[SQUEAKING]

[RUSTLING]

[CLICKING]

**JACK HARE:** So today, and for the remainder of this semester, we will be discussing Thomson scattering. Please hold your applause. You won't like it. [LAUGHS]

So Thomson scattering is an extremely powerful and subtle diagnostic, and it can be used over a huge range of different plasma parameters to provide local information about our plasma. And so that makes it extremely powerful in comparison to a large number of line integration techniques that we've seen so far.

The basic idea of Thomson scattering is very simple. We have our vacuum chamber with some plasma inside that we would like to diagnose. And we take a laser beam, and we focus it inside the plasma.

And the light from the laser scatters off the particles in the plasma, and it will scatter off in lots of different directions. Our job, then, is simply to place some sort of detector to look at this scattered light here. That detector is most likely to be something like a spectrometer.

If we use a spectrometer, we can see that the initial laser that we put in, which we will choose to have a very narrow line width-- so it will be at just one wavelength or just one frequency-- that narrow line width will undergo Doppler drift due to the velocity of the plasma.

And it will undergo Doppler broadening due to the thermal motion of the plasma. So this will give us out two key quantities. Cool. Could you mute, please, Yang?

**AUDIENCE:** Yes, I can hear you.

**JACK HARE:** Yes, but could you mute yourself? Because we can hear something in the background.

**AUDIENCE:** Oh, sorry.

**JACK HARE:** Thank you very much. Ah, tediousness of the Zoom. OK, good.

And so this is going to give us things like the flow velocity within the plasma and the temperature of the plasma-- so the ion temperature and the electron temperature. And so this seems simple enough. And yet we're going to spend four lectures on it. And so it is not particularly simple at all.

And, in fact, if you keep analyzing the Thomson scattering, you can also get out quantities like the average ionization state of the ions. We can get out the electron density. We can even start measuring the local current within the plasma.

We can measure the magnetic field locally within the plasma. And we may be able to get out the full electron and ion distribution function, which is particularly important when these distribution functions are not Maxwellian.

If they were Maxwellian, of course, they would fully characterized, once we knew the temperature and the flow velocity and the density.

Right. So the nice thing about Thomson scattering, from my point of view, is it unites a lot of plasma physics. We have electromagnetism that we need to understand to do this. We need this to get the particle orbits and the radiation from these particles.

We need to understand our statistical mechanics. In particular, we need kinetic theory because, although we're going to end up a lot of the time with gross fluid quantities like temperature and flow velocity, in order to get them, we still need to first do the theory using distribution functions, even if those distribution functions turn out to be Maxwellian.

And, of course, this involves lasers. Lasers are just simply very cool. It also unites a lot of plasmas.

So I'll give you some ideas of the sorts of plasmas that we can measure stuff in. If I have a little table here with the electron temperature and the electron density, we can start out with a pretty low-temperature plasma, 1 electron-volt, and also a pretty low density, 10 to 17 per meter cubed.

This sort of density could be a low-temperature plasma used, for example, for converting carbon dioxide into carbon monoxide and oxygen. This is an interesting plasma-catalyzed process. It could be used for doing carbon capture. The plasma catalysis here is because the electron distribution function is non-Maxwellian.

And it's actually the fast electrons in the tail of this distribution which do the plasma catalysis process and make it thermodynamically and economically feasible. But in these sorts of plasmas, people have been able to characterize the non-Maxwellian nature of this distribution function extremely well by repetitively pulsing this plasma and doing Thomson scattering and acquiring signals over hours or days.

And so even in this very low-density, low-temperature plasma, Thomson scattering has given us a lot of information. We could go up to a plasma which is much hotter, 6 KeV, and a density of around about 10 to the 20. This will be a plasma-- the specific example here is TFTR, which was a fusion reactor prototype at Princeton. And this was a D-T plasma.

And it produced a significant number of alpha particles. There was quite a lot of fusion going on. It got to very high temperatures and moderately high densities. Here they did time-resolved Thomson scattering. And they were able to image an event called a sawtooth crash, which is a catastrophic loss of confinement due to reconnection. And they made a movie of this, effectively using the Thomson scattering. So they were able to better understand this process.

And then we can go to similar temperatures here but much higher densities-- 10 to the 27 per meter cubed. And this is a measurement using Thomson scattering inside an inertial confinement fusion hohlraum, which is that little gold cylinder that has 192 laser beams hitting the inside of it to produce an X-ray bath. And it was in this hohlraum with all the lasers coming in.

There's a large amount of plasma on the walls. Remember, the eventual goal is that the X-rays compress this capsule. But the plasma on the walls is pretty hard to diagnose. But folks were able to use Thomson scattering to diagnose the temperature and density here, which is very important if you want to know what your X-ray flux is going to be onto the capsule.

So this is 10 orders of magnitude in terms of density, but it is fundamentally the same technique. So how are we going to tackle this? Oh, no, I've got another bit of reasons why you should care about Thomson scattering. Let's keep going.

OK, so Thomson scattering also changed the course of fusion research forever. Now, I may have hopped on about this a little bit already. Those of you who took 2262 with me will remember the famous or infamous ZETA Reactor which was in the UK. So this was back in the '50s or so.

1958, they say they've got fusion. It turns out that the reason that they think they saw fusion is they didn't understand their diagnostics. So this is poor diagnostic interpretation. They interpreted their neutron spectra as being due to isotropic neutrons when, in fact, it was beam-target fusion.

This, of course, was rather humiliating. But it did lead to a rather good result, which was the development of Thomson scattering, because it was clear that we needed much better diagnostics in order to measure our plasma conditions and actually show that fusion is happening.

