. It drops for a bit, and then levels off. And so you just eyeball it as like, it's here. Maybe you would fit some sort of curve.

But it's not easy to do, so that gives you a lot of error. And that error won't affect your interpretation of Te, but it will affect your interpretation of the density.

AUDIENCE: Why wouldn't it affect your interpretation of Te if it depends on Iis ?

JACK HARE: Oh, you're quite right. Yes, you're quite right. I guess it's inside a logarithm, so it's less important there. But it will still have some effect. You're quite right, yeah. I just want to point out-- once when a student did this exercise, they forgot to subtract the ion saturation current. And this doesn't really work properly there.

So this is a common pitfall. Make sure when you're doing this, you subtract the ion saturation current from your current curve. So it should look like that, right? The thing that you're working with that you're trying to find. It doesn't matter for the slope, of course-- or it does matter a little bit for the slope, so yeah.

OK. We're going to move on. How fast are these mechanisms? Oh, thank you. My notes didn't update, and I'd actually made a new note on this that I didn't want to say. So let's talk about limitations.

So one limitation is you might draw I greater than the ion saturation current. So you don't know where Vf is to start with. So when you put your probe in and you start adjusting the voltage, you might accidentally draw too much potential and melt the probe. And we'll look at some other designs which are safe and don't ever do that. And so this is a disadvantage of this design.

The other disadvantage is the sweep time. So we're talking about literally sweeping the potential. I wish I could draw straight lines. There we go. So if we've got time here, and we've got voltage here, and this is the floating potential, maybe we have a sweep pattern that looks like this.

But it takes some time, delta t, to sweep. And that might be limited by how quickly your voltage source can sweep voltage. But it might also be limited by how quickly your digitizing can digitize data, because effectively, as you do this sweep, you're digitizing this curve here. And you want to have enough data points on that to do all of your fitting.

And so you are limited. You're very much temporarily limited. If there's an event which takes place on a time scale that's faster than this, you won't be able to resolve that. And in fact, it will do horrible things to your data analysis. So you want to make sure that your plasma is not fluctuating on rapid time scales. Yeah, Daniel. I see your hand.

AUDIENCE: Yeah. Do you also run into situations where the actual response of the plasma is ever a limiting factor for your measurement time there?

JACK HARE: So response being this whole setting up the sheath and things like that?

AUDIENCE: Yeah.

JACK HARE: Yeah. I think that happens really quickly. I think that's going to happen on electron time scales over a Debye length. And so those are probably very small time scales. So I would guess that delta t of the perturbation is going to be on the order of lambda d over VTe. That's the sort of time scale I can think of. And that seems like a really short time scale.

In fact, I know what that time scale is. It is 1 over the electron plasma frequency. And so that's really fast. So maybe in some plasmas, you could reach that timescale. But probably that's all going to happen very quickly. So I think from the point of view of the plasma, it responds instantaneously to the change in the potential on your probe, and we don't have to worry about that. But if you have a really funky plasma, that could be problematic.

AUDIENCE: OK, cool.

JACK HARE: I think omega p is probably the fastest timescale-- one of the fastest, probably the fastest timescale we see in most plasmas. Did I see another hand? OK. I'm going to keep moving because I've got a few more things to get through in the next 20 minutes or so. So let's power on.

So the next probe we can consider is called the double probe. The previous one, we're going to retroactively rename the single probe. And the double probe-- once again, we've got some sort of plasma. But now we have two probes sticking into the plasma.

And the clever thing is that the two probes are attached together, and they are biased with respect to one another by a bias potential, VB. And again, we measure the current. This is just like a resistor. So I can measure the voltage across this resistor, and I'm going to get out the current here. And that goes to an oscilloscope.

Now, the cool thing about this setup is that if probe 1 draws a current I1, then probe 2 is going to have to discharge a current, I2. And from Kirchhoff's law, we know straight away that I1 plus I2 is equal to 0. So any current that comes in one probe has to be ejected out the other probe. There's no other place for it to go.