We need to know if the plasma is even hot enough to do fusion. The problem with the ZETA machine is it was far too cold anyway, wherever we've got fusion happening. And the British should have realized this before they claimed that fusion was occurring here.

But they did develop this Thomson scattering diagnostic with some of the first lasers. I'll point out that the laser was just invented in 1961. So they didn't really have a chance to do it before then. But after then, there was really no excuse.

And this was very handy because only a few years later, there was a new device, T3. This was in the USSR. And it was something that they were calling a tokamak. Some of you may have heard of this.

And in 1968, these guys claimed that they got temperatures in excess of 1 kiloelectron volt, which was significantly better than any other machine at the time. So this was really remarkable. And, of course, there was a huge amount of skepticism, both from the UK, who felt a little bit burnt after the ZETA incident, and also from the US.

And so there were needed to be a way to verify the claim of the Soviets. And what happened was kind of a remarkable piece of Cold War history, which was a team from the UK went to the USSR with their Thomson scattering diagnostic in a load of crates, consisting of a laser and a load of spectrometers here. And they used it to confirm these temperatures.

And there's a beautiful *Nature* paper by Peacock, the name of the lead scientist on this group, showing the Thomson scattering spectra from the T3 tokamak. And this caused an absolute firestorm across the world of fusion. Instantly, everyone wanted to build tokamaks instead.

So at the Princeton Plasma Physics Lab, they had the model C stellarator, which sort of looks like this kind of racetrack-shaped thing, like this. And within one year, they'd gone, all right, get rid of that. Now look. It's a tokamak. They converted the machine into a tokamak and immediately started to get very good performance out of it.

So if you work on tokamaks, which many of you do, you should thank Thomson scattering. And it's the reason why everyone was so convinced straight away that they were the way to go. All right, question?

**AUDIENCE:** What diagnostics did they [AUDIO OUT]?

**JACK HARE:** Mostly neutron diagnostics. They had neutron detectors, and they were looking for neutrons because they were like, if we have neutrons, there must be fusion. And there were fusion reactions happening, but they were beam-target fusion reactions caused by [INAUDIBLE]. They were not isotropic. If they had more neutron detectors, they could have detected the anisotropy, and that would have been a key signature that they didn't have thermonuclear fusion. Yeah?

**AUDIENCE:** Did the [INAUDIBLE]?

**JACK HARE:** Well, I mean, I don't know. But I do know that Peacock wrote a book about it called *Lasers over the Cherry Orchard* or something like that, which I've not read but I'm told is quite good. And I should read it at some point. So maybe he would have more insight than I do into geopolitics in 1968. OK, other questions?

Good. I haven't really said anything yet. So let's do some math. Is it math next? Yes. OK, good.

OK, so I gave you a very handwavy explanation of it through plasmas and lasers. Let's break this down to just a single charge. Here it is.

And this charge is initially at rest. And there's some wave packet. So think of this as roughly a plane wave, but I'm just going to draw it as a wave packet. It increases in amplitude and decreases in amplitude again. And this wave packet is traveling along in this direction.

So this is our laser beam going through. It's got some finite extent because the pulse of the laser beam has some finite width. There would obviously be many, many more oscillations within the wave packet. But then you won't be able to see them when I drew them on the board.

So just to be clear, particle, and this is an EM wave. So, for example, this could be electric field here.

What does this particle do in this electric field? Yeah?

It will oscillate. This particle will see the electric field going up and down, and it will join it. It will continue going up and down.

OK, now let's imagine some time a little bit later, we now have our particle oscillating still. And the wave packet has passed on. What does this particle do there?

Hmm? It's an accelerating charge. So it will continue to radiate away. Presumably, it'll be radiating away little wave packets of its own. Yeah?

**AUDIENCE:** [INAUDIBLE].

**JACK HARE:** Yeah, I guess we're still dealing with a state where there is still some electric field trailing off. So this is radiating while the electric field is going past.

But because it's being accelerated by the electric field, it will be emitted. So this is the geometry of the problem that we're going to consider here. We're going to consider just plane waves here because it's much easier than doing the Fourier-- well, it's much easier when we go into doing the Fourier analysis. So we're just going to consider some plane waves.

We're going to consider a plane wave coming in. And this plane wave has some wave vector Ki, which is in the i hat direction. That is just simply Ki divided by the size of Ki.

And there's a load of parallel wavefronts like this. So these are just constant phase. You can't get away with a geometric optics picture of Thomson scattering.

We've got some particle. And this particle is moving in some direction with a velocity v, which is a vector. And it's doing it at a time t prime. We'll get back to this t prime in a moment. There's some subtleties here that we have to deal with.

We also need to have an origin in our system. This is the point where we measure all of our displacement vectors from. And so we can say that this particle is at some distance. I'm going to draw it somewhere slightly different so I don't have an accidental coincidence between my vectors.

This particle is at some position r t prime, like this. So this specifies this single particle very well, and this specifies our incoming electromagnetic wave.

Now, we assume that there's going to be scattering in lots of different directions. We haven't exactly solved where all the scattered light is going to go to, but maybe it goes lots of different directions here. So we're just going to consider what happens if we have an observer down here looking at the scattered light from this particle.

Well, if we have an observer, that means that the light that we are seeing must be heading towards it with a wave vector Ks that is in the s hat direction. And these are more plane waves coming towards our observer.

And we also need a position vector for our observer. And that is a position capital R. We can assume that the observer is stationary in time. And so I don't need to put a time coordinate here. We're not going to move our spectrometer during this experiment.