This probably addresses Daniel's question about the plasma accidentally charging up. It turns out-- and Hutchinson talks about this in great depth in his book-- that almost every probe is really a double probe. It's just that either you have a double probe or you have a single probe, and the entire vacuum vessel as the other probe surfaces. And I'm not going to go into that in much detail, but that's a reasonable thing to think about. Yeah, I see a question from Nigel.

AUDIENCE: OK. This might be a little out there. But effectively, because they're in a plasma, there is a pseudo resistor that we could draw on a circuit diagram between the two tips of the probe. Is there a problem with that creating a closed loop now? And that magnetic flux could go through and then make Kirchhoff's law not entirely valid.

JACK HARE: What do you mean by magnetic flux? Because we haven't really talked about magnetic fields very much.

AUDIENCE: I guess I've been assuming that these are on a tokamak this whole time.

JACK HARE: So is your question, is there a problem if there's a current flowing inside the plasma here?

AUDIENCE: No, it's more like if those are connected and there's a resistance between them, if we're just looking at this like a circuit, we've created a closed loop. And so now if you get a change in magnetic flux through the loop closed by that, you could induce an EMF that could affect--

JACK HARE: Yeah, OK. I see what you're saying. I see what you're saying. We're not actually measuring the potential on the probes, though. So we're measuring the current through the probes through this resistor. So if we induce a voltage around the whole loop, I don't know whether we would see that show up across this resistor.

That's a really interesting question. I'm going to say that I don't know the answer, and I haven't thought about it much. So I'll certainly have to think about it and get back to you if I have any interesting thoughts on it. But I get what you're saying.

You're saying, potentially, this could have some sort of plasma current channel here. And then that looks like a little V dot probe that we've stuck in the plasma, and something could induce-- yeah, I see what you're saying. Cool. I'll have a think about it. Cool.

OK. And so there's other things that we can say about the currents here. So we know that the current going into-- excuse me-- probe 1 is going to be equal to the ion saturation current plus the electron current modified by a factor e V1 over Te. So that's just our Boltzmann factor.

And the current through probe 2 looks very similar, except that we modify this now by e V2 over Te, like that. So we haven't used Kirchhoff's law yet. This is just saying if we had two random probes, this is the current that we would draw.

Now we use Kirchhoff's law here, and we note the nice thing about this is that the current through any probe is now strictly less than the ion saturation current here. So we can't accidentally draw the electron saturation current and melt probe. So this is a definite advantage of this system. I'll do this a little bit more mathematically in a moment if you're not convinced yet.

So then what we do is we continue and we say, we've got these equations separately for I1 and I2. We've got this equation that links I1 and I2 together. And we're also going to operate this so that the probes are floating near the floating potential. In fact, that will happen as soon as we put these probes into the plasma.

We're not biasing the probes themselves. We're biasing them with respect to each other so the whole setup will float towards the floating potential. And that means that our ion current is going to be equal to our electron current, like this. Then we can put all of this together, and we can say that I1 is equal to Isi 1 minus exponential of e V1 over Te. So that's just using this fact here.

And then that is going to be equal to minus I2 from Kirchhoff's law. And so that's going to be equal to minus Isi 1 minus exponential of e V2 over Te. And then we're going to say that this bias voltage that we're applying is simply, by definition, the difference between V1 and V2. And if we do a little bit of magic on this-- and this is actually an exercise in Hutchinson's book, which I had to do to try and work out where on Earth this came from. You find out that the current that you're measuring, which is the current flowing into one probe and out the other probe, is going to be equal to the ion saturation current times by the hyperbolic tangent of e times the bias voltage over 2Te, like that.

And so now if I sketch the IV curve for this, this is B bias and this is I. I end up with something that looks like-- if I could draw straight lines. Give me a moment. OK. And so this is asymptoting at the ion saturation current, or at minus the ion saturation current here.

So this is sort of the proof of what I said earlier, that we can't draw more than the ion saturation current here. And this is very nice and symmetric. And I'll show you in a moment how we use that to actually measure all the properties of the plasma.