And there also has to be a vector which joins the particle through the observer here. And that is just capital R prime, like that. And so just from standard vector algebra, we have R equals R prime plus r of t prime is just forming this little triangle here.

And now we need to think a little bit about the time because when the observer sees things, it's happening at a time t. But because it's taken the light some time to get from the particle to the observer, the observer is observing the state of the particle at a time t prime. And that time t prime is t, but it's earlier by a factor of R prime upon c, like this.

So this is just simply the distance. Whenever I drop the vector notation, I'm just taking the size of the vector here. So this is no longer a vector.

OK, and this is often called the retarded time.

We have sort of seen this setup before when we're talking about radiation from a moving charge. I just want to run through it again because it's very important for understanding Thomson scattering.

So what we do at this point is we write down two of Maxwell's equations again-- curl of curl of E plus 1 upon c squared, second partial derivative of the electric field with respect to time. And this is equal to minus mu 0, partial J, the electric current, partial t.

Normally, when we're solving Maxwell's equations for light in a vacuum, we just get rid of this left-hand-- right-hand side and set it to 0 because there's no current in a vacuum. But now we're dealing with a plasma here. And the current in this system here-- well, there's only a single particle. So this J is simply equal to qv t. And I guess we should really have a delta function saying that the current is localized at that point there.

I haven't specified whether this is an electron or an ion, but we'll get on to that later on. OK, questions?

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** [CHUCKLES] We will be more precise later on. Does it matter? Yes. [LAUGHS]

But this is on the subtlety. So this is one of the things where-- if you go and look at Jackson, he's very precise about all of these. Here I'm going to be a little bit handwavy until it really matters. But yeah. Oh, yeah?

Well, that's true for this single particle. What if I have a second particle and I want to look at the scattering from two particles? I can't put the origin in both the same place, both of them. They're in different places. So I'm better off just starting with a more generalized way of looking at it.

OK, yeah. We could do this. This is slightly more general. What if have multiple observers? So, for example, if I'm doing Thomson scattering, I might observe from multiple different angles.

But then I couldn't get away with that trick. So this more general formula will probably be useful. Yeah?

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** No. So, really, at the moment, we're just being like, say we've got some v of t, which, of course, has to be changing. It has to be changed in time for us to get radiation at all. And we're going to go back, and we'll solve it for the oscillations due to the electric field.

So this is really the very general "imagine you have a moving accelerating charge, what's the radiation from it," which you have probably seen before. But we're just going through it to set up the mathematics and the diagram and all the vectors so that I can then use them for doing the Thomson scattering derivation. But yeah. Yeah?

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** It doesn't matter for this because as we found, the refractive index only varies by a very, very small amount from 1. So, in fact, you're still going to-- for most reasonable cases, if we're operating below the critical density-- if you're operating near the critical density, then your laser beam will probably refract before it gets to the center of the plasma and can scatter. So this is-- I guess we're making an assumption here. Yeah.

No. Yeah. And I will say, actually, in this example I'm giving you right now, there is no plasma. There's just vacuum. So it is just it. Yeah.

But that's a good point. Maybe when we get into plasma, we should be a little bit more careful about what we're doing. And, in fact, we will substitute out a dispersion relationship in the plasma that takes into account how the wave propagates through it. Yeah, OK. Other questions?

OK. So, again, we're not going to solve these equations. We're just going to remind ourselves that we have a near-field term where the electric field drops off as 1 upon R cubed.

And we're going to have a far-field term where the electric field drops off as 1 upon R. And we're going to be far enough away that we don't see the near-field term. We only see the far-field term.

We're also going to make some approximations here. We're going to say that the length of R prime is approximately equal to R. I have a terrifying feeling that this means that I've effectively put my origin on the particle. So maybe Brian was right all along that. We could have just saved some time here.

But this has the effect of us being able to simplify our expression for the retarded time to be t prime is equal to or approximately equal to minus R s hat dot R over c.

This, again, is just looking at this and using some of the vectors we've got. You see there's an R here, s here, this sort of thing. I think this is actually saying that this is an isosceles triangle, right?

So we're just saying these two distances are similar. We're trying to calculate this small term here. This is similar to putting our observer far away from the particle and the origin of our coordinate system here.

OK, and if we do all of that, we can go back and take a formula from something like Jackson for the far-field radiation.

So the scattered radiation-- so this is the one coming towards our observer. And I'm going to use subscript s to show that it's the scattered radiation observed at the observer R prime at some time t-- so at the observer's time, not at the particle's time-- is going to be equal to q over 4 pi epsilon 0 capital R, which, again, is approximately equal to half of R prime.

And then we get this interesting term, s hat cross s hat cross s hat minus theta. I'll define beta again in a moment, but some of you will remember it from before. And all of that inside these brackets is crossed with theta dot.

And I'm going to put a little subscript right on here. We'll get back to it in a moment. And this is divided by 1 minus s hat dot theta cubed, like this.

OK, so two things in here that I didn't define straight away-- beta is simply defined as the velocity of the particle normalized to the speed of light. This is particularly useful because a lot of the time, we can just take a nonrelativistic approximation, and this makes the betas quite small. And we can drop pesky terms in the denominator here.

And this is evaluated at the retarded time because we're seeing the light some time later at our observer, which is emitted some time earlier at our particle. And if we do take this nonrelativistic approximation, where v upon c is much less than 1, then we end up with a formula for the scattered radiation, which is Es at R prime t is equal to q upon 4 pi epsilon 0 c s hat cross s hat cross theta dot over R, where the numerator is still evaluated at the retarded time.

The key thing here is that in order to get the scattered electric field, we now need to know this beta dot, which is the particle's trajectory or orbit. And this is where, as Sean was asking, what exactly v are we using, we'll then go back and we'll work out how the particle oscillates in the electric field of our probe.

But you might already be thinking, what if my particle is also gyrating due to some magnetic field? We'll not get on to that straight away, but it is a good question to ask yourself. OK, how are we doing so far?

OK, let's keep going. I'm just going to move this word "trajectory" because it's bit of board.

So now we want to find the trajectory of the particle beta for our incident electric field. So this is the electric field that is coming in and setting the whole thing in motion. It's got a subscript i for incident.

So Ei at some position R at some time t prime-- so this is looking at the retarded time here, this is where things will get complicated-- is equal to whatever the strength and polarization of that electric field is. And we'll just have it be a plane wave. So it's oscillating sinusoidally, and it's got a phase factor Ki dot r minus omega i t prime.

We know that the equation of motion for this particle is m dv dt is equal to minus q Ei. I don't see the need to write all the rest of it. Good?

And we will assume that the particle is initially stationary. So v at time 0 is equal to 0 here. So this is equivalent to saying the thermal motion is pretty negligible compared to the motion that we get from this. And we will relax that assumption later on. If we want to measure the temperature, we're going to need to care about the thermal motion. But for now, we're just going to deal with this [INAUDIBLE].

And so we solve this to get out the beta dot, which is effectively the acceleration of the particle is equal to minus q upon mc times the i here. The particle is being accelerated with the electric field, parallel or antiparallel, depending on the charge here.

And we can put all of that back into our previous equation, and we can get out that the scattered light now that's observed at a time at a position prime at time t is equal to q squared upon 4 pi epsilon 0 mc squared times s hat cross s hat cross Ei 0, which was the strength and polarization of the initial electric field.

This is all evaluated at the retarded time. And then there's also a factor out here, cosine Ki dot r minus omega i t prime.

This term here we're going to say is roughly constant. And the reason is that r is not going to change very much. Remember, our particle is just oscillating up and down.

We assume it doesn't oscillate very far. Then its mean position doesn't change very much. And so, therefore, we can just take this as a constant term here. So the only term we have to worry about actually changing here is omega i t prime. OK, someone tell me something interesting about this formula. Yeah?

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** Yeah, I'm trying to work out if I've dropped it or whether it disappeared for some other reason. I can't see a good reason to drop it. So I'm going to put it back in for now. Thank you.

We would have trouble conserving energy if we didn't put it there-- so probably for the best. OK. Tell me something interesting about this equation.

Do I care more about scattering in a plasma from the electrons or the ions? Why?

OK. So electron scattering dominates. And we will no longer-- from this point onwards, we will not consider the ion scattering at all, because it'll be smaller by a factor of at least 2,000 in the electrons. It's irrelevant. It may be there, but it won't be easy to measure over the electrons.

Later on, we will now be talking about scattering from the ion feature and the electron feature. I want to be very clear. Whenever we're talking about Thomson scattering, we are scattering off electrons.

Some of those electrons know about what the ions are up to. Some of those electrons know about what the electrons are up to. It's to do with the bi-shielding. We'll talk about it more.

But if anyone ever asks you, what is Thomson scattering from, it is from the electron. You cannot measure easily Thomson scattering from the ions.

The second interesting thing about this formula is, of course, it's a mess. We're trying to evaluate this at some time t. But there's t primes here. There's a retarded time.

So if we want to make any good use of this, we're going to have to sort all of this out and write it in terms of some consistent time here. But before we get on to that, I want to just look at how much power is being scattered from this.

So this is the electric field which is being scattered. But we also want to know what the power being scattered is. Is that all I have to say on that? Yeah.

So this is the scattered power. So this has units of watts per steradian, per solid angle. And we often write it as dt d omega s. This omega here is some differential solid angle. So this is ds like that. We have some scattering source here scattered through some area at some distance. That's the solid angle.

And so this ds can change. We can have-- for example, if our electron is oscillating up and down like this, there's no good reason to expect that it's the same amount of energy scattered into this volume as there is scattered into this volume. We'll talk about that in just a moment. So this is what this means. You have to actually know which direction you're scattering to.

And there's an equation from this that you can find, Jackson or similar, which looks like r squared c epsilon 0, and then the time average of the scattered electric field averaged with itself, dotted with itself.

Yeah, it's a good thing I did put that r back in there because that's going to cancel out with this r squared. So now I don't have to think about it anymore. And the key thing here is we're going to end up with a term that looks like s hat cross s hat cross Ei 0 squared.

And if we say that our electric field is in this direction, which is also the direction the particle is oscillating in, and we define some angle with respect to that electric field, phi here-- so this is like a polar angle, I don't care about the azimuthal angle, just the polar angle here-- then you can do some vector algebra and find out that this can be written as sine squared of phi Ei 0 squared, like that.

So our total scattered power looks like-- did I skip the thing about the classical electron radius? Oh, I did. Excuse me.

So if we say that these are electrons-- so we say that q equals minus e and m equals m e, then this whole term here looks like the classical electron radius. So this is, using classical physics, how big you would estimate an electron to be.

There is nothing that is actually the size of the classical electron radius. It's just a length scale. And this is a useful length scale that we'll be using here.

Notice this kind of has to be the case because dimensionally, we've got electric field here. We've got electric fields here. These are unit vectors. We've got some of these dimensions of length. So to make this balance, you knew this was going to have some length scale. And we just call it the classical electron radius-- just some length scale, not very big.