So I'm happy to take questions on this, but if you're like, I don't understand the derivation, the reason is I didn't really do the derivation. There's lots of steps missing here. I'm just giving you a couple of intermediate steps. And if you want to go get it, you should go work through the exercise in Hutchinson's book.

I'm going to try and write this on this page because I think it'll be more useful. If we have for small bias voltages-- so we're operating just in this little regime here. We find similarly to the probe that we had before that di d the bias around B bias equals 0 is e times Isi over 2Te. And we're also going to be able to measure Isi by sweeping just a little bit larger, like this. And so we'll get, again, Isi is equal to whatever our equation we had before, which has a proportionality to ne and Te to the 1/2.

So from this probe characteristic here, we can now get out both the density and the temperature-- again, from looking at the gradient near the bias voltage of 0 and looking at the ion saturation current as we did before. But the advantage of this is that we can no longer draw a large current by accident. But the limitation is still the sweep time.

So we still need to sweep this bias voltage up and down so that we can trace out this tank function so that we can fit it. So you still can't resolve very fast moving things, but this at least is a nice and safe probe. So any questions on this?

So no prizes for anyone who guessed after the double probe we go to the triple probe. I promise I will stop after this. So a triple probe is simply a double probe whilst a floating probe. So let me sketch that and explain what that means. That's meant to read floating.

So we have our double probe as before, like this. And we bias it with some bias potential. And we measure the current across a resistor. But we also have-- stuck somewhere nearby, we have our single probe. So that is by floating. There we go. OK.

Sorry. For this probe, we actually don't measure the current through it because we're going to leave it floating. So we actually measure the voltage that it's at here. So for the double probe, we set our bias voltage to be greater than a few electron temperatures. And maybe this takes a bit of trial and error, but we find it in the end.

And then we find out that one of the probes is going to draw positive ion saturation current. Second probe is going to draw a negative ion saturation current because it's at a potential greater than a floating potential. This one is at a potential less than the floating potential. And finally, the floating probe is going to allow us to measure the floating potential itself directly.

And the reason that this is neat and fundamentally different from the other probes that we've looked at is if I redraw our IV characteristic-- so I and V. And again, we have a floating potential. We have a plasma potential here. We have the ion saturation current and we have the electron saturation current. These three probes are actually representing three different places on this curve.

So probe number one is representing a place down here, probe number three is representing this point, and probe number two is representing the opposite point up here, where we're drawing minus the ion saturation current here. And this means we're measuring three points on an exponential curve. And three points on an exponential curve is enough to specify it. And so we can just fit the IV curve, and we can get out straight away what the density is and what the temperature is.

And we can do that without sweeping any voltage here. Remember, we are not sweeping the bias voltage here. We've just set it to a few times eTe. So the whole system floats near the floating potential. So we can't draw the electron saturation current by design.

We also have fast time resolution, because now we're no longer sweeping anything but just digitizing this in time. So we can resolve any short-lived phenomena. But the only problem with this is that it assumes implicitly that your IV curve model is good. And I think that is a big assumption.

So you've only got three points on there. So if it starts deviating from exponential, you're effectively not overfitting, but precisely fitting it. And so you won't be able to see any deviations. And that's when people start using quadruple probes, and all sorts of exciting things like that, which we're not going to go into. But you can keep adding probes.

And one way to think about adding probes is that you keep adding more points along this line by biasing probes in different places. And if you measure their current and their potential, you can reproduce that curve instantaneously. Of course, if you try and crowd too many probes together in the same place, you're going to cause lots of perturbations and the probes are going to interfere with each other. So this is not flawless, but people do all sorts of clever things with this. OK. So questions? And I see Nigel.

AUDIENCE: Yeah, I wanted to actually ask about the last point you brought up about-- could you talk more about the trade-off between having the probes close together so you ensure that they're measuring the same plasma versus them interfering with each other like they're sheets overlapping and things like that?

JACK HARE: Yeah. It's fair enough. So I can talk a little bit about it, but I'm not an expert. So one thing I'll point out is our sheath thickness is going to be a few Debye lengths. So you might think, well, that's OK. I can keep my probes more than 60 microns apart or something like that.