And so then we can write the scattered power as classical electron radius squared sine squared phi speed of light epsilon 0, size of or strength of the incident electric field here.

This amount of light we're scattering into some solid angle in some direction. And if we sketch this-- so this is the electric field-- and we do a polar plot of this, we find out that most of the light is scattered, actually, at 90 degrees electric field.

You imagine this being rotated around. This is sort of like a red blood cell type shape, where you have most of your emission out here, you have a very small emission in this direction, and you have no emission along E-- so dp d omega s parallel to E is equal to 0.

This is for the case with nonrelativistic. Does anyone know what happens if we look at the scattering from a relativistic particle? Yeah?

**AUDIENCE:** You get more scattering [INAUDIBLE].

**JACK HARE:** Yeah, it sort of starts looking like this. So this is often called beaming. And it goes in the direction of the electric field and, therefore, in the direction of the motion of the particle.

I guess when the particle reverses, this also reverses as well. So this is important if you're doing Thomson scattering in plasmas which are relativistic. You might want to think about this.

If you're not in a relativistic regime, if you're a nice classical regime, then it's important to note that your choice of polarization of your laser beam dictates where the light will be scattered. If you fire a laser beam like this, and it's polarized in this direction pointing upwards, and you put a spectrometer above looking down, you will see no light.

And I have done this experiment, accidentally. And it's very frustrating. So you want to make sure-- honest-- that if you want to collect upwards, you want to have your polarization like that. Otherwise, you just won't see anything at all.

But it doesn't matter, with respect to the electric field, where you collect, because this has no as azimuthal dependence here. So I have my electric-- my light going through this, my electric field pointing up, I'm going to get scattering in every direction like this.

And so I can put lots of spectrometers at lots of different positions around the plasma, and they will all see light. OK, question?

**AUDIENCE:** So where is [INAUDIBLE]?

**JACK HARE:** Well, there's this t here, and there's this t prime here.

**AUDIENCE:** [INAUDIBLE] because if the cosine term is [INAUDIBLE].

**JACK HARE:** Yeah.

**AUDIENCE:** --the cross product--

**JACK HARE:** This is being evaluated at t prime as well.

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** Yeah, but this is the equation at t. So we still need to do a little bit of substitution to get between the two of these. And this just gets a little bit subtle because we're going to start Fourier transforming them soon. Yeah.

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** Yes, and we see stuff that happened R upon c earlier, right? But you could imagine, especially-- this becomes especially important if you don't have a plane wave, which is just a continuous wave in time, but if you have a pulse. Then it starts to get very complicated to keep track of all these t primes. So you're going to see a lot of them come up, and I'm going to do my best to get it right but will probably make a mistake somewhere. Yeah?

**AUDIENCE:** What [INAUDIBLE]?

**JACK HARE:** Yes. But, again, it becomes complicated when you do the Fourier transform because what we need to do is find out a spectra. So at the moment, this is a time-varying electric field. But we don't tend to measure that.

So imagine that your laser beam is going through the plasma. It's a visible green laser beam. And so its frequency is extremely large.

We don't have a detector that can digitize that. What we have is a spectrometer that disperses light depending on its frequency. So we have to do all of this instead in the Fourier domain. I'm setting it up in the time domain, but we're going to Fourier transform later on.

And that's when you have to really start thinking, like, am I doing E to the i omega t or E to the i omega t prime? And if it's t prime, all of a sudden, there's an R inside there. So now I'm doing a Fourier transform with respect to space as well as time in the same go. This gets complicated.

So I'm just sort of setting you up to think this will be hard, though I agree right now it looks quite trivial. Yeah. Was there a question online? Oh, maybe not. Other questions?

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** Yeah, so you're saying that as the particle starts to oscillate, it will have some velocity, and then it should feel the magnetic field. We are going to assume that the velocity is still quite small so that we don't have to worry about that.

So that's kind of saying that the particle doesn't actually get accelerated very far Before the electric field reverses and sends it the other way. So its velocity never gets large enough that we have to worry about that, because that sounds like a nightmare. But, yeah, you could, in theory, extend this treatment to include that. Yeah.

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** Right, yeah. So the electromagnetic wave has a magnetic field. And the force on it is v cross B. So if I don't want that force to be large, I just keep v small. I keep v small by making my frequency large enough that the electron never really has a chance to accelerate very fast or the electric field turns around and turns it the other way.

And this very simple treatment to get us through just scattering of a single particle is 0. And, of course, very soon, we need it to be nonzero. So, yeah, great. Any other questions?

Ooh, boy. OK, another quantity we want to calculate is the total cross-section. So this is the differential cross-section here in terms of power. Ah, sorry. This is the-- what's the right word for this?

This is the scattered power. We also want to calculate a quantity called the differential cross-section. This is where I wished had a bit more space, but I think I'm just going to use this.

So the differential cross-section is the probability of this scattering event happening and scattering a photon in a certain direction through a certain solid angle, d mu s. And it's basically equal to the scattered power, but we're no longer interested in the power. So we just divide through by c epsilon 0 Ei squared. So that's the incident power.

And we're just dividing it out. So now we'll have something that just simply looks like r e sine squared phi. The reason to do that is to say, well, this is the probability of a photon scattering into some solid angle in some direction omega s. What's just the probability of it scattering?

Well, that probability, the total probability of any scattering event happening, is then equal to our E squared sine squared phi d omega, integrating over all the solid angle, that the omega is 2 pi sine phi d phi. So this is just the infinitesimal area element for a cylindrically symmetric system. And when you perform this integral, you get out 8 upon 3 pi r e squared.

OK, this is extremely important because these are all constants. It doesn't depend on your laser or your plasma or anything. This is just the chance of scattering off a particle, off an electron. And this number is 6.7 times 10 to the minus 29 meters squared.

So if we have a plasma with a density of any, any particles that scatter off, and it's got some length L-- so where the laser beam is going through some distance L with some number of particles per unit volume ne, and we times this by the scattering cross-section here, and if I put this in some numbers here-- so ne is maybe 10 to 14 per meter cubed. I don't know why I picked this. It seems kind of dense for a tokamak. But-- I don't know, not very-- yeah, kind of dense we're talking about, not very dense for other things.

That's not that dense. Yeah, it is meant to be sent to me centimeters because then the number I write down doesn't work either. OK, that's a pretty dense for a tokamak, right? Yeah? OK.

And then let's say this length looks like 1 meter. So this is looking a lot more like a tokamak here. Then this is equal to 10 to the minus 8. So this is scattered photons-- scattered-- over incident photons.

So if I put 10 to the 8 photons in, I get 1 photon out. Can someone tell me something interesting about Thomson scattering?

So that's very true. So, yeah, we definitely don't want to put our detector somewhere where the beam will hit it. What if we put the detector somewhere else? Is this an easy technique? Is it a hard technique?

I'm putting a lot of laser-- so if I put in a joule of laser energy, I'm going to get out 10 to the minus 8, 10 nanojoules of laser energy distributed in 4 pi. So when I put my detector there and it's got a solid angle that is not 4 pi, that number will go down even further.

So this is a very hard technique. Thomson scattering is difficult. There's not very much scattered light. You need a very powerful laser source in order to do it. Can someone tell me something else interesting about this result? Yeah?

So denser plasmas, it's easier, definitely. Yeah. That's true. But, of course, yeah, it depends on this length as well a little bit, and how you're collecting the light because you're not collecting it-- if you have a spectrometer looking here, you're only collecting it over a relatively short length.

You're not probably collecting it over the total length of the plasma. So this is an upper bound on the total number of photons scattered. And as we said with solid angle and your small collection volume, you'll scatter many fewer photons.

What about double scattering? Should I ever worry about a photon scattering off an electron and then, having done some weird shift in frequency, scattering off a second electron? Do I need to account for that in my calculation? Never, ever, ever, ever, right?

I mean, maybe if you got to extremely high densities, but this is not something that we need to worry about. So we're just looking for scattering of single particles. This is very important.

And this is why this is a local measurement, because if you scatter off a particle here, the photon will leave the plasma without scattering again. It's not like radiation transport, where there was a chance of the scattered photon being absorbed. Well, it could still be absorbed, I guess. But it's not going to be scattered twice.

So we know if we see a scattered photon that it came from the volume we think it came from and then straight out the plasma without scattering again. OK, any questions? We're going to relax B 0 equals 0. Yes?

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** Yes.

**AUDIENCE:** Why is that?

**JACK HARE:** I think if you take the nonclassical version, there is a dependent frequency there. But in this treatment here-- yeah, again, this is very classical treatment. This is the classical electron radius. And we dropped relativistic effects to get this formula here.

So we don't know about quantum. We don't know about those. I suspect there is. Thomson scattering turns into Compton scattering and the limit of high-energy photons. So we're not going to cover Compton scattering. But this is technically a limit of it.

So I guess I could derive Compton scattering and then get Thomson scattering from it. But this is hard enough already that we don't need to inflict that extra pain. Yeah. OK, any other questions? OK. Yeah? Go on.

Yes. Yes. Almost all of your laser light will pass directly through the plasma. And then, on the other side of the plasma, you have to work out what happens to that beam. If it reflects off the wall of your chamber, the reflected light will be quite large, even from a Thomson scattering perspective, because the electron density of metal is extremely high, 10 to the 30 or so.

And so that reflected light will bounce a few times and eventually end up in your spectrometer, where it will completely drown out scattered light. So instead, you have to do something with that unscattered light, which is most of your light. You have to have a beam dump, which is normally a very, very long pipe with a very, very dark box at the far end, which you hope absorbs all the light and stops the backscatter. So, yeah, that's a quite important practical consideration. Yeah?

**AUDIENCE:** So if the scattering probability is so low, how could you do time-resolved Thomson scattering? How could you get sufficient [INAUDIBLE] time? Because--

**JACK HARE:** A big laser.

**AUDIENCE:** OK.

**JACK HARE:** Yes. OK. Let's make life more complicated. Yeah?

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** So you could, in principle, do something clever with it. I've seen people do Thomson where they also look at the rotation of the polarization by magnetic fields in the plasma. So they've done Faraday at the same time with the same proton beam. So there are some things you can do.

In general, it's better to have dedicated laser beams with different angles. Yeah. OK.

So let's now say we have 0 is not 0. Our position at time t prime is now equal to wherever we started at at time 0 plus velocity t prime later. And this means that our retarded time is now equal to t minus R minus s dot r over c, like this.

By the way, we've assumed at this point this particle is just going in a straight line. So this is before the electric field gets here and mucks things up-- just going to let this particle move.

Now let me see what I'm saying here. No, that's not true. This particle is going roughly in a straight line, but the electric field is causing it to deviate only a very small amount here. So there's some small deviation delta v from the incident electric field.

But that delta v is much, much smaller than this velocity here. So the particle is roughly going in a straight line, and it's oscillating ever so slightly. OK.

Now, I have a load of algebra here, which I worked out because I couldn't find it in the book. But I'm not going to go through it. You can have the joy of doing it yourself.