The trouble is that there's actually a region here called the presheath, and although we tried our absolute hardest not to model the presheath very much, if the presheath starts to overlap, it can be a big deal. And the presheath is much larger than the Debye length. It's maybe hundreds of Debye lengths. In fact, it's all a bit subtle. And if you start looking at Hutchinson's book, you realize that I haven't mentioned collisions or magnetic fields at all, and these make this extremely subtle and complicated to work out.

So you really want to have your probe spaced far enough apart the presheaths don't interfere. So if I was doing this in an experiment, I would have one probe, and then I'd put another probe in next to it and see if my signal on the first probe changed. And if it changed significantly, by 10% or more, I would say those probes are too close together. And so you might have to do this a little bit quasi experimentally in order to work out how close you can put all the probes.

AUDIENCE: Thank you.

JACK HARE: You're welcome. Any other questions on this? I have just got two bullet points before we finish up. And these are just due to what we would call added complications.

So the first complication are magnetic fields. And you definitely might want to use a Langmuir probe in a magnetic field. You might want to put it in the scrape off layer of a tokamak or something like that. And the big problem with magnetic fields is they alter particle orbits.

If you have very strong magnetic fields, this may not seem like too big of a deal. Here's my probe here. I've got a magnetic field that's oriented like this. And I'll say, OK, well, all my particles are going to gyrate around this magnetic field.

And so what you end up with is a sheath that's very close in like this, but you end up with a presheath that is effectively swept out by the projection of the probe area along this magnetic field. And you have particles which enter the presheath and then travel down the magnetic fields until they hit our probe. So I've probably drawn too many lines, but this is the rough picture here.

So this is the presheath, this is the sheath, and this is the probe. It turns out that if you assume your plasma is completely collisionless, this all falls apart because you need some process to knock the particles from being outside the presheath to inside the presheath to keep fueling the particle flux onto it. And so even if the collisionality is very, very weak, this presheath keeps expanding until it reaches a length scale where the collisionanilty is important.

So the exact size of the presheath is very hard to calculate. And this is only in the case that I've sketched here for very strong magnetic fields. Of course, if your magnetic fields are weaker, your particles are going to be doing things like this, where they spend some of their time inside the presheath and some of the time outside. And then you have to start doing single particle orbit calculations to see how many particles hit your probe, and it becomes very complicated. So probes in magnetic fields are very hard, and people spend their whole careers trying to work on that.

The other thing we haven't discussed is collisions here. So in the sheath theory that I presented, I didn't derive it, as I said. But I had assumed that there were no collisions here so all the particles would just be streaming inwards without interacting. In a real plasma, especially the sorts of temperatures the plasma is at when you tend to work with Langmuir probes, collisions are actually quite important.

And these collisions are going to modify your flux, which, if you remember, was the first thing we wrote down-- gamma. And they're going to modify it to include particle diffusion here. And so you need to go back and rederive all these results with this modified gamma in this case. And again, Hutchinson spends a very long time on this in the book because this is something that he cares about deeply.

And if you're working with Langmuir probes, then one very good resource is to go look at his book. And then Lieberman and Lichtenberg also deal with magnetized sheaths and collisional sheaths in a great deal of detail. So there's more information there. So I'm very happy to take questions. But I actually have to go teach another class virtually in 4 minutes, so please keep them brief.

AUDIENCE: Are there games to play when you sample? Instead of doing a perfect sweep, if you're clever about trying to change the frequency with which you do that, you can get a little bit better idea of how consistent your response is in time or something? Make sure you're not having sampling effect.

JACK HARE: I can imagine you could do all sorts of clever things with some fast sweeps and some slow sweeps and not sweeping the whole range, if you only need a few points to fit the characteristic. So yeah, I imagine there are some things that you can do in order to optimize that. Any other questions?

Cool. Well, thank you very much, everyone. This is likely to be online on Thursday as well, as I won't be able to stop self-isolating by then. And we'll be talking about refractive index diagnostics. So have a good day, and I'll see some of you again in a couple of minutes at the next class. So bye for now.