You can rewrite this equation. And this is actually much less trivial than it looks as t minus R minus s dot r of 0. We're taking this expression for r, substituting it into here. All of this is divided by c. And this whole thing is divided by 1 minus s hat dot beta.

This term down here is a Doppler shift. We'll see another one of those occur in a moment.

So now we want to calculate the phase factor inside the cosine that we had for our incident electric field. Remember, previously, this was Ki dot r of t prime minus omega i of t prime.

And we said that this was roughly constant because the particle didn't move. Well, now the particle's moving. So it's not constant anymore.

This factor becomes equal to Ki dot r of 0 plus v prime of K dot v minus omega i. This is from substituting this into here.

I think that should be a Ki. I'm going to put it as a Ki. It doesn't have a subscript for my notes-- definitely a mistake. OK.

And then we can rewrite this altogether as Ks R. This is just some phase that doesn't change in time, which is just due to the fact that the scattered wave has some distance to go to get to our detector. So this is just another constant phase-- minus a term that looks like scattered frequency, omega s times t minus a term that looks like Ks minus Ki dot r of 0.

And I've introduced several new terms here. One of them is this omega s. And this is the frequency of the scattered wave which is seen at the observer. So by switching from t prime to t, I've had to take into account this change in time. And the Doppler shift starts coming in here.

So this scattered frequency looks like 1 minus i hat dot beta over 1 minus s hat dot beta times by omega i. This is a scattered frequency, and it's interesting because it's got two Doppler shifts inside it.

It's got a Doppler shift due to the light coming in being Doppler shifted by this particle's motion. So the particle sees a different frequency of the incoming light, and then the particle scatters that incoming light in some other direction. And that light is Doppler shifted again to the observer. So we get two Doppler shift factors here.

And the other new term I introduced is this Ks here. And this is simply from the dispersion relationship for electromagnetic radiation in a vacuum. So it's omega equals tK, and this is going in the s hat direction here.

So if you put this equation into this equation, turn through all the mathematics, you find out you get a new phase term. In a moment, I'll explain what's going on here. OK, any questions on that?

This is the phase factor inside the cosine for our scattered radiation. Previously, we just had this term here. So our scattered radiation just looked like a plane wave with the same wavelength and frequency as the incoming wave.

But now it's oscillating at the frequency of the scattered wave, which it should do because that's how we've defined it. But interestingly, we've got this extra phase term that comes from the difference between these two. So we can now define two more variables which don't have subscripts.

So this is K, and this is simply equal to Ks minus Ki, which is this term here. And so it looks like a plane wave which is actually moving with a wave number K because we've got the difference between these two. And we can also define a frequency, omega with no subscript, which is equal to the difference between omega s and omega i.

And this is simply K dotted with v. So this is the Doppler shift. This is the shift in frequency between the incident wave and the scattered wave-- that's omega. And that Doppler shift, as we expect, is the wave number, this difference wave number K, dotted into the velocity. This is what we normally expect for the Doppler shift here.

What's interesting about these equations is this is a vector equation. And it implies that we have some particle with velocity v, like this. We have some input wave of light, our probing laser beam, that crosses that particle in this direction. And we observe the scattered light at some other point, Ks, like this.

Then it appears that what we're actually measuring are the properties of some wave that goes from-- well, has this wave number K, which is the difference between these two wave vectors here, and it has some frequency, omega, which is that K dot v here.

So in reality, what we're doing when we measure with our spectrometer at omega s and at Ks is we're measuring the properties of this wave, which is transferring us from the incident radiation to the scattered radiation. So this looks an awful lot like an equation for the conservation of momentum and energy. If I just times these by h bar and think about them as photons, you can see that straight away.

This is the mathematics. We'll get to an interpretation of it quite soon. Hold on if you're struggling. We will get there. Any questions on the mathematics so far? OK.

So a couple of things to say about these new vectors we introduced-- the size of this new vector K, which is simply equal, again, to the size of the difference between these two vectors, is equal to Ks squared plus Ki squared minus 2 Ki Ks cosine of theta to the half, where theta is the angle between these two. So this is just vector algebra here.

Now, for nonrelativistic particles, we find that the incident light is going to have a frequency very similar to the scattered light. And so these two wave numbers will also be similar in magnitude here. So let me write that more explicitly.

For nonrelativistic systems, the Doppler shift is small. So omega equals K dot v is much, much less than omega s or omega i. That means omega s is roughly omega i. We haven't seen a significant Doppler shift here. And that means that the size of these two K vectors, Ks, is roughly equal to the size of Ki, like this.

And if we put that into this equation, then we find that the size of K is roughly equal to 2 times Ki sine theta upon 2. This is a very useful formula to remember because it relates the scattering angle that we're observing with respect to the incident laser beam and the wavelength of the incident laser beam to the wavelength of the mode that we're scattering on.

And we'll get back to what this mode with scattering off means physically in a moment. Let's make it clear that this is sine of theta upon 2 within the brackets here.

And the other thing to note is that we actually choose several of these things, right? We choose our initial laser frequency. And we choose what direction our laser beam goes in. And we choose what direction the light scatterers in.

You might say, no, we don't choose that. It scatters everywhere. We choose which light to see. We choose where to place our spectrometer. So we set Ks, right?

So all of these three things, we choose these. Again, I've got some plasma with some laser going through it, going along Ki, and I set up my spectrometer to look at Ks. And I'll redraw these so they're the same length.

I have controlled most of the parameters here. And so that means that because I've chosen these two things, when I'm measuring my spectrum, I know that I'm measuring modes, which, again, correspond to this K here.

So just to write that again, if we launch a wave with omega i, Ki, and we detect a wave being scattered at a certain frequency in a certain direction here, then this implies that some wave omega K, where omega is omega s minus omega i, and K is Ks minus Ki, implies that this wave exists.

But there is something within the plasma that is capable of taking away or adding the momentum and the energy we need in order to be able to see this mode here. So, for example, if we have a spectrum on our spectrometer here-- and this is in units of omega, omega s minus omega i, so I'm just showing you the shift-- then our incident laser is going to be at omega i, which is just 0.

So if we accidentally reflect this laser beam off the metal inside the chamber and catch the scattered light on our spectrometer, we will see it at omega i unshifted, like that. If we now put a detector at 10 degrees, we might, for example, see scattering here.

And this scattering here, omega equals K dot v, equals 2 Ki sine theta over 2 v, will be proportional not only to the velocity of the particle that we scattered off, but it will also be related to the angle.

So now if we put another detector at a different angle, we will see an omega which is larger because the angle is larger as well.

So what exactly are we scattering off? Well, in this picture here, what we've drawn above, we're talking about something called incoherent scattering. And we'll get back to what this means later on. But incoherent scattering, we scatter off a particle.

That's what we've drawn here. There's some particle. And that particle has a velocity v. And that v causes a shift, omega equals K dot v.

So the particle is responsible for balancing this momentum and energy equation. It is the K, and it is effectively the omega here as well.

There's also another regime of Thomson scattering, though, and this is called coherent Thomson scattering. And in coherent scattering, we scatter off waves in the plasma. And waves also have omega and K. And they have some distribution which links these-- for example, omega equals Cs K.

This Cs, the sound speed, square root of ZTe plus Ti upon Mi means that if we now measure a shift on our spectrometer, we can interpret that omega as being due to, for example, this sound wave. And from the shift that we see, we can now infer the plasma temperature.

So this very roughly is giving you an idea of how we will use coherent waves. It will make this very, very mathematical. OK, there's a lot going on there. Question? Yes?

**AUDIENCE:** How-- you said that [INAUDIBLE].

**JACK HARE:** Yes.

**AUDIENCE:** How [INAUDIBLE]?

**JACK HARE:** Yeah, absolutely. So there's your background. You've got bremsstrahlung, and you've got line emission, and you've got synchrotron. You've got all sorts of things.

You pick a probe, as in you pick a laser with an incident frequency which is not near any other strong features in your plasma. Bremsstrahlung is a nice background. So if you have bremsstrahlung-- this is wavelength here. If you have bremsstrahlung, and you're doing Thomson scattering, you should see a very clear peak from it.

If you've got bremsstrahlung, and you've got lots of lines, then it is a bit difficult to tell the difference between the lines and your Thomson scattering. So you want to pick your Thomson scattering probe so it's not close to any other features there. Then it's usually pretty clear because it's like, what else could it be? If we see scattered light there, we know there aren't any atomic lines. So it is likely [INAUDIBLE].

**AUDIENCE:** [INAUDIBLE] very common.

**JACK HARE:** Yes. Yes. There are also advantages. For example, a lot of the radiation that's emitted from the plasma is unpolarized, whereas the Thomson scattering radiation is polarized. And so we can use a polarizer to help reject some of that unpolarized light-- half of it.

So there are various techniques you can do. The Thomson scattering probe is usually very short in time. So if you have a gated camera, you only collect light while the probe is active. So you don't collect so much of the self-emission from the rest of the time that the plasma is there.

But it is challenging to get above the background. And, again, this is the answer to Lansing's question. You need a big laser. You need a lot of scattered light. Yeah.

Joules in nanoseconds? Yeah. That's really big. [LAUGHS] Yeah.

Yeah, for example, on the [INAUDIBLE] tokamak, where they're trying to do this in a sort of time-resolved way, they have a huge bank of Nd:YAG lasers that they trigger sequentially and send down the same beams. They're like, pulse, pulse, pulse, pulse, pulse, and then they have a load of spectrometers which are timed with those pulses as well.

And so then they're doing Thomson scattering at, like, I think, 100 kilohertz or something like that using the staggered bank of lasers. In my experiments, I can't get the laser to go twice during a microsecond because you can't easily get, like, a joule at a megahertz from a laser. It's terrifying.

So then we just have one time. And then we have to gate around that time and make sure that we only collect Thomson scattering light during the time the laser is on.

**AUDIENCE:** [INAUDIBLE]

**JACK HARE:** Yeah, so the technique I talked about was the low-temperature plasmas with the CO2. For example, they had a pulsed system. So they were constantly making the plasma. What they did is they had the laser synchronized with that-- so at whatever rep rate the plasma was being produced-- and they integrated.

So they just added up the photons from every single experiment until they had enough photons to get a good spectrum because the trouble is the density is so low there that just from a single shot, you don't really get very much scattered light. But if you do it for hours at a kilohertz, you start to get enough photons that you build up your spectrum.

For that to work, you have to be really sure your plasma is the same every time. And they were, or they felt that they were sure that it was the same every time. Obviously, if your plasma is changing, if we're dealing with a very unstable system where the plasma is moving around, and then the Thomson scattering probe will be sampling a different bit of plasma every time, that's not really reasonable.

But if you've got a nice homogeneous plasma, you've got it. So that's a nice situation to be in, actually, because if you want better signal-to-noise, you can just run the system for, like, another day. Of course, your signal-to-noise usually goes to square root n. So you get diminishing returns. But yeah